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H∞ control of discrete-time uncertain linear systems with quantized feedback☆

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ABSTRACT

This paper studies the problem of H_∞ control for uncertain linear discrete-time systems with quantized state feedback. Consider that the uncertain parameters are supposed to reside in a polytope. The system state is quantized by a logarithmic static and time-invariant quantizer. Via giving a new control law and using parameter dependent Lyapunov function approach, new results on the quantized H_∞ state feedback control are expressed in terms of linear matrix inequalities (LMIs). A numerical example is introduced to illustrate the effectiveness and applicability of the proposed methodology.

1. Introduction

In recent years, quantization in feedback control systems has attracted a growing interest [1–17]. This is mainly due to the wide application of digital computers in control systems and the rapid development of network based control. Comparing with classical control theory, quantized feedback control is a common source of errors, which may degrade the system performance as described by Kalman [1], the effect of quantization in a sampled data control system and pointed out that if a stabilizing controller was quantized using a finite-alphabet quantizer, the feedback system would exhibit limit cycles and chaotic behavior. Consequently, a lot of works have focused on understanding and mitigating the quantization effects in the early. While in recent studies, a general practice is to treat the quantizers as information coders. Among these results, there are mainly two approaches for studying control problem with quantized feedback. The first approach handles static quantizers such as uniform and logarithmic quantizers [2–12], while the second approach considers the dynamic quantizers which scales the quantization levels dynamically in order to improve the steady-state performance [13,14].

For the problem of quantized feedback control, many important achievements have been obtained. Elia [2] had proven that a logarithmic quantizer is needed for stabilization of discrete-time single-input–single-output (SISO) linear time-invariant systems. Fu [3] gave a comprehensive study on feedback control systems with logarithmic quantizers by the sector bound approach. Both stabilization and H_∞ performance issues have been considered. Following this work, Gao [7] noticed that the constant Lyapunov function is conservative for quantized feedback problem and proposed a new general framework based on quantization dependent Lyapunov functions. Recently, Zhou [8] revisited the absolute stability approach, and gave a less conservative result. For some new results about quantized feedback control, see [15–17]. Considerable attention, however, have been paid toward the study of H_∞ control for linear systems [18-24], nonlinear systems [25-28]. For polytopic uncertain systems, refs. [19–23] focus on the improved problem of the bounded real lemma (BRL) for the polytopic uncertainty systems, that is, how to find a less conservative LMI-based method of designing H_∞ controller. It was noted that the basic idea behind these papers is based on constant feedback matrix K. However, the constant feedback matrix is independent of polytopic uncertainty parameters, the results obtained with constant feedback matrix are conservative when extended to polytopic uncertainty system with quantized feedback. Our main objective is to propose a new

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parameter dependent control law and to obtain less conservative results for polytopic uncertainty quantized feedback systems.

In this paper, we present a new way to deal with the quantized feedback problem for polytopic uncertainty systems, that is, change constant feedback matrix K into parameter dependent $K(\eta)$ by Lagrange's interpolation. Obviously the result obtained by parameter dependent feedback matrix is less conservative than the ones by constant feedback matrix for polytopic uncertainty systems with quantized feedback. Finally, we will illustrate the effectiveness and reduced conservatism of our main results by a numerical example.

Notations: The symbol $*$ induces a symmetric structure in LMIs. Generally, for a square matrix A, A^T denote its transpose and $He\{A\}$ denotes $(A+A^T)$. Matrices are assumed to have compatible dimensions.

2. Problem statement and preliminaries

Consider the following linear discrete-time system with polytopic uncertainties:

$$x(k+1)=A(\theta)x(k)+B(\theta)u(k)+E(\theta)w(k),$$
$$z(k)=C(\theta)x(k)+D(\theta)u(k)+F(\theta)w(k),$$
$$x(0)=0, \tag{1}$$

where $x(k)\in\mathcal{R}^n$ is the state variable, $u(k)\in\mathcal{R}^m$ is the control input, $z(k)\in\mathcal{R}^q$ is the control output and $w(k)\in\mathcal{R}^v$ is the noise signal that is assumed to be the arbitrary signal in $l_2[0,\ \infty)$. The uncertain matrices $A(\theta)$, $B(\theta)$, $C(\theta)$, $D(\theta)$, $E(\theta)$ and $F(\theta)$ belong to the polyhedron

$$\Omega=\left\{[A(\theta),B(\theta),C(\theta),D(\theta),E(\theta),F(\theta)]=\sum_{i=1}^{r}\theta_i[A_i,B_i,C_i,D_i,E_i,F_i],\sum_{i=1}^{r}\theta_i=1,\theta_i\geq 0\right\} \tag{2}$$

For a given scalar $\gamma>0$, the H_∞ performance of the system (1) is defined to be

$$\sum_{k=0}^{\infty}z(k)^Tz(k)<\gamma^2\sum_{k=0}^{\infty}w(k)^Tw(k). \tag{3}$$

The next lemma is necessary to establish our main results.

Lemma 1 (*Petersen [29]*). *Given matrices Γ, Λ and symmetric matrix Ω, we have that $\Omega+\Gamma F\Lambda+\Lambda^TF^T\Gamma^T<0$, for any $F^TF\leq I$, if and only if there exists a constant scalar $\varepsilon>0$ such that $\Omega+\varepsilon\Gamma\Gamma^T+\varepsilon^{-1}\Lambda\Lambda^T<0$.*

3. Main results

As we know well that for state feedback problem the constant feedback matrix K renders the condition to be conservative when matched with the polytopic uncertainties described in (2). Our main objective is to change K into parameter dependent by Lagrange's interpolation, i.e., by using Lagrange's interpolation estimate, the system parameter θ described in (2) further gives a new control law parameter dependent on the estimation of θ. First, with the quantized error considered, a robust H_∞ control analysis is given, which is based on parameter dependent Lyapunov function. Then, the main result is obtained in terms of LMIs.

Definition 1 (*Meijering [30]*). Given a set of $k+1$ data points $(x_0,y_0),\ldots,(x_j,y_j),\ldots,(x_k,y_k)$, where no two x_j are the same, the interpolation polynomial in the Lagrange form is a linear combination as follows: $L(x)=\sum_{j=0}^{k}y_jl_j(x)$ of Lagrange basis polynomials

$$l_j(x)=\prod_{\substack{0\leq m\leq k\\ m\neq j}}\frac{x-x_m}{x_j-x_m}=\frac{x-x_0}{x_j-x_0}\cdots\frac{x-x_{j-1}}{x_j-x_{j-1}}\frac{x-x_{j+1}}{x_j-x_{j+1}}\cdots\frac{x-x_k}{x_j-x_k},$$

where $0\leq j\leq k$. Note how, given the initial assumption that no two x_j are the same, $x_j-x_m\neq 0$, so this expression is always well-defined.

The pairs $x_i=x_j$ with $y_i\neq y_j$ are not allowed is that no interpolation function L such that $y_i=L(x_i)$ would exists; a function can only get one value for each argument x_j. On the other hand, if also $y_i=y_j$, then those two points would actually be one single point.

For all $j\neq i$, $l_j(x)$ includes the term $(x-x_i)$ in the numerator, so the whole product will be zero at $x=x_i$

$$l_{j\neq i}(x_i)\prod_{m\neq j}\frac{x_i-x_m}{x_j-x_m}=0,$$

on the other hand

$$l_i(x_i)\prod_{m\neq i}\frac{x_i-x_m}{x_j-x_m}=1.$$

From Definition 1 we have the estimate of y_j

$$l_j(x)=\prod_{\substack{0\leq m\leq k\\ m\neq j}}\frac{x-x_m}{x_j-x_m},$$

then

$$L_k(x)=\sum_{j=0}^{k}\prod_{\substack{0\leq m\leq k\\ m\neq j}}\frac{x-x_m}{x_j-x_m}y_j,$$

we have

$$y_j=f(x_i)=L_k(x_j)+R_{n-1}(x_j),\quad R_{n-1}(x_j)=\frac{f^n(x_j)}{k!}\prod_{m=0}^{k}(x_j-x_m) \tag{4}$$

$R_{n-1}(x_j)$ is the error of estimate.

Assume that parameter θ described in (2) is distributed on a curve and a set of data points are known that is $x_0, x_1, \ldots, x_k$; θ_0, θ_1, …, θ_k; $(k<<r)$. Now, we can get the estimation of θ from the known data points by using Lagrange's interpolation described in Definition 1

$$\eta=\sum_{j=0}^{k}\theta_jl_i(x), \tag{5}$$

where $l_i(x)$ is defined in Definition 1.

Then we can use η to design a new parameter dependent control law. For the system (1), a new state feedback control law with quantization is given by

$$u(k)=Q(v(k)), \tag{6}$$

$$v(k)=K(\eta)G^{-1}(\eta)x(k), \tag{7}$$

where

$$K(\eta)=\sum_{j=1}^{r}\eta_jK_j,\quad G^{-1}(\eta)=\left[\sum_{j=1}^{r}\eta_jG_j\right]^{-1}.$$

Here $Q(\cdot)=[Q_1(\cdot)Q_2(\cdot)\cdots Q_f(\cdot)]^T$ is a static time-invariant logarithmic quantizer given by Fu et al. [3]

$$Q_j(y)=\begin{cases}v_i^{(j)} & 0\leq\frac{1}{(1+\delta_j))v_i^{(j)}}<y\leq\frac{1}{(1-\delta_j))v_i^{(j)}}\\ 0 & y=0\\ -Q_j(-y) & y<0\end{cases} \tag{8}$$

$$\delta_j=\frac{1-\rho_j}{1+\rho_j},\quad 0<\rho_j<1,\quad v_i^{(j)}>0. \tag{9}$$

Then, for (7), we can get that for any $v(k)\in R^q$, $|Q(v(k)-v(k)|\leq\delta v(k)$ and $\delta=\delta_1,\delta_2,\ldots,\delta_f$. Therefore, $u(k)=Q(v(k))=(I+\Delta(k))v(k)$, $|\Delta(k)|\leq\ \delta$, where $\Delta=(\Delta_1,\Delta_2,\ldots,\Delta_f)$.

Then, we have the following closed-loop system:

$$x(k+1)=\tilde{A}x(k)+E(\theta)w(k),$$