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Dissipativity and passivity analysis for uncertain discrete-time stochastic Markovian jump neural networks with additive time-varying delays

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ABSTRACT

In this paper, the problem of dissipativity and passivity analysis for uncertain discrete-time stochastic Markovian jump neural networks with additive time-varying delays is investigated. By introducing a triple-summable term in the Lyapunov functional and by applying stochastic analysis technique, the dissipativity and passivity criteria are established for discrete-time neural networks with additive time-varying delays. The reciprocally convex approach is utilized to bound the forward difference of the triple-summable term. The proposed criteria that depend on the upper bounds of the additive time-varying delays are given in terms of linear matrix inequalities, which can be solved by MATLAB LMI Control Toolbox. Two numerical examples are given to demonstrate the effectiveness of the proposed method.

1. Introduction

Neural networks (NNs) have received extensive attention in the past decades due to their extensive applications in a variety of areas, such as signal processing, image processing, optimization problems, pattern recognition, associative memory, model identification, fixed-point computation, and other scientific areas [1–7,18,23,28,35,37,39]. It is worth noting that, for numerical simulation and practical implementation of the continuous-time neural networks, it is necessary and essential to formulate a discrete-time system that is an analogue of the continuous-time system. The discretization may not preserve the dynamics of the continuous-time counterpart even for a small sampling period. Moreover, the connection weights of the neurons are inherent dependent on certain resistance and capacitance values that inevitably bring in uncertainties during the parameter identification process. Therefore, a study on the dynamics of discrete-time neural networks is crucially needed. For this study, many effective methods utilized in standard state-space systems have been extended to time-delay neural networks, for example, the free weighting matrices method [2], the delay-partitioning method [3], the triple-integral technique [4], and the reciprocally convex combination method [5]. Taking this into account, in [43], the author investigated the stability analysis of discrete-time neural networks with delays. In addition, exponential filtering for discrete-time switched neural networks with random delays has been studied in [43]. Recently, finite-time stability analysis of discrete-time neural networks is addressed in [44].

Markov jump systems described by a set of linear systems with commutations generated by a finite-state Markov chain are very appropriate and powerful to model changes induced by external causes, e.g., random faults, unexpected events, and uncontrolled configuration changes [40]. Therefore, the study of Markov jump systems with or without time-delay is of great significance both theoretically and practically, and a lot of relevant results have been reported in the literature over the past decades [22,35,42]. On the other hand, the word “stochastic” means “pertaining to chance” (Greek roots), and is thus used to describe subjects that contain some element of random or stochastic behavior. For a system to be stochastic, one or more parts of the system has randomness associated with it. Unlike a deterministic system, for example, a stochastic system does not always produce the same output for a given input. A few components of systems that can be stochastic in nature include stochastic inputs, random time-delays, noisy (modeled as random) disturbances, and even stochastic dynamic processes. A stochastic process is one whose behavior is non-deterministic, in which a system's subsequent state is determined both by the process's predictable actions and by a random element [40,41]. Additionally, in real neural networks, synaptic transmission is an

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inherent noisy process induced by random fluctuations along with released factors from neurotransmitters or other probabilistic causes.

Dissipativeness was initially introduced by Willems (1972) in terms of an inequality involving the storage function and supply rate. Dissipativity theory provides a fundamental framework for the analysis and design of control systems using input–output description based on system energy related considerations. Dissipativity theory is an important idea which has been used in many areas of sciences and control engineering. This provides strong connection between Physics, system theory and control engineering. Dissipativity has proven to be essential and very useful tool for control applications like robotics, active vibration damping, electromechanical systems, combustion engines, circuit theory, and for control techniques like adaptive control, and inverse optimal control [10,32]. The main idea behind this is that many important physical systems have certain input–output properties related to the conservation, dissipation and transport of energy. Therefore, dissipativity analysis of the dynamical systems has become an active area of research in both theoretical and practical point of view.

Passivity, as a particular case of dissipativity, was introduced in [11] and later generalized in [12]. The passivity theory, intimately related to the circuit analysis method [13], means that the systems cannot generate more energy than what they absorb. That is to say, the passive property can keep a system internally stable, and thus play a critical role in the analysis of the stability of dynamical systems for nonlinear control and other research fields. In [25], the author investigated dissipativity analysis for discrete stochastic neural networks with Markovian delays and partially known transition matrix. The problem of dissipativity of discrete-time neural networks with time delay has been investigated in [27]. In [45], robust passivity analysis of neural networks with discrete and distributed delays is addressed. Over the past decade, the problems of dissipativity and passivity analysis for continue-time neural networks and discrete-time neural networks have been extensively studied and many dissipativity and passivity conditions have been reported [21–26,36,38]. However, to the best of our knowledge, there is no work in the available existing literature that considers the problem of dissipativity and passivity analysis for uncertain discrete-time stochastic Markovian jump neural networks with additive time-varying delays. Thus, the main purpose of the present research is to linkage such a gap by making the first attempt to deal with the dissipativity analysis problem for discrete-time stochastic neural networks with additive time-varying delays. Moreover, in [22] the author investigated the problem of passivity analysis for discrete-time stochastic Markovian jump neural networks with mixed time delays. In addition, the problem of stochastic dissipativity analysis on discrete-time neural networks with time-varying delays is addressed in [26].

Meanwhile, time delay is one of the most important parameters in delayed neural networks. In hardware implementation of neural networks, time delay is an unavoidable factor due to finite switching speed of the amplifiers and communication time. The existence of time delay may affect dynamic behaviors such as oscillation, instability, divergence, manufacturing systems, telecommunication and economic systems, and is a major cause of instability and poor performance of neural networks [6]. In networked systems, signals transmitted from one point to another may experience two segments of networks, and the resulting time delays have different properties due to variable network transmission conditions. This model has a physically powerful application background in remote control and networked control. Followed by this, the network-induced delay can be represented as the sum of two additive time-varying delay components [7,8,19,20]. For NNs with two additive time-varying delay components, a stability criterion is presented in [9] by using the free-weighting matrix method. In this paper, we consider d_{k2} as the time-delay induced from sensor to controller and d_{k2} as the delay induced from controller to the actuator. The stability analysis of such system was earlier carried out by adding up all the successive delays into single delay (i.e. $d_{k1}+d_{k2}=d(k)$) to develop sufficient dissipativity and passivity conditions. Taking this into account, in this paper, we handle both lower and upper bounds of the additive delays (i.e. $0 \le d_{11} \le d_{k1} \le d_{12}$ and $0 \le d_{21} \le d_{k2} \le d_{22}$) for obtaining the dissipativity and passivity results. Furthermore, time delay in the leakage term also has great impact on the dynamics of neural networks because time delay in the stabilizing negative feedback term has a tendency to destabilize a system. In practice, the leakage delay is not a constant, so we ought to consider the neural networks with time-varying leakage delay. It is worth noting that, in most of the available existing literatures, only continuous-time neural networks with leakage delay have been studied [29,30]. However, it appears that very little attention is devoted to the investigation of dissipativity and passivity for discrete-time neural networks with time-varying leakage delay. Therefore, it is necessary to further investigate the dissipativity and passivity problem for neural networks with both leakage delay and stochastic effects.

Motivated by the above discussions, in this paper, the problem of dissipativity and passivity analysis of discrete-time stochastic neural networks with additive time-varying delays is considered. By construction of newly augmented Lyapunov–Krasovskii functional and utilization of reciprocally convex combination approach [27] employed to bound the forward difference of a triple-summable term, a sufficient condition is established to ensure the $(\mathcal{Q},\mathcal{S},\mathcal{R})-\gamma$ -dissipativity and passivity criteria which depends on the additive time-varying delays. Such condition is demonstrated in terms of linear matrix inequalities (LMIs) to guarantee the dissipativity and passivity conditions of delayed neural networks, which can be easily checked by MATLAB-LMI toolbox. Later, based on the results of Theorem 3.1, passivity criteria for discrete-time stochastic neural networks with additive time-varying delays have been introduced in Corollary 3.5. Finally, the effectiveness and advantages of the derived results are demonstrated by two illustrative examples.

The rest of this paper is organized as follows. Section 2 formulates the problem under consideration. Dissipativity and passivity analysis for uncertain discrete-time stochastic Markovian jump neural networks with additive time-varying delays are presented in Section 3. Illustrative examples and their simulation results for dissipativity have been given in Section 4. Finally conclusions are drawn in Section 5.

Notations: Throughout this paper, I represents the unitary matrix with appropriate dimensions, $\mathbb{N}$ denotes the set of non-negative integers. $\mathbb{R}^n$ stands for the n-dimensional Euclidean space, and $\mathbb{R}^{m\times n}$ denotes the set of all $m\times n$ real matrices. The notation $X>0$ (respectively, $X\ge 0$), for $X\in\mathbb{R}^{n\times n}$ means that the matrix X is a real symmetric positive definite (respectively, positive semi-definite), X^T represents the transpose of matrix X, we use an asterisk ($*$) to represent a term that is induced by symmetry. The matrix $0_{n,m}$ denotes the null matrix of order $n\times m$. $(\Omega,\mathcal{F},\mathcal{P})$ is a probability space, Ω is the sample space, $\mathcal{F}$ is the σ-algebra of subsets of the sample space Ω. $\mathcal{P}$ is the probability measure on $\mathcal{F}$, $\mathcal{E}[\cdot]$ denotes the expectation operator with respect to some probability measure $\mathcal{P}$. For integers a and b with $a<b$, let $\mathbb{N}[a,b]=\{a,a+1,\ldots,b-1,b\}$. $l_2[0,+\infty)$ denotes the space of square integrable vector functions over $[0,\infty)$.

2. Problem description and preliminaries

Throughout this paper, let $r(k)(k\ge 0)$ be a Markov chain taking values in a finite state space $\mathfrak{S}=\{1,2,\cdots s\}$ with probability