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Recurrent neural network for solving model predictive control problem in application of four-tank benchmark

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ABSTRACT

Based on model predictive control techniques, this paper presents a discrete-time recurrent neural network for solving four-tank benchmark problem which is reformulated to a convex programming problem. If the weighting matrices are positive definite symmetric, it is shown that the proposed neural network is globally exponentially stable and exponentially convergent to the exact optimal solutions. Finally, the experimental results have testified the effectiveness of the proposed approach and shown that the four-tank benchmark problem can be well resolved.

1. Introduction

Model Predictive Control (MPC) is a control method based on optimization, by solving an open-loop optimal control problem at each sampling instant computes the current control input. MPC is referred as receding horizon control, which is a senior strategy of multivariable control systems for optimizing the performance. The optimal problem's initial state is treated as the current system state, and future states are predicted by a model of the system. Dating back to late 1970s, the development and applications of MPC strategy were presented. Richalet et al. [1] proposed the first MPC technology, based on quadratic programming (QP). In recent years, MPC leads to the formulations of nonconvex optimization problems [2]. However, there are no reliable optimization procedures for solving such problems which would find exact solutions reliably and quickly [3]. It has large-scale applications in the process industry [22–27], chemical, food processing industries, economics, aerospace industries, and robotics. The application in the four-tank plant can be considered as a model of many industrial applications in process industry, for instance, chemical and petroleum chemistry. Moreover, four-tank benchmark has been made use of as an educational tool to teach advanced multivariable control techniques. The hot problem of the hierarchical and distributed model predictive control (HD-MPC) four-tank benchmark [11,28] is used to test, compare and evaluate different control system design approaches. The HD-MPC four-tank benchmark has different properties: (1) there is strong coupling between subsystems and it can manipulate the degree of coupling expediently; (2) the plant has nonlinear dynamics; (3) the states can be measured; (4) hard states and input constraints subordinate to the plant; (5) the plant can be safely operated.

Neurodynamic optimization using recurrent neural networks (RNNs) has become one applicable and promising approach. In the past two decades, varieties of neural networks have been applied for MPC. In general, the applications of RNNs for MPC branch out into two classifications: solving system modeling [15] and optimization [16]. The applications of RNNs for optimization have been widely investigated since the seminal study of Hopfield and Tank [9,10]. They played a significant role in neural network field, their work vitalized many researchers to exploit neural network models to figure out linear and nonlinear optimization problems. Kennedy and Chua made the Tank–Hopfield network expanding and reformative, they used the penalty method for solving nonlinear programming problem [5]. Zhang [17] solved nonlinear programming problems with equality constraints by utilizing the Lagrangian neural network. To the latest development such as Xia and Wang, in [6], a recurrent neural network was proposed for solving nonlinear convex optimization subject to nonlinear inequality constraints. In [7], it discussed the exponential convergence of the neural network and solved convex quadratic programming. In [8], a recurrent neural network was presented for solving nonlinear convex programs with linear constraints. In particular, Forti et al. [20] have introduced a generalized neural network for solving

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non-smooth nonlinear programming problems by means of penalty function method and it has a constraint neuron with high-gain nonlinearities. And He et al. [32] solved bilevel linear programming problem using a recurrent neural network. Hu et al. [36,37] proposed neural networks for solving synchronization issues. Also, there is another intelligent method of swarm neural networks for solving equalities-constrained nonconvex optimization in [21]. There are some applications that utilize the neural network such as multiuser power control [18], optimal real-time price in smart grid [19], and ultra-thin shape memory alloy wire [29]. Nowadays, some constrained optimization problems have been proposed by Liu and Wang [30,31]. Regarding optimal problems, there are a lot of researches about it [33–35]. Unlike several numerical optimization methods, recurrent neural networks for solving optimization problems are readily hardware-implementable. RNNs as parallel computational models for real-time optimization and applications are more effective. In the neural-network literature, there exist a few RNN models for solving quadratic optimization problems with bound constraints [14]. RNNs can be divided into continuous-time recurrent neural networks and discrete-time recurrent neural networks. Compared with continuous-time RNNs, the discrete-time RNN models have some advantages, such as numerical simulation and digital implementation.

In this paper, motivated by the effectiveness and efficiency of neural network optimization method, we have attempted to solve four-tank benchmark problem using neural network approach. Compared with iterative algorithm [4], our main contribution is to design a discrete-time recurrent neural network for solving the four-tank benchmark problem. The presented neural network optimization method is effective and has a faster convergence rate. Using the MPC scheme, the four-tank problem is formulated to a quadratic optimization problem, and a recurrent neural network is presented for solving this problem. Also, we have eliminated the formulated four-tank benchmark problem's state error and control error to make the system stable. Moreover, global exponential convergence can be certified in solving the reformulated optimization problem. In addition, a simple circuit is established to depict the neural network. Finally, simulation result shows the effectiveness and performance of the neural network for solving four-tank benchmark problem.

This paper is organized as follows. In Section 2, the four-tank benchmark about MPC is stated, and a recurrent neural network is proposed to solve the four-tank benchmark. In Sections 3, a set of sufficient conditions are derived for the global exponential stability. Simulation result is presented in Section 4. In Section 5, we summarize the main results of the paper and make a concluding remark.

2. Problem formulation and model description

Fig. 1 shows structure of the four-tank benchmark. The tanks 3 and 4 (at the top of the plant) discharge into the corresponding tanks 1 and 2 (at the bottom of the plant), respectively. The three-way valves are emulated by a proper calculation of the setpoints of the flow control loops according to the considered ratio of the three-way valve. Hence, the inlet flows of the three-way valves q_a and q_b can be considered as the manipulated variables of the real plant.

The plant is simplified and described as the following differential equations [4]:

$$\frac{dh_1}{dt} = -\frac{a_1}{S}\sqrt{2gh_1} + \frac{a_3}{S}\sqrt{2gh_3} + \frac{r_a}{S}q_a,$$
$$\frac{dh_2}{dt} = -\frac{a_2}{S}\sqrt{2gh_2} + \frac{a_4}{S}\sqrt{2gh_4} + \frac{r_b}{S}q_b,$$

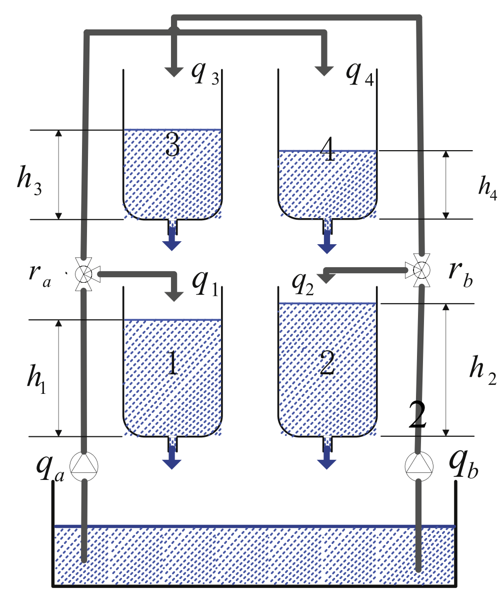

Fig. 1. Simplified model of the four-tank process.

$$\frac{dh_3}{dt} = -\frac{a_3}{S}\sqrt{2gh_3} + \frac{1-r_b}{S}q_b,$$
$$\frac{dh_4}{dt} = -\frac{a_4}{S}\sqrt{2gh_4} + \frac{1-r_a}{S}q_a, \tag{1}$$

where a_i, h_i, $i=1,\ldots,4$, indicate the discharge constant of tank i and the water level, respectively; g is the gravitational acceleration ($g = 9.806\ \mathrm{m/s^2}$ in this study), S is the cross section of each tank, and r_j, q_j, $j \in \{a,b\}$ refer to the ratio of the three-way valve of pump j and the flow of pump j, respectively. Some parameters of the plant, such as the ratio γ of each three-way valve and the cross section of the outlet hole a_i, can be manually regulated by the researcher. Moreover, the level of tanks as well as the inlet flows are physically constrained. Therefore, the dynamics of the plant can be adjusted by the researcher.

Define the deviation variables, and use linear model at an operating point given by the equilibrium levels and flows, shown as follows:

$$x_i = h_i - h_i^0, \quad i \in \{1,2,3,4\},$$
$$u_j = q_j - q_j^0, \quad j \in \{a,b\}, \tag{2}$$

then we can acquire the continuous-linear model:

$$\begin{cases} \dot{x} = A_c x + B_c u, \\ y = C_c x, \end{cases} \tag{3}$$

where $x = (x_1, x_2, x_3, x_4)$, $u = (u_a, u_b)$, $y = (x_1, x_2)$.

We can get the discrete-time model through the zero-order hold method with a sampling period of 5 s, which is shown below

$$\begin{cases} x(k+1) = Ax(k) + Bu(k), \\ y(k) = Cx(k). \end{cases} \tag{4}$$

By using an MPC scheme, we can depict an online finite horizon open-loop optimal control problem,

$$\min \quad J(k,\mathbf{u}) = \sum_{i=0}^{N-1}\left[\frac{1}{2}\|x(k+i+1|k)\|_Q^2 + \frac{1}{2}\|u(k+1|k)\|_R^2\right],$$
$$s.t. \quad x(k+i+1|k) = Ax(k+i|k) + Bu(k+i|k)),$$
$$x(k|k) = x(k),$$
$$\underline{x} = [h_{min}, h_{min}, h_{min}, h_{min}]^T \le x(k+i|k) \le [h_{1max}, h_{2max}, h_{3max}, h_{4max}]^T$$
$$= \overline{x},$$