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Robust fuzzy observer-based fault detection for nonlinear systems with disturbances

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ABSTRACT

This paper studies the integrated design of fuzzy observer-based fault detection (FD) for nonlinear systems in the presence of external disturbances. To this end, we first approximate the nonlinear systems by a set of Takagi–Sugeno (T–S) fuzzy models. Then, the observer-based FD systems are studied with the aid of $\mathcal{L}_2$ stability theory. In the end, a numerical example is given to show the efficiency of the proposed method.

1. Introduction

Over the past decades, fault detection and diagnosis (FDD) have received considerably increasing attention [1–7], for the purpose of meeting high requirements on system performance and reliability from industrial processes. Since nonlinearity is considered to exist universally in real plants, a great deal of efforts have been paid to the observer-based fault detection (FD) and fault-tolerant control (FTC) for nonlinear systems [8–12]. More particularly, the integrated design of the nonlinear observer-based FD systems, which consists of an observer-based residual generator, a residual evaluator and a decision maker with an embedded threshold, have been extensively investigated in recent years [13–16].

Takagi–Sugeno (T–S) Fuzzy technique, as an alternative approach to conventional control techniques, has been proved to be a powerful tool to analyze and approximate complex nonlinear systems [17–20]. Encouraged by these studies, significant research efforts have been devoted to solve the controller [21] and filtering design problems [22–25] for nonlinear systems. Besides, as reported in the literatures [26–29], a great deal of attention has been paid to the construction of nonlinear residual generators based on the obtained universal T–S fuzzy model. Over the past few years, a trend of approximating general type of nonlinear systems to any degree of accuracy on any compact set by invoking control variables in the premise variables have been observed [30]. In spite of the result mentioned in [30], the synthesis and construction of universal fuzzy controllers for general nonlinear systems have been investigated in [31,32].

The main objective of this paper is to devoted to the integrated design of T–S fuzzy observer-based FD systems for general type of complex nonlinear systems involving external disturbances. To this end, the nonlinear plant is first interpreted in light of a class of generalized T–S dynamic fuzzy models. Then the integrated design procedure of FD systems is carried out by developing a fuzzy observer-based residual generator, an evaluator and a decision logic with the aid of $\mathcal{L}_2$ stability theory.

The paper is organized as follows. The needed preliminaries and problem formulation are addressed in Section 2. In Section 3, robust fuzzy observer-based fault detection for nonlinear systems with external disturbances are presented. A numerical example is given in Section 4 to illustrate the effectiveness of the proposed method. Conclusions and future work are stated in Section 5.

In addition, standard notations are adopted in this paper. $\mathcal{R}_+ = [0, \infty)$. $\|\cdot\|$ denotes the Euclidean norm of a vector in some Euclidean space. $\|u\|_2$ denotes $\mathcal{L}_2$-norm of $u(t)$ which is defined as $\|u\|_2 = \left(\int_0^\infty \|u(t)\|^2\, dt\right)^{1/2}$. $\mathtt{Sym}\{A\}$ denotes $A+A^T$. In a symmetric matrix, $\star$ represents the symmetric elements.

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2. Problem formulation

Generally speaking, a standard observer-based FD system consists of an observer-based residual generator, a residual evaluator and a decision maker with a threshold [5]. Along this line, numerous results have been proposed for the integrated design of observer-based FD systems for linear time-invariant (LTI) systems [5,6].

Motivated by the integrated design schemes of observer-based fault detection systems for LTI systems, the major objective of this paper is to investigate the integrated design of robust fuzzy observer-based FD issues for general type of nonlinear systems with external disturbances, which is described by

$$\Sigma : \begin{cases} \dot{x} = f(x,u) + g(x,u)d \\ y = h(x,u) + k(x,u)d \end{cases} \tag{1}$$

where $x(t) = [x_1(t) \cdots x_{k_x}(t)]^T \in \mathcal{R}^{k_x}, u = [u_1(t) \cdots u_{k_u}(t)]^T \in \mathcal{R}^{k_u}, y = [y_1(t) \cdots y_{k_y}(t)]^T \in \mathcal{R}^{k_y}$ denote the state, output and input vectors, respectively.

$f(x,u)$, $h(x,u)$, $g(x,u)$ and $k(x,u)$ are continuously differentiable nonlinear functions with appropriate dimensions. $d \in \mathcal{R}^{k_d}$ denotes the disturbances which is assumed to be $\mathcal{L}_2$-bounded with

$$\|d\|_2 \le \delta_d \tag{2}$$

Inspired by the Takagi–Sugeno (T–S) Fuzzy modelling technique [30,31] for nonlinear systems, our first objective is to establish the T–S Fuzzy model for nonlinear systems (1). Then, the robust $\mathcal{L}_2$ fuzzy observer-based residual generator will be first investigated. As a result, an integrated FD system will be constructed, which consists of a residual generator, an evaluation function and a decision logic with a dynamic threshold. Our further task is dedicated to the robust $\mathcal{L}_2$ observer-based FD system in the presence of external disturbances for nonlinear systems (1).

3. Design of $\mathcal{L}_2$ robust fuzzy observer-based FD systems

3.1. Fuzzy dynamic modelling

For our purpose, the following class of general T–S fuzzy models are employed to approximate nonlinear systems (1) first:

Plant rule $\Re^i$: **IF** $\theta_1(t)$ is ${N_1}^i$ and $\theta_2(t)$ is ${N_2}^i$ and $\cdots$ and $\theta_p(t)$ is ${N_p}^i$

$$\textbf{THEN} \begin{cases} \dot{x}(t) = A_i x(t) + B_i u(t) + E_i d(t) \\ y(t) = C_i x(t) + D_i u(t) + F_i d(t), \quad i \in \{1, 2, \ldots, \kappa\} \end{cases} \tag{3}$$

where $\Re^i$ represents the ith fuzzy inference rule; κ denotes the number of inference rules; $\theta(t) = [\theta_1(t) \cdots \theta_p(t)]$ denotes the premise variables assumed measurable; $N_j^i (j = 1, 2, \ldots, p)$ indicates the fuzzy sets; A_i, B_i, C_i, D_i, E_i and F_i are system matrices with appropriate dimensions; $x(t)$, $u(t)$ and $y(t)$ denote the system state, input and output variables, respectively. $d(t)$ represents the external disturbance variables.

Let $\mu_i(\theta(t))$ be the normalized fuzzy membership function of the inferred fuzzy set $N^i := \prod_{l=1}^p N_l^i$, which is defined as

$$\mu_i(\theta(t)) = \frac{\prod_{l=1}^p N_l^i}{\sum_{j=1}^{\kappa} \prod_{l=1}^p N_l^j}. \tag{4}$$

Consequently, we have

$$\mu_i(\theta(t)) \ge 0, i = 1, 2, \ldots, \kappa, \sum_{i=1}^{\kappa} \mu_i(\theta(t)) = 1. \tag{5}$$

Thus by using a center average defuzzifier, a singleton fuzzifier and product inference, the T–S fuzzy system in (3) can be inferred as follows:

$$\begin{cases} \dot{x}(t) = \overline{f}(x,u) \\ y(t) = \overline{h}(x,u) \end{cases} \tag{6}$$

where

$$\overline{f}(x,u) = \sum_{i=1}^{\kappa} \mu_i(\theta(t))(A_i x(t) + B_i u(t) + E_i d(t))$$
$$\overline{h}(x,u) = \sum_{i=1}^{\kappa} \mu_i(\theta(t))(C_i x(t) + D_i u(t) + F_i d(t)). \tag{7}$$

In what follows, the approximation capability of the T–S fuzzy models in (3) will be addressed based on the results given in [30,31]. To this end, the following lemma is introduced first.

Lemma 1 (*Zeng, et al., [30]*). *If vector value function $s(z) = [s_1(z_1, \ldots, z_N) \cdots s_n(z_1, \ldots, z_N)]^T$ is $m(\ge 1)$ time continuously differentiable on $\mathcal{Z}$ with $s(0) = 0$, then for i = 1,…,N, the vector value function*

$$Q_i(z) = q_i(z_1, \ldots, z_N) = \begin{cases} \dfrac{s(0, \ldots, 0, z_i, z_{i+1}, \ldots, z_N) - s(0, \ldots, 0, z_{i+1}, \ldots, z_N)}{z_i}, & z_i \ne 0 \\ \dfrac{\partial s(0, \ldots, 0, z_{i+1}, \ldots, z_N)}{\partial z_i}, & z_i \ne 0 \end{cases} \tag{8}$$

is $m-1$ continuously differentiable on $\mathcal{Z}$ and

$$s(z) = \sum_{i=1}^{N} Q_i(z) z_i = \sum_{i=1}^{N} q_i(z_1, \ldots, z_N) z_i \tag{9}$$

Theorem 1. *Consider nonlinear systems given in (1), where $f(x,u)$ and $g(x,u)$ are continuously differentiable on the compact set $\mathcal{X} \times \mathcal{U}$ and $f(0,0) = 0, g(0,0) = 0$. Then, for any positive $\epsilon_i, i = 1, \ldots, 4$ and any $(x,u) \in \mathcal{X} \times \mathcal{U}$, there exist T–S fuzzy models (7) such that*

$$f(x,u) + g(x,u)d = \overline{f}(x,u) + \Delta_A(x,u)x + \Delta_B(x,u)u + \Delta_E(x,u)d$$
$$h(x,u) + k(x,u)d = \overline{h}(x,u) + \Delta_C(x,u)x + \Delta_D(x,u)u + \Delta_F(x,u)d \tag{10}$$

with

$$\begin{aligned} &\|\Delta_A(x,u)\| \le \epsilon_1, \quad \|\Delta_B(x,u)\| \le \epsilon_2 \\ &\|\Delta_C(x,u)\| \le \epsilon_3, \quad \|\Delta_D(x,u)\| \le \epsilon_4 \\ &\|\Delta_E(x,u)\| \le \epsilon_5, \quad \|\Delta_F(x,u)\| \le \epsilon_6. \end{aligned} \tag{11}$$

Proof. The proof follows directly from [31] and thus is omitted here.□

As a result, the nonlinear systems (1) can be also rewritten as the following T–S fuzzy models with the norm bounded uncertainties:

Plant rule $\mathcal{R}^i$: **IF** $\theta_1(t)$ is ${N_1}^i$ and $\theta_2(t)$ is ${N_2}^i$ and $\cdots$ and $\theta_p(t)$ is ${N_p}^i$

$$\textbf{THEN} \begin{cases} \dot{x}(t) = A_i x(t) + B_i u(t) + E_i d(t) + \Delta_A(x,u)x(t) + \Delta_B(x,u)u(t) + \Delta_E(x,u)\, d(t) \\ y(t) = C_i x(t) + D_i u(t) + F_i d(t) + \Delta_C(x,u)x(t) + \Delta_D(x,u)u(t) + \Delta_F(x,u)\, d(t), \\ i \in \{1, 2, \ldots, \kappa\} \end{cases} \tag{12}$$

Thus the final state of the fuzzy system can be inferred as follows:

$$\dot{x}(t) = \sum_{i=1}^{\kappa} \mu_i(\theta(t))(A_i x(t) + B_i u(t) + E_i d(t)) + \Delta_A(x,u)x(t) + \Delta_B(x,u)u(t) + \Delta_E(x,u)\, d(t)$$

$$y(t) = \sum_{i=1}^{\kappa} \mu_i(\theta(t))(C_i x(t) + D_i u(t) + F_i d(t)) + \Delta_C(x,u)x(t)$$