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Distributed multi-agent optimization with inequality constraints and random projections

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ABSTRACT

In this paper, we consider a multi-agent convex optimization problem whose goal is to minimize a global convex objective function that is the sum of local convex objective functions, subject to global convex inequality constraints and several randomly occurring local convex state constraint sets. A distributed primal-dual random projection subgradient (DPDRPS) algorithm with diminishing stepsize using local communications and computations is proposed to solve such a problem. By employing iterative inequality techniques, the proposed DPDRPS algorithm is proved to be convergent almost surely. Finally, a numerical example is illustrated to show the effectiveness of the theoretical analysis.

1. Introduction

Distributed cooperative control has attracted great attention from researchers due to its broad applications in areas such as formation control [1–4], network synchronization [5–11], neural network optimization [12–15] and so on. Nowadays, distributed multi-agent optimization, as an application area of distributed cooperative control, has gradually become a research spotlight, due to the emergence of large-scale networks such as internet networks, mobile ad hoc networks, wireless sensor networks.

The goal of distributed multi-agent optimization with or without constraints is to construct distributed algorithm to minimize the global objective function that is composed of a sum of local objective functions, each of which is known to only one agent. Due to a lack of centralized authority in the network of agents, more and more recent works are focused on designing distributed multi-agent optimization algorithms. These distributed algorithms share two common features: (1) each agent only accesses its own objective function and exchanges limited information with its local neighbor agents only; (2) the objective function and the constraints depend upon a global decision vector, which requires the agents reach an agreement on the optimal solution. These features are highly related to the concept of consensus in multi-agent networks, whose goal is designing control mechanisms such that the group of autonomous agents reaches an agreement via local communication [16–24].

Most related literature on parallel and distributed computation builds on the seminal works [25–27], which are focused on optimizing a global objective function among multiple processors. Most of the recent works, for example, see [28–31,33,34,32,35–39] are focused on multi-agent environments and study consensus algorithm for achieving a cooperative behavior in a distributed manner. In [30–32], the authors propose distributed subgradient methods for cooperative optimization in multi-agent networks. In [33–36], the authors propose the distributed primal-dual subgradient methods for cooperative optimization with inequality constraints in multi-agent networks. Distributed algorithms proposed in [30,31,33–36] rely on deterministic projections, except in [30] where the unconstraint optimization problem is considered. Due to the uncertainty of the online transmission, the local constraint sets may not be explicitly observed in advance, in this case, the projection operators randomly project the information to the component of the constraint sets. In [37–39], the authors consider distributed multi-agent subgradient algorithm with random projections, but none of them takes the inequality constraints into account.

In this paper, a multi-agent optimization problem where the goal is to minimize a global convex objective function that is the sum of local convex objective functions, subject to global convex inequality constraints and several randomly occurring local convex state constraint sets is considered. To solve this problem, the

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distributed primal-dual random projection subgradient (DPDRPS) algorithm is proposed to highlight the effects of randomly occurring local constraints. This algorithm involves each agent performing a local weighted averaging, which is time-varying, to combine his estimate with the other agents' estimates he has access to, taking a subgradient step along his local Lagrange dual function, and randomly projecting the estimates on the local constraint sets. The convergence of these algorithms with a diminishing stepsize is provided. Finally, a numerical example is illustrated to show the effectiveness of the theoretical analysis. The contribution of this paper is mainly in two directions. First, we propose a novel DPDRPS algorithm for convex optimization with inequality constraints and randomly occurring local constraint sets, which consists of a subgradient descent step with a local time-varying weighted averaging step and a random projection step, while in [33,34], the considered communication networks are time-invariant. Second, we study the almost sure convergence of the DPDRPS algorithm with a diminishing stepsize and its variant using a mini-batch of random projections. To the best of our knowledge, there is no previous work on distributed primal-dual optimization algorithms using random projections. Finally, a numerical example is given to show effectiveness of the theoretical analysis.

Notations: A vector is viewed as a column. For a vector x, $\|x\|$ denotes the Euclidean norm. For a vector x and a closed convex set $\mathcal{X}$, $\text{dist}(x,X)$ denotes the distance of x from $\mathcal{X}$, i.e., $\text{dist}(x,X)=\min_{y\in\mathcal{X}}\|x-y\|$. For a vector x and a closed convex set $\mathcal{X}$, $\mathcal{P}_{\mathcal{X}}[x]$ denotes projection of x on $\mathcal{X}$, i.e., $\mathcal{P}_{\mathcal{X}}[x]=\arg\min_{y\in\mathcal{X}}\|x-y\|$. $\mathbb{R}$ denotes the real number set and $\mathbb{R}^n=\underbrace{\mathbb{R}\times\cdots\times\mathbb{R}}_{n}$. $\mathbb{R}^n_+$ denotes a vector in $\mathbb{R}^n$ whose components are nonnegative. For matrix A, $[A]_{ij}$ denotes the (i,j)-th component of matrix A. The superscript T denotes the transpose of the vector and a matrix. $Pr[Z]$ and $E[Z]$ denote the probability and expectation of a random variable Z, respectively.

2. Problem formulation and assumptions

2.1. Network model

We consider the multi-agent network operates synchronously. The topology of the network at time $k\ge 1$ is represented by a directed weighted graph $\mathcal{G}=(V,E(k),W(k))$, where $V=\{1,2,\ldots,N\}$ denotes the set of vertices, $E(k)=\{(i,j):\{i,j\}\in V\}$ denotes the set of edges, in which $(i,j)\in E(k)$ indicates that agent i receives information from agent j at time slot k, $W(k)\in\mathbb{R}^{N\times N}$ is the adjacency matrix with $[W(k)]_{ij}\ge 0$ being the weight assigned to edge (j,i). The set of neighbors of vertex i at time slot k is denoted as $N_i(k)=\{j:(i,j)\in E(k)\}$, where $i\in N_i(k)$ for all $i\in V$ and $k\ge 1$. We here make the following assumptions on the network communication graphs, which are standard in the analysis of distributed convex optimization [30,31].

Assumption 1 (*Network connectivity*). There exists a scalar Q such that the graph $(V,\bigcup_{l=0,1,\ldots,Q-1}E(k+l))$ is strongly connected for all $k\ge 0$.

Assumption 2 (*Non-degeneracy*). There exists a constant $0<\eta<1$, such that $[W(k)]_{ii}\ge\eta$ for all $i\in V$ and $k\ge 0$; if $[W(k)]_{ij}>0$, then $[W(k)]_{ij}\ge\eta$ for all $i,j\in V$ and $k\ge 0$.

Assumption 3 (*Double stochasticity*). It holds that $\sum_{j=1}^{N}[W(k)]_{ij}=1$ for all $i\in V$ and $k\ge 0$; $\sum_{i=1}^{N}[W(k)]_{ij}=1$ for all $j\in V$ and $k\ge 0$.

Remark 1. In [33,34], the authors investigated the multi-agent optimization problem with inequality constraints, where the considered local constraint sets are fixed and the communication network is time-invariant. In this paper, we consider the randomly occurring local constraint sets and time-varying communication networks.

2.2. Problem formulation

We are interested in solving the following problem over the multi-agent network

$$\text{minimize}\quad f(x)\triangleq\sum_{i=1}^{N}f_i(x)$$
$$\text{subject to}\quad g(x)\le 0$$
$$x\in\mathcal{X}\triangleq\bigcap_{i=1}^{N}\mathcal{X}_i \tag{1}$$

where $x\in\mathbb{R}^n$ is a global decision vector; $f_i:\mathbb{R}^n\to\mathbb{R}$ is the convex objective function of agent i, which is only known by agent i; $g:\mathbb{R}^n\to\mathbb{R}^m$ are convex functions which are known by all the agents in the network, where $g(x)\le 0$ means that each component $g_l(x)\le 0$, $l=1,2,\ldots,m$; $\mathcal{X}_i\subseteq\mathbb{R}^n$ represents the nonempty closed convex compact constraint set of the global decision vector x of agent i, which is only known by agent i. We denote $\mathcal{Y}\triangleq\{x\in\mathbb{R}^n:g_l(x)\le 0,\ l=1,2,\ldots,m\}$. We assume that the feasible set is nonempty, i.e., $\mathcal{X}\bigcap\mathcal{Y}\ne\varnothing$. On the other hand, it can be observed that $\mathcal{Y}$ is a closed subset of $\mathbb{R}^n$, which means that $\mathcal{Y}$ is compact. Hence, $\mathcal{X}\bigcap\mathcal{Y}$ is compact. The convexity of f_i implies that of f, thus, f is continuous. So, the optimal value f^* of problem (1) is finite and $\mathcal{X}^*$, denoted as the set of primal points, is nonempty.

In some applications, the local constraint of agent i may not be explicitly given in advance due to online constraints or uncertainty [39]. In such a case, we assume the constraint set $\mathcal{X}_i$ can be realized by the intersection of finitely many simple nonempty closed convex constraints, i.e., $\mathcal{X}_i=\bigcap_{j\in I_i}X_i^j$, where I_i is the index set of simple nonempty closed convex constraints of agent i.

Remark 2. The phase "randomly occurring" can be interpreted as follows [40]: a wide class of practical systems are influenced by disturbances that are caused by environmental circumstances. For multi-agent systems with state constraints, such state constrains themselves may experience random abrupt changes, which may result from abrupt phenomena such as noisy communications and repairs of the components. In a real-time networked environment, due to the limited bandwidth, network-induced packet losses, congestions, as well as quantization could be interpreted as a kind of external disturbances that occur in a probabilistic way and are randomly changeable in terms of their types and/or intensity.

Remark 3. Distributed algorithms proposed in [30,31,33–36] rely on deterministic projections. In this paper, the random projection is considered. On the other hand, in [37–39], the authors consider distributed multi-agent subgradient algorithm with random projections, but none of them takes the inequality constraints into account. In this paper, the inequality constraints are considered.

For a convex function $f:\mathbb{R}^n\to\mathbb{R}$, a vector $s_f(\bar{x})\in\mathbb{R}^n$ is called the subgradient of f at $\bar{x}\in\mathbb{R}^n$ when the following relation holds

$$s_f(\bar{x})^T(x-\bar{x})\le f(x)-f(\bar{x}),\quad \text{for } x\in\text{dom}(f),$$

where $\text{dom}(f)=\{x\in\mathbb{R}^n:f(x)\le+\infty\}$ is the domain of f.

For a concave function $g:\mathbb{R}^n\to\mathbb{R}$, a vector $s_g(\bar{x})\in\mathbb{R}^n$ is called the subgradient of g at $\bar{x}\in\mathbb{R}^n$ when the following relation holds

$$s_g(\bar{x})^T(x-\bar{x})\ge g(x)-g(\bar{x}),\quad \text{for } x\in\text{dom}(g),$$

where $\text{dom}(g)=\{x\in\mathbb{R}^n:g(x)\ge-\infty\}$ is the domain of g.