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H_∞ filtering for uncertain systems with time-delay and randomly occurred sensor nonlinearities[☆]

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ABSTRACT

This paper investigates the H_∞ filtering problem for discrete time-delay systems with quantization and stochastic sensor nonlinearity. The problem of quantization considered in this paper is logarithmic quantization and the quantization error is processed into an uncertain term. The sensor nonlinearity is supposed to occur randomly on the basis of a stochastic variable obeying the Bernoulli distribution and the nonlinear decomposed technique is introduced to deal with the nonlinear term. The time-varying delay of systems is processed by using the Scaled Small Gain (*SSG*) theorem. The *LMI*-based sufficient conditions in view of the *SSG* and the Lyapunov–Krasovskii functional (*LKF*) approach are presented to guarantee the error system mean square stable and with the prescribed H_∞ performance index γ. An example is given to illustrate the effectiveness of the proposed method.

1. Introduction

As is known to all, in the past few decades, the filtering technology for discrete-time systems has been playing a decisive role in various fields including control and signal processing, target tracking, and image processing [1–3]. The Kalman filtering is the first filtering technology that scholars have begun to study. But for the main constraint of Kalman filtering which holds a linear system model with Gaussian distributions of all noise terms and measurements, the H_∞ filtering is introduced instead of the classical Kalman filtering. So, the H_∞ filtering problem has attracted a lot of scholars to study. Up to now, the main results for the research and analysis on the H_∞ filtering problem have been yielded, see [4–7] and the references therein.

In recent years, the study of the networked control systems (*NCSs*) is a hot research field in the international academic community. The stable performance of *NCSs* is affected by many factors, which include delay, quantization, interference, uncertainty, data loss and nonlinear, etc. Therefore, under the influence of these factors, how to ensure the stability of systems is the focus of research. For example, the design problem of H_∞ filter for discrete-time systems with time-varying delay has been investigated in [7], where a method for designing a mode-dependent filter has been proposed. Reference [8] gives a simple stability criteria for systems with time-varying delays. The problem of sampled-data synchronization for Markovian jump neural networks with time-varying delay is considered in [9]. The problem of quantization has been discussed in [10–12]. A new control strategy with on-line updating the quantizer's parameter is proposed in [13]. The achievements about the analysis of uncertainty have been considered [4,14–16]. The research on nonlinear problems has also achieved remarkable results, see [5,17,18].

Recently, another interesting problem is that how to use the *SSG* to research the stable performance of systems. The *SSG* is the theoretical basis of the input and output stability method. This approach could give results with mush less conservatism. And it has been effectively used to deal with the time-delay systems. A new model transformation of discrete-time systems with time-varying delay by the lower and upper delay bounds has been introduced in [19]. Then, the results of [19] have been further improved in [20]. In [21], the application of the *SSG* in H_∞ filtering problems has been reported. The problem of the filtering with sensor nonlinearity has been discussed in [18]. However, to the best of the author's knowledge, up to now, little work regarding the application of the approach in the H_∞ filtering problems with stochastic sensor nonlinearity and quantization has been reported yet, which is the purpose of this paper.

In this paper, the quantized H_∞ filtering problem for discrete time-delay systems with quantization and stochastic sensor

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nonlinearity is investigated. First, the quantization error of logarithmic quantization is expressed in the form of uncertainty, and the nonlinear function of the system will be decomposed into a linear term and a nonlinear term by the nonlinear decomposed technique. Then, a model transformation is applied to the original system. In order to ensure the approximation error to be as small as possible, a two-term approximation constructed by the lower and upper delay bounds is employed for the time-varying delay. Combined with the *LKF* and the *SSG* approach, the new sufficient conditions for the existence of the H_∞ filter are established. Finally, an illustrative example is presented to show the feasibility of the proposed method.

The organization of this paper is as follows. Section 2 presents the problem of the H_∞ filtering under consideration and some preliminaries. Section 3 gives a design method of the H_∞ filtering strategies. In Section 4, an example is presented to illustrate the effectiveness of the proposed method. Finally, Section 5 gives some concluding remarks.

Notation: Throughout this paper, R^n represents the n-dimensional Euclidean space, $R^{n\times m}$ is the set of all $n\times m$ real matrices. Given a matrix U, U^T and U^{-1} denote its transpose, and inverse when it exists, respectively. The notation $P>0$ (≥ 0) for $P\in R^{n\times n}$ means that P is real symmetric and positive definite (semidefinite). The symbol $*$ within a matrix represents the symmetric entries, and the block-diagonal matrices are denoted by diag$\{\cdots\}$. $M_1 \circ M_2$ means the series connection of mapping M_1 and M_2, $\|x\|_{L_2} \triangleq \sqrt{\sum_0^\infty x^T(k)x(k)}$ represents the L_2 norm of series $x(k)$ and $\|\cdot\|_\infty$ denotes the L_2-induced norm of a general operator, and $E\{\cdot\}$ denotes the expectation operator with respect to some probability measure.

2. Problem statement and preliminaries

2.1. Problem statement

Suppose the following system with time-delay and stochastic sensor nonlinearity:

$$\begin{aligned} x(k+1) &= Ax(k)+A_t x(k-t(k))+B\omega(k),\\ y(k) &= (1-\beta(k))\varphi(Cx(k)+C_t x(k-t(k)))\\ &\quad +\beta(k)(Cx(k)+C_t x(k-t(k)))+D\omega(k),\\ z(k) &= Lx(k), \end{aligned} \tag{1}$$

where $x(k)\in R^n$ is the state, $y(k)\in R^m$ is the measured output, $\omega(k)\in R^q$ is the disturbance input that belongs to $L_2[0,\infty)$ and $z(k)\in R^p$ is the signal to be estimated. $A\in R^{n\times n}$, $A_t\in R^{n\times n}$, $B\in R^{n\times q}$, $C\in R^{m\times n}$, $C_t\in R^{m\times n}$, $D\in R^{m\times q}$ and $L\in R^{p\times n}$ are constant matrices with appropriate dimensions. $t(k)$ denotes the time-varying delay and satisfies

$$t_1\leq t(k)\leq t_2,\quad t_{12}=t_2-t_1 \tag{2}$$

where $t_1>0$ and $t_2>0$ denote the lower and upper bounds of the delays, respectively. The symbol φ represents the sensor nonlinearity with the following sector condition [17]:

$$(\varphi(h)-R_1 h)^T(\varphi(h)-R_2 h)\leq 0,\quad h\in R^m, \tag{3}$$

where R_1 and R_2 are diagonal matrices satisfying $R_2>R_1\geq 0$. The stochastic variable $\beta(k)$ obeys the Bernoulli distribution, which is introduced to account for the phenomena of randomly occurred sensor nonlinearity and taking the values of 0 and 1 with

$$\Pr\{\beta(k)=1\}=\beta,\quad \Pr\{\beta(k)=0\}=1-\beta, \tag{4}$$

where $\beta\in[0,1]$ is a known constant of probability density. In order to technical convenience, $Cx(k)+C_t x(k-t(k))$ is expressed by $\odot$, and the nonlinear function $\varphi(\odot)$ can be decomposed into a linear term and a nonlinear term as

$$\varphi(\odot)=\varphi_r(\odot)+R_1\odot, \tag{5}$$

where the nonlinearity $\varphi_r(\odot)$ satisfies

$$\varphi_r(\odot)^T(\varphi_r(\odot)-R\odot)\leq 0,\quad R=R_2-R_1>0. \tag{6}$$

The problem considered here is to estimate the signal $z(k)$. Due to the introduction of the network, the signal needs to be quantized before entering the filter. Therefore, considering the following quantized filter described by

$$\begin{aligned} x_f(k+1) &= A_f x_f(k)+B_f q(y(k)),\\ z_f(k) &= C_f x_f(k), \end{aligned} \tag{7}$$

where $x_f(k)\in R^n$ is the filter state, $z_f(k)\in R^p$ is the estimation of $z(k)$, and the constant matrices A_f, B_f and C_f are filter matrices to be determined. $q(y(k))$ is the quantized measured signal, and the symmetric logarithmic quantizer [10] is employed, which is denoted as $q(\cdot)=[q_1(\cdot),q_2(\cdot),\ldots,q_m(\cdot)]$ and $q_j(u)=-q_j(-u), j=1,\ldots,m$. For each $q_j(\cdot)$, the set of quantized levels is described by

$$U_j=\{\pm\sigma_j^i \mid \sigma_j^i=(\rho_j)^i\cdot\sigma_j^0,\ i=\pm1,\pm2,\ldots\}\cup\{0\}, \tag{8}$$

where $0<\rho_j<1, \sigma_j^0>0$, and ρ_j is the quantizer density of the sub-quantizer $q_j(\cdot)$, and σ_j^0 denotes the initial quantization values of the jth sub-quantizer $q_j(\cdot)$, which is defined as

$$q_j(u)=\begin{cases}\sigma_j^i, & \dfrac{1}{1+\upsilon_j}\sigma_j^i<u\leq\dfrac{1}{1-\upsilon_j}\sigma_j^i,\\ 0, & u=0,\\ -q_j(-u), & u<0,\end{cases} \tag{9}$$

where $\upsilon_j=\frac{1-\rho_j}{1+\rho_j}$ is the quantizer parameters.

Furthermore, $q(y(k))$ can be rewritten in the following form:

$$q(y(k))=(I_m+\Sigma(k))y(k), \tag{10}$$

where $\Sigma(k)=diag\{\Sigma_1(k),\Sigma_2(k),\ldots,\Sigma_m(k)\}$, $\Sigma_j(k)\in[-\upsilon_j,\upsilon_j]$, $j=1,2,\ldots,m$.

Then, the quantized filter (7) can be expressed as

$$\begin{aligned} x_f(k+1) &= A_f x_f(k)+B_f(I_m+\Sigma(k))y(k),\\ z_f(k) &= C_f x_f(k). \end{aligned} \tag{11}$$

By combining (1) with (11), and defining $\zeta(k)=[x^T(k)\ x_f^T(k)]^T$, $e(k)=z(k)-z_f(k)$, the filtering error system can be written as

$$\begin{aligned} \zeta(k+1) &= (\overline{A}+A_\Sigma H)\zeta(k)+(\overline{A}_t+A_{t\Sigma})H\zeta(k-t(k))\\ &\quad +(A_r+A_{r\Sigma})H^T\varphi_r(\odot)+(\overline{B}+B_\Sigma)\omega(k)\\ &\quad +(\beta(k)-\beta)[(\overline{A}_1+A_{1\Sigma})H^T H\zeta(k)\\ &\quad +(\overline{A}_2+A_{2\Sigma})H^T H\zeta(k-t(k))+(\overline{A}_3+A_{3\Sigma})H^T\varphi_r(\odot)],\\ e(k) &= \overline{C}\zeta(k), \end{aligned} \tag{12}$$

where

$$\overline{A}=\begin{bmatrix} A & 0\\ \beta B_f C+(1-\beta)B_f R_1 C & A_f\end{bmatrix},\quad A_\Sigma=\overline{A}_\Sigma\Sigma(k)C,$$

$$\overline{A}_t=\begin{bmatrix} A_t\\ \beta B_f C_t+(1-\beta)B_f R_1 C_t\end{bmatrix},\quad A_{t\Sigma}=\overline{A}_\Sigma\Sigma(k)C_t,$$

$$A_r=(1-\beta)B_f,\quad A_{r\Sigma}=(1-\beta)B_f\Sigma(k),$$

$$\overline{B}=\begin{bmatrix} B\\ B_f D\end{bmatrix},\quad B_\Sigma=\begin{bmatrix}0\\ B_f\end{bmatrix}\Sigma(k)D,\quad H=[I\ \ 0],$$

$$\overline{A}_1=B_f(1-R_1)C,\quad A_{1\Sigma}=B_f\Sigma(k)(1-R_1)C,$$

$$\overline{A}_2=B_f(1-R_1)C_t,\quad A_{2\Sigma}=B_f\Sigma(k)(1-R_1)C_t,$$

$$\overline{A}_3=-B_f,\quad A_{3\Sigma}=-B_f\Sigma(k),\quad \overline{C}=[L\ \ -C_f],$$