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Finite-time stochastic boundedness of discrete-time Markovian jump neural networks with boundary transition probabilities and randomly varying nonlinearities ☆

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ABSTRACT

This work studies the problem of finite-time stochastic boundedness of discrete-time Markovian jump neural networks with boundary transition probabilities and randomly varying nonlinearities. The partly unknown and uncertain transition probabilities (TPs) are included in the paper, and more general nonlinearities are introduced with both upper and lower bounds due to the nature of its probability information. By employing the free-weighting matrix technique, finite-time stability theory and boundary incomplete TPs, the solvability sufficient conditions of finite-time stochastic boundedness are given. Finally, numerical examples are presented to demonstrate the effectiveness of the proposed approach.

1. Introduction

As a special class of nonlinear dynamical systems, neural networks have been received much attention because of its applications in a variety of fields, including pattern recognition, signal processing and other areas. The stability analysis of neural networks is required due to the existence of time delays and other disturbances, which cause the oscillations and instability. Therefore, the stability problem of neural networks with time delay [1–3] and disturbance [4,5] has been widely studied by many researchers. Furthermore, the finite modes of neural networks always switching from one to another with respect to varying time, which can be considered into the theoretical framework of Markovian jump systems (MJSs). In the past decades, particular research interests have been devoted to Markovian jump neural networks with time delay for their extensive applications [6–9].

On another research forefront, the jump rules of each modes are often determined by transition probabilities (TPs) in the MJSs, which are always assumed to be completely known. However, this situation is actually questionable and probably hard to satisfied. In the Markov chains, both packet dropout and variation of time delay are random and vague at different, which leads to TPs matrix is costly to get. Therefore, the focus has been switched to verify the stability of MJSs with partially unknown TPs by some researchers [10–18], such as in [10], a very general system, namely a traditional Markovian jump nonlinear quadratic system with both the completely known transition probabilities and arbitrary switching, is considered. In [12], a relaxation scheme is proposed for a class of discrete-time singular Markovian jump systems with time-varying delays and incomplete (i.e., unknown and uncertain) transition probabilities. In [15], with definition of finite-time stochastic stability for discrete-time MJSs and the relationship of TPs $\sum_{j\in\mathcal{R}_{\mathcal{K}}}\pi_{ij} = 1-\sum_{j\in\mathcal{R}'_{\mathcal{UK}}}\pi_{ij}$ are given, the problems of finite-time stochastic stability and stabilization with partly unknown transition probabilities for linear discrete-time Markovian jump systems (MJP) have been discussed.

Furthermore, the nonlinearities are extensive exist in dynamic systems which caused by environmental circumstances [19–23] and in other engineering application [36–41]. It should be pointed out that the nonlinear disturbances may cause random abrupt changes for its random changes, which is said to be randomly varying nonlinearities. However, to acquire the stability conditions, the existing results only focus on either sector bounds or the upper bound of nonlinearity. The inner information between lower and upper bounds is neglect, which brings the conservativeness.

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In the practical dynamic systems, the behavior of dynamic systems in a fixed finite-time interval, for example, the state trajectories stay a given bound from the perspective of real-world applications, which attracts our attention. Up to now, considerable results of dynamic systems relate to finite-time stability, finite-time stabilization and finite-time boundedness have been reported in [24–32]. In [28], the solutions to the finite-time H_∞ synchronization problem are formulated by employing the properties of Kronecker product combined with the Lyapunov–Krasovskii method. In [31], finite-time state-feedback stabilization is addressed for a class of discrete-time nonlinear systems with conic-type nonlinearities and additive disturbances. Haddad and L'Afflitto [32] addressed the problem of optimal nonlinear analysis and feedback control design for finite-time partial stability and finite-time, partial-state stabilization.

Motivated by the above discussion, the problem of finite-time stochastic boundedness of discrete-time Markovian jump neural networks with boundary transition probabilities and randomly varying nonlinearities is discussed in this paper. The main contribution of this paper lies first in bring the general mode transition probabilities are set, namely, partly unknown and uncertain in the Markov chain to derive the less conservative stability condition. Markovian jump neural networks jump from one to others have a finite number of modes. Second, more general probability nonlinearity model with both upper and lower bounds is introduced. Third, by employing the free-weighting matrix technique, finite-time stability theory and bounded incomplete transition probabilities, sufficient conditions are given for the solvability of the problems, which can be tackled in form of linear matrix inequalities (LMIs). Fourth, numerical examples are presented to demonstrate the effectiveness of the proposed method.

2. Preliminaries

Consider the following Markovian jump neural networks with n neurons defined on a probability space (Ω, F, Ψ):

$$x(k+1)=A(r_k)x(k)+B(r_k)x(k-\tau(k))+D(r_k)\omega(k)+h(x(k),r_k) \tag{1}$$

where $x(k)$ is the state, $\omega(k)$ is the external exogenous disturbances, and for given $N\in\mathbb{Z}^+$, $\omega(k)$ satisfies

$$\sum_{k=0}^{N}\omega^\top(k)\omega(k)\le d, \quad d\ge 0 \tag{2}$$

The transmission delay $\tau(k)$ is an unknown time-varying function that satisfies $\tau_m\le\tau(k)\le\tau_M$, where τ_m and τ_M are prescribed positive integers representing the lower and upper bounds of the delay. r_k represents a discrete-time, discrete-state Markov chain taking values in a finite set $\mathcal{N}=\{1,2,\dots,N\}$, with transition probability matrix $\pi=[\pi_{ij}]_{i,j\in\mathcal{N}}$ and $\pi_{ij}\ge 0$ is defined as

$$\pi_{ij}=\Pr\{r_{k+1}=j\,|\,r_k=i\} \tag{3}$$

where $\sum_{j=1}^{N}\pi_{ij}=1$, and $\pi_{ij}\in[0,1]$, the Markov process transition rate matrix π is defined by

$$\pi=\begin{bmatrix}\pi_{11} & \pi_{12} & \cdots & \pi_{1N}\\ \pi_{21} & \pi_{22} & \cdots & \pi_{2N}\\ \vdots & \vdots & \ddots & \vdots\\ \pi_{N1} & \pi_{N2} & \cdots & \pi_{NN}\end{bmatrix}$$

$A(r_k)$, $B(r_k)$ and $D(r_k)$ are known real constant matrices for all $r_k=i\in\mathcal{N}$. We denote the matrices associated with $r_k=i\in\mathcal{N}$ by $A(r_k)=A_i$, $B(r_k)=B_i$, $D(r_k)=D_i$.

It should be mentioned that transition probabilities of the jumping process are always assumed to be some are known, some are unknown. If Markovian jump delayed systems with four operation modes, the transition probability matrix with four operation modes can be represented as follows:

$$\pi=\begin{bmatrix}\pi_{11} & ? & \pi_{13} & ?\\ ? & ? & ? & \pi_{24}\\ \pi_{31} & ? & \pi_{33} & ?\\ ? & ? & \pi_{43} & \pi_{44}\end{bmatrix},$$

where ? represents the completely unknown transition probabilities. Therefore, the transition probability matrix can be represented as follows:

$$\begin{cases}\mathcal{R}_{\mathcal{K}}=\{j\,|\,\pi_{ij}\text{ is completely known for } i\}\\ \mathcal{R}_{\mathcal{UK}}=\{j\,|\,\pi_{pq}\text{ is unknown but bounded as }\alpha_{ij}\le\pi_{ij}\le\beta_{ij}\text{ for } i\}\\ \mathcal{R}^0_{\mathcal{UK}}=\{j\,|\,\pi_{ij}\text{ is unknown for } i\}\end{cases} \tag{4}$$

To express the transition probability matrix π, the defined sets can be denoted as $\mathcal{N}=\mathcal{R}_{\mathcal{K}}\cup\mathcal{R}_{\mathcal{UK}}\cup\mathcal{R}^0_{\mathcal{UK}}$. However, in the discrete-time Markov process, $\sum_{j=1}^{N}\pi_{ij}=1$, and $\pi_{ij}\in[0,1]$. For all $j\in\mathcal{R}^0_{\mathcal{UK}}$, the probabilities π_{ij} is bounded as follows:

$$\alpha_{ij}=0,\quad \beta_{ij}=1-\sum_{\zeta\in\mathcal{R}_{\mathcal{K}}}\pi_{i\zeta}-\sum_{\zeta\in\mathcal{R}_{\mathcal{UK}}}\alpha_{i\zeta}$$

It should be noted that, for the given i, it is not correct with π_{ij} is completely unknown, it belongs to π_{ij} is unknown but bounded as $\pi_{ij}\in[\alpha_{ij},\beta_{ij}]$, namely, $\mathcal{R}^0_{\mathcal{UK}}\in\mathcal{R}_{\mathcal{UK}}$. Then, one has

$$\mathcal{N}=\mathcal{R}_{\mathcal{K}}\cup\mathcal{R}_{\mathcal{UK}},\quad \forall i\in\mathcal{N}$$

For example, for any S_j, one has

$$\sum_{j\in\mathcal{N}}\pi_{ij}=\sum_{j\in\mathcal{R}_{\mathcal{K}}\cup\mathcal{R}_{\mathcal{UK}}}=\sum_{j\in\mathcal{R}_{\mathcal{K}}}\pi_{ij}Q_j+\sum_{j\in\mathcal{R}_{\mathcal{UK}}}\pi_{ij}Q_j,\quad\forall i\in\mathcal{N}$$

Furthermore, for the incomplete transition probabilities, we denote

$$\Pi_{\mathcal{K}}=\sum_{j\in\mathcal{R}_{\mathcal{K}}}\pi_{ij},\quad \Pi_{\mathcal{UK}}=[\pi_{ij}]_{j\in\mathcal{R}_{\mathcal{UK}}}\in\mathbb{R}^g$$

where g represents the number of elements in $\mathcal{R}_{\mathcal{UK}}$.

In [33], the sector bounded nonlinearities are given as $(\Psi(\nu)-H_1\nu)^\top(\Psi(\nu)-H_1\nu)\le 0$, in which $\Psi(\nu)$ is nonlinear function, H_s $(s=1,2)$ are known constant matrices with $\mathrm{tr}(H_1)<\mathrm{tr}(H_2)$. Similarly, the sector bounded nonlinearities are extended with both state and time-delayed state vectors. In this paper, $h(x(k),r_k)$ is the nonlinear function with both state and time-delayed state vectors and is given as follows:

$$\begin{aligned}&[h(x(k),i)-(M_{im}x(k)+N_{im}x(k-\tau(k)))]^\top[h(x(k),i)-(M_{iM}x(k)\\ &\quad+N_{iM}x(k-\tau(k)))]\le 0\end{aligned} \tag{5}$$

where M_{im}, N_{im}, M_{iM} and M_{iM} are real matrices with compatible dimensions. Furthermore, $\mathrm{tr}(M_{im})\le\mathrm{tr}(M_{iM})$ and $\mathrm{tr}(N_{im})\le\mathrm{tr}(N_{iM})$. In such case, it is said that $h(i,x(k))\in[\{M_{im},N_{im}\},\{M_{iM},N_{iM}\}]$.

In this paper, new probability of nonlinearities is given as

$$\kappa_1(k)=\begin{cases}0 & \text{if } h(x(k),i)\in[\{M_{im},N_{im}\},\{M_i,N_i\}]\\ 1 & \text{if } h(x(k),i)\in[\{M_i,N_i\},\{M_{iM},N_{iM}\}]\end{cases}$$

$$\kappa_1(k)+\kappa_2(k)=1$$

where M_i and N_i are constant matrices with $\mathrm{trance}(M_{im})\le\mathrm{trance}(M_i)\le\mathrm{trance}(M_{iM})$ and $\mathrm{trance}(N_{im})\le\mathrm{trance}(N_i)\le\mathrm{trance}(N_{iM})$. $\kappa_1(k)$ Bernoulli distributed sequence and satisfying

$$\Pr\{\kappa_1(k)=1\}=\kappa_1,\quad \Pr\{\kappa_1(k)=0\}=1-\kappa_1$$
$$\Pr\{\kappa_2(k)=1\}=1-\kappa_1=\kappa_2,\quad \Pr\{\kappa_2(k)=0\}=\kappa_1=1-\kappa_2.$$

Then, one has

$$h_1(x(k),i)=\begin{cases}h(x(k),i), & \kappa_1(k)=1\\ M_{im}x(k)+N_{im}x(k-\tau(k)), & \kappa_1(k)=0\end{cases}$$