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Cooperative output regulation problem for linear time-delay multi-agent systems under switching network ☆

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ABSTRACT

In this paper, we study the cooperative output regulation problem for linear time-delay multi-agent systems subject to jointly connected switching network. We first establish two lemmas that lay the foundation for solving the problem and then present the solution to the problem by both the dynamic state feedback control law and the dynamic measurement output feedback control law. Our results not only extend the existing results on linear multi-agent systems without time-delay to linear time-delay multi-agent systems, but also lead to the solution of the leader-following consensus problem of some typical linear time-delay multi-agent systems.

1. Introduction

The cooperative output regulation problem aims to design a distributed control law for a multi-agent system to drive the tracking error of the output of each follower to the origin asymptotically while rejecting some classes of external disturbances. The problem can be viewed as a generalization of some cooperative control problems such as leader-following consensus, synchronization, and formation in that it handles heterogeneous multi-agent systems, rejects disturbances, allows the leader's signal to be a class of signals generated by a so-called exosystem. The cooperative output regulation problem has been studied by two approaches, namely distributed observer approach [1–3] and distributed internal model approach [4,5]. However, so far, all these papers have assumed that the systems under consideration do not contain any type of time-delays. In practice, time-delays are inevitable in a networked system, and excessively large time-delays may destroy the stability of a well designed control system. Therefore, in this paper, we will further study the cooperative output regulation problem for linear multi-agent systems with input time-delays.

An advantage of the distributed observer approach is that it can effectively handle the case where the communication network is switching satisfying the jointly connected condition in [2]. A distributed observer is a networked dynamic compensator and has the property that the state of each subsystem of the observer will asymptotically approach the state of the leader system. As a result, those followers which cannot access leader's signal can make use of the estimated leader's signals in their control loop. In this paper, we will also adopt the distributed observer approach to studying our problem. However, due to the presence of input time-delays, the method developed in [2] cannot be applied directly to our current problem. We need to establish some general theory for handling the cooperative output regulation problem of linear time-delay multi-agent systems. In particular, we need to establish some lemmas to guarantee the exponential stability of a class of time-delay lower triangular systems.

For the past decade, the consensus problem of multi-agent systems with time-delays has been studied in numerous papers. The attention was first given to the leaderless consensus problem. For example, for single-integrator and double-integrator multi-agent systems with constant communication time-delay, the leaderless consensus problem was studied in [6] and [7] under static and undirected communication network, respectively, and for high-order linear multi-agent systems with both input and communication time-delays, the leaderless consensus problem was studied in [8] under static and directed communication network, and the leaderless consensus problem of single-integrator and second-order multi-agent systems with the communication time-delay under switching communication network was studied in [9] and [10,11], respectively. On the other hand, some results on the leader-following consensus problem of multi-agent systems with time-delays were worked out. For example, the leader-following

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consensus problem of double-integrator multi-agent systems with diverse constant input time-delays under static communication network was studied in [12], and the leader-following consensus problem of double-integrator multi-agent systems with time-varying communication time-delays was dealt with in [13] and [14] under balanced switching communication network and frequently connected communication network, respectively.

To our knowledge, there is no result on the leader-following consensus problem of linear multi-agent systems with input time-delays subject to jointly connected communication network. Thus, as will be seen in Corollaries 5.1 and 5.2, a special case of our main result solves, for the first time, the leader-following consensus problem for a class of linear multi-agent systems with input time-delays under the jointly connected communication network.

The rest of this paper is organized as follows. Section 2 gives the precise problem formulation for cooperative output regulation for linear time-delay multi-agent systems. In Section 3, some technical lemmas are established. Section 4 gives our main result. Section 5 specializes the main result to the leader-following consensus problem. Two examples are used to illustrate our design in Section 6. The paper is concluded by some concluding remarks in Section 7. Finally, we note that a preliminary version of the paper appeared in [15].

Notation: $\mathbf{1}_N$ is a column vector of dimension N with all its elements being 1 and $\mathcal{C}$ denotes the complex plane. For $x_i \in R^{n_i}$, $i=1,\ldots,m$, $\mathrm{col}(x_1,\ldots,x_m)=(x_1^T,\ldots,x_m^T)^T$. For any positive scalar τ, $\mathcal{C}([-\tau,0],R^w)$ denotes the Banach space of continuous functions mapping the interval $[-\tau,0]$ into R^w endowed with the supremum norm.

2. Problem formulation and preliminaries

2.1. Graph

We first give a brief introduction to some notation of graph as found in [16]. A digraph $\mathcal{G}=(\mathcal{V},\mathcal{E})$ is composed of a node set $\mathcal{V}=\{1,\ldots,N\}$ and an edge set $\mathcal{E}=\{(i,j), i,j\in\mathcal{V}, i\neq j\}$. The neighbor set of node i is defined as $\mathcal{N}_i=\{j,(j,i)\in\mathcal{E}\}$. If there exists a set of edges $\{(i_1,i_2),\ldots,(i_{k-1},i_k)\}$ in the digraph $\mathcal{G}$, then i_k is said to be reachable from node i_1. A digraph $\mathcal{G}_s=(\mathcal{V}_s,\mathcal{E}_s)$, where $\mathcal{V}_s\subseteq\mathcal{V}$ and $\mathcal{E}_s\subseteq\mathcal{E}\cap(\mathcal{V}_s\times\mathcal{V}_s)$, is a subgraph of the digraph $\mathcal{G}=(\mathcal{V},\mathcal{E})$. Given a set of m digraphs $\{\mathcal{G}_i=(\mathcal{V},\mathcal{E}_i), i=1,\ldots,m\}$, the digraph $\mathcal{G}=(\mathcal{V},\mathcal{E})$, where $\mathcal{E}=\bigcup_{i=1}^m\mathcal{E}_i$, is called the union of digraphs $\mathcal{G}_i$, and is denoted by $\mathcal{G}=\bigcup_{i=1}^m\mathcal{G}_i$. Associated with the digraph $\mathcal{G}$, we define $\mathcal{A}=[a_{ij}]\in R^{N\times N}$ such that $a_{ii}=0$, $a_{ij}>0 \Leftrightarrow (j,i)\in\mathcal{E}$. It is called the weighted adjacency matrix of $\mathcal{G}$.

To define a switching graph $\mathcal{G}_{\sigma(t)}$, let $\mathcal{P}=\{1,2,\ldots,\rho\}$ for some positive integer ρ. We call a time function $\sigma(t):[0,+\infty)\to\mathcal{P}=\{1,2,\ldots,\rho\}$ a piecewise constant switching signal if there exists a sequence $0=t_0<t_1<t_2<\cdots$, satisfying $t_{k+1}-t_k\geq\tau>0$ for some constant τ and all $k\geq0$ such that, for any $k\geq0$ and for all $t\in[t_k,t_{k+1})$, $\sigma(t)=i$ for some $i\in\mathcal{P}$. $\mathcal{P}$ is called the switching index set and τ is called the dwell time.

Given $\sigma(t)$, we can define a switching digraph $\mathcal{G}_{\sigma(t)}=(\mathcal{V},\mathcal{E}_{\sigma(t)})$, where $\mathcal{E}_{\sigma(t)}\subseteq\mathcal{V}\times\mathcal{V}$ for all $t\geq0$. The other way around, given a matrix $\mathcal{A}_{\sigma(t)}=[a_{ij}(t)]\in R^{N\times N}$, where $a_{ii}(t)=0$ and $a_{ij}(t)\geq0$, $i,j=1,\ldots,N$, we can define a switching digraph $\mathcal{G}_{\sigma(t)}$ such that $\mathcal{A}_{\sigma(t)}$ is the weighted adjacency matrix of $\mathcal{G}_{\sigma(t)}$. Then, $\mathcal{G}_{\sigma(t)}$ is called the digraph of $\mathcal{A}_{\sigma(t)}$.

2.2. Problem formulation

In this paper, we will study the cooperative output regulation problem of linear time-delay multi-agent systems of the form

$$\begin{aligned}
\dot{x}_i(t)&=A_ix_i(t)+\sum_{j=0}^{r}B_{ij}u_i(t-\tau_j)+E_iv(t), \quad t\geq0\\
y_{mi}(t)&=\overline{C}_ix_i(t)+\sum_{j=0}^{r}\overline{D}_{ij}u_i(t-\tau_j)+\overline{F}_iv(t), \quad t\geq0\\
e_i(t)&=C_ix_i(t)+\sum_{j=0}^{r}D_{ij}u_i(t-\tau_j)+F_iv(t), \quad t\geq0\\
x_i(\theta)&=x_{i0}(\theta), \quad v(\theta)=v_0(\theta), \quad \theta\in[-\tau_r,0]
\end{aligned} \tag{1}$$

where for $i=1,\ldots,N$, $x_i(t)\in R^{n_i}$, $u_i(t)\in R^{m_i}$, $y_{mi}(t)\in R^{p_{mi}}$, and $e_i(t)\in R^{p_i}$, are the state, input, measurement output, and tracking error of the ith subsystem, respectively, τ_j, $j=1,\ldots,r$ are positive real numbers representing the time-delays in the system input and satisfying $0=\tau_0<\tau_1<\cdots<\tau_r$, and $v(t)\in R^q$ is the exogenous signal representing the reference input to be tracked or/and disturbance to be rejected and is assumed to be generated by the exosystem of the form

$$\dot{v}(t)=Sv(t), \quad t\geq0. \tag{2}$$

We assume $x_{i0}\in\mathcal{C}([-\tau_r,0],R^{n_i})$, $i=1,\ldots,N$, and $v_0\in\mathcal{C}([-\tau_r,0],R^q)$.

The plant composed of (1) and (2) can be viewed as a multi-agent system with the exosystem (2) as the leader and the N subsystems of (1) as the followers. Associated with this multi-agent system, and a given piecewise constant switching signal $\sigma(t)$, we can define a nonnegative matrix $\overline{\mathcal{A}}_{\sigma(t)}=[a_{ij}(t)]\in R^{(N+1)\times(N+1)}$, $i,j=0,1,\ldots,N$, where $a_{ii}(t)=0, i=0,1,\ldots,N$ and $a_{ij}(t)>0, i=1,\ldots,N, j=0,1,\ldots,N$ if and only if $u_i(t)$ can access the information of agent j for control at the time instant t. Let $\overline{\mathcal{G}}_{\sigma(t)}=(\overline{\mathcal{V}},\overline{\mathcal{E}}_{\sigma(t)})$ be a digraph of $\overline{\mathcal{A}}_{\sigma(t)}$. Then, $\overline{\mathcal{V}}=\{0,1,\ldots,N\}$ with the node 0 associated with the exosystem (2) and all the other nodes associated with the N subsystems (1), and $(i,j)\in\overline{\mathcal{E}}_{\sigma(t)}$ if and only if $a_{ji}(t)>0$. We use $\overline{\mathcal{N}}_i(t)$ to denote the neighbor set of node i at the time instant t. If $\overline{\mathcal{A}}_{\sigma(t)}$ is constant for all $t\geq0$, we say that the digraph $\overline{\mathcal{G}}_{\sigma(t)}$ is static.

As in [2], we will consider two classes of control laws as follows:

(1) Distributed dynamic state feedback:

$$\begin{aligned}
u_i(t)&=K_ix_i(t)+\overline{K}_i\eta_i(t), \quad i=1,\ldots,N\\
\dot{\eta}_i(t)&=S\eta_i(t)+\mu\left(\sum_{j\in\overline{\mathcal{N}}_i(t)}a_{ij}(t)\big(\eta_j(t)-\eta_i(t)\big)\right)\\
\eta_i(\theta)&=\eta_{i0}(\theta), \quad \theta\in[-\tau_r,0]
\end{aligned} \tag{3}$$

(2) Distributed dynamic measurement output feedback:

$$\begin{aligned}
u_i(t)&=K_iz_i(t)+\overline{K}_i\eta_i(t), \quad i=1,\ldots,N\\
\dot{\eta}_i(t)&=S\eta_i(t)+\mu\left(\sum_{j\in\overline{\mathcal{N}}_i(t)}a_{ij}(t)\big(\eta_j(t)-\eta_i(t)\big)\right)\\
\dot{z}_i(t)&=A_iz_i(t)+\sum_{j=0}^{r}B_{ij}u_i(t-\tau_j)+E_i\eta_i(t)\\
&\quad+L_i\left(\overline{C}_iz_i(t)+\sum_{j=0}^{r}\overline{D}_{ij}u_i(t-\tau_j)+\overline{F}_i\eta_i(t)-y_{mi}(t)\right)\\
\eta_i(\theta)&=\eta_{i0}(\theta), \quad z_i(\theta)=z_{i0}(\theta), \quad \theta\in[-\tau_r,0]
\end{aligned} \tag{4}$$

where $\eta_0(t)=v(t)$, μ is some positive number, $K_i\in R^{m_i\times n_i}$, $\overline{K}_i\in R^{m_i\times q}$, $L_i\in R^{n_i\times p_{mi}}$ are gain matrices to be determined later, $\eta_{i0}\in\mathcal{C}([-\tau_r,0],R^q)$ and $z_{i0}\in\mathcal{C}([-\tau_r,0],R^{n_i})$.