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Nonlinear 2D shape registration via thin-plate spline and Lie group representation

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ABSTRACT

Thin-plate spline for robust point matching (TPS-RPM) algorithm is a famous and widely used approach in nonlinear shape registration. In this paper, we improve this approach by adopting an alternatively iterative strategy of globally affine and locally nonlinear registration. Concretely, in the affine registration step, we apply the Lie group parameterization method to globally align two shapes to assume the global similarity. In which, some suitable constraints are introduced to improve the robustness of algorithm. Then, in the locally nonlinear deformation step, we apply the thin-plate spline approach. By alternatively iterating these two steps, the proposed method not only preserves the advantages of spline methods, but also overcomes an overmatching phenomenon in shape registration. Finally, we test the proposed method on several conventional data sets with comparison of TPS-RPM. The experimental results validate that our method is really effective for nonlinear shape registration as well as more robust.

1. Introduction

Nonlinear shape registration is an important but still difficult task in pattern recognition and shape analysis. For example, Belongie et al. consider the object recognition by using the shape similarity [2]. Huang and Li propose a new shape context and apply it in airborne multi-sensor image registration [17], Albrecht et al. apply the shape prior to the nonlinear medical image registration [1], as well as Gao et al. for image classification and recognition [14,15]. Therefore, as a significant feature of the complex image, shape plays more and more important roles, while shape registration, especially nonlinear shape registration, becomes one of fundamental techniques for image processing and analysis.

The aim of shape registration is to find the best deformation that warps the source shape to the target shape as similarly as possible. To address this issue, recently, there are amounts of approaches have been proposed. According to the representation of deformation between two shapes in solving process, they can be classified into two categories: the finite dimensional methods and the infinite dimensional methods.

In the case of infinite dimensional methods, the deformation between two shapes are considered as a certain function on a fixed spatial region and the optimal deformation is evolved though minimizing an energy functional on such functional space, where the variational method and numerical partial differential equations methods are always adopted. Therefore, the energy function and the regularity term of the registration model as the key factors are widely studied. For example, Paragios et al. modeled the shape registration as the global affine and local nonlinear registration, and solved the local nonlinear registration by variational method [20]. Later, Huang and her authors proposed a shape registration method by variational and statistical approach, where shapes are embedded into a higher-dimensional space [18]. Albrecht et al. introduced the regularity term with prior knowledge to the statistical deformation model [1]. El Munim et al. proposed a closed form for shape registration by using a linear system of equations to approximate the solution of local nonlinear deformation [12] while Rouhani and Sappa used a single linear least squares framework [22]. At the same time, Domokos et al. established a framework for nonlinear shape registration by solving a system of nonlinear equations with respect to the diffeomorphism [8]. Although these variational-based methods offer a fine mathematical theory for the solutions and do shape registration well, it should be pointed out that there are two common drawbacks. The first is that they may not find the exact correspondence between two shapes. That is, there is only the similarity of shapes. The second is that they have more local minimum because the space of solutions is infinite dimensional and the energy functional is always nonconvex. In addition, it is always time-consuming because they need update all nodes in the spatial region. Therefore, in recent years the finite dimensional methods are more popular because of their simplicity. In these methods, the

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deformation are always represented based on some bases, especially, the spline-based methods. For example, radial basis functions (RBF) [29], octree-spline [25], geometric splines [13], free form B-spline [23], wavelet [7] and Robust point matching with thin-plate spline algorithm (TPS-RPM) [6]. It is worth mentioning that although the TPS-RPM, which is designed based on the TPS [4], the softassign technique [16,5] and the deterministic annealing strategy [30], becomes a famous and widely used approach in nonlinear shape registration, it is still not robust for some shape registration. For example, it may not do registration well between two shapes with a large deformation and/or with some missing parts. Therefore, how to improve the TPS-PRM algorithm is necessary and valuable.

It should be pointed out that one main reason of failure registration by TPS-PRM algorithm is the affine transformation is not estimated accurately, even it may be degenerated [28]. That is, the shape may be degenerated to a line or a point without any constraint. In [9–11,24] and [21,26,27], authors tried to improve the robustness of registration algorithms. In recent, to deal with the degenerated problem in linear registration, we proposed an affine ICP algorithm by using Lie group parametrization method based on the ICP framework [3,31], where we introduced some suitable constraints to transformations [19,28]. Therefore, in this paper, we will improve the TPS-RPM by using these techniques.

The rest of this paper is organized as follows. In Section 2, we will briefly recall the relative model and algorithm of TPS-RPM, and then we develop a novel model by using the Lie group representation and introducing some suitable constraints in Section 3 In Section 4, we demonstrate the effectiveness of our proposed method, and compare it with the conventional TPS-RPM algorithm. Finally, the whole paper is concluded in Section 5.

2. Relative works

In this section, we will briefly recall one famous nonlinear shape registration algorithm, which is named as TPS-RPM algorithm. For more detail, we refer to [6].

Given two 2D shape/point data sets $V = v_a, a = 1, \dots, K \subset R^2$ and $X = x_i, i = 1, \dots, N \subset R^2$, then the goal of nonlinear registration is to find the best spatial deformation $f^* : R^2 \to R^2$ and the best correspondence between two data sets. The conventional TPS-RPM method modeled this problem by minimizing a fuzzy linear assignment-least squares energy function

$$E(M,f) = \sum_{i=1}^{N}\sum_{a=1}^{K} m_{a,i}\|x_i - f(v_a)\|^2 + \lambda\|Lf\|^2 + T\sum_{i=1}^{N}\sum_{a=1}^{K} m_{a,i}\log m_{a,i} - \varsigma\sum_{i=1}^{N}\sum_{a=1}^{K} m_{a,i} \tag{1}$$

where $M = (m_{a,i})_{K\times N}$ is an assigned matrix which is used to describe the correspondence between two data sets after being deformed and is satisfied with $\sum_{i=1}^{N+1} m_{a,i} = 1$ and $\sum_{a=1}^{K+1} m_{a,i} = 1$ for all $m_{a,i} \in [0,1]$. The extra $N+1$th row and $K+1$th column of M are added to handle the outliers. f is any nonlinear spatial deformation. L is a Laplacian operator and $\|Lf\|^2$ is a regularity term which is used to assure a certain smoothness of the deformation and defined by

$$\|Lf\|^2 = \iint\left[\left(\frac{\partial^2 f}{\partial x^2}\right)^2 + 2\left(\frac{\partial^2 f}{\partial x\partial y}\right)^2 + \left(\frac{\partial^2 f}{\partial y^2}\right)^2\right] dx\, dy$$

Besides, T is a temperature parameterize, which is used to control the degree of correspondence between two points. In fact, one point is at most connected with one point when T is zero. The parameter λ is used to balance the smoothness and the accuracy of the deformation and ς is a parameter which is used to prevent rejecting too many points as outliers.

Then the TPS-RPM algorithm can be summarized as following alternating iterations:

(S1) Update the correspondence. First, calculate

$$m_{a,i} = \begin{cases} \frac{1}{T}\exp\left(-\frac{(x_i - f(v_a))^T(x_i - f(v_a))}{2T}\right) & \forall a \neq K+1; i \neq N+1. \\ \frac{1}{T_0}\exp\left(-\frac{(x_i - v_{K+1})^T(x_i - v_{K+1})}{2T_0}\right) & a = K+1; \forall i \neq N+1. \\ \frac{1}{T_0}\exp\left(-\frac{(x_{N+1} - f(v_a))^T(x_{N+1} - f(v_a))}{2T}\right) & i = N+1; \forall a \neq K+1. \\ 0 & a = K+1; i = N+1. \end{cases}$$

where v^{K+1} and x^{N+1} in the second and third rows are the outlier cluster centers. Then, normalize the rows or columns of M by

$$m_{a,i} = \frac{m_{a,i}}{\sum_{b=1}^{K+1} m_{b,i}}, \quad i = 1, 2, \dots, N,$$

$$m_{a,i} = \frac{m_{a,i}}{\sum_{j=1}^{N+1} m_{a,j}}, \quad a = 1, 2, \dots, K,$$

(S2) Update the transformation. Minimize the following TPS energy function:

$$E_{\text{TPS}}(f) = \sum_{i=1}^{N}\sum_{a=1}^{K} m_{a,i}\|x_i - f(v_a)\|^2 + \lambda\|Lf\|^2,$$

In this energy function, the deformation f is decomposed into two matrices d and ω,

$$f(v_a, d, w) = v_a \times d + \phi(v_a) \times \omega,$$

where d is an affine matrix describing the linear transformation, and ω is a warping coefficient matrix representing non-affine deformation. The vector $\phi(v_a)$ is related to the TPS kernel. Therefore, the deformation f is parameterized and we can update f via updating d and ω.

However, recently some numerical experiments show that the TPS-RPM algorithm is not always robust, especially when the data have a large deformation and/or missing parts. The main reason is the affine transformation is not estimated accurately. It should be pointed out that the affine transformation can be estimated more robust by using Lie group parametrization method and introducing some suitable constraints to transformations in our recent work [28]. Therefore, in the next section, we will combine the advantages of TPS and our affine registration method to form a novel and more robust nonlinear shape registration model and algorithm.

3. Lie-TPS registration model and algorithm

In this section, we will rewrite the objective function (1) to make it possible to use Lie group parametrization method and introduce the suitable constraints to the parameters. To this end, different from TPS-RPM, we decompose f the deformation into two parts A and g by $f = A \oplus g$, which is defined by

$$f(v_a) = A(v_a) + g(A(v_a)),$$

where A is a nondegenerated matrix representing the global linear transformation between two data sets while g is the local nonlinear deformation. $A(v_a)$ and $g(A(v_a))$ mean the transformation A and the local nonlinear deformation g act on the point v_a and $A(v_a)$, and they are defined by $A(v_a) = Av_a$ and $g(A(v_a)) = (g_1(A(v_a)), \dots, g_n(A(v_a)))$, respectively.