Neurocomputing 174 (2016) 921–927

Contents lists available at ScienceDirect

Neurocomputing

journal homepage: www.elsevier.com/locate/neucom

Non-Gaussian noise quadratic estimation for linear discrete-time time-varying systems

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ARTICLE INFO

Article history:
Received 21 July 2015
Received in revised form
26 September 2015
Accepted 2 October 2015
Communicated by Ma Lifeng Ma
Available online 22 October 2015

Keywords:
Input noise quadratic polynomial estimation
Kronecker algebra
Deconvolution filter
Fixed-lag smoother

ABSTRACT

This study deals with the input noise quadratic polynomial estimation problem for linear discrete-time non-Gaussian systems. The design of the non-Gaussian noise quadratic deconvolution filter and fixed-lag smoother is firstly converted into a linear estimation problem in a suitable second-order polynomial extended system. By employing the Kronecker algebra rules, the stochastic characteristics of the augmented noise in the augmented system are discussed. Then a solution to the non-Gaussian noise quadratic estimator is obtained through applying the projection formula in Kalman filtering theory. In addition, the stability is proved by constructing an equivalent state-space model with uncorrelated noises. Finally, a numerical example is given to show the effectiveness of the proposed method.

1. Introduction

The input noise estimation (also known as deconvolution) has a rich history and a wide range of applications in image restoration, oil exploration, speech signal processing, fault detection and so on [1–4]. The task of the deconvolution problem is to estimate the intended unknown input noise of a system by utilizing the obtainable outputs. For the first time, an optimal white noise smoother with application to seismic data processing in oil exploration was presented in [2]. Applying the polynomial approach in frequency domain, the optimal deconvolution estimator was derived based on spectral factorization in [5]. Later, both input and measurement white noise estimators were designed by using the modern time series analysis method in [6]. Recently, the deconvolution theory was successfully applied to the multi-sensor linear discrete time systems [7,8] and the systems with packet dropouts [9–11]. Note that the above results were obtained based on the input Gaussian noise assumption, however, in many important technical areas the input noise is non-Gaussian (see for instance [12–14]). This is the motivation to develop a new algorithm which permits us to find a satisfactory non-Gaussian noise estimator for linear discrete-time time-varying systems.

The estimation problem for non-Gaussian systems has received more and more attention and some fundamental results have been developed, refer to [12,15] and the references therein. For linear non-Gaussian systems, the conditional expectation giving the minimum mean square error estimate is an infinite dimensional problem, and its solution cannot be easily numerically computed [15]. Although the Kalman filter is the best affine estimator for the non-Gaussian case, its estimated accuracy is inadequate in some cases. Note that the polynomial filtering algorithm [12,15], which employ both the observations of the original system and their Kronecker products, is more accurate than the classical Kalman filter, while maintaining the characteristics of easy calculability and recursivity. Therefore, an increasing number of authors have focussed on the polynomial estimator design for the non-Gaussian systems. The pioneer work can be traced back to the recursive arbitrary-degree finite-memory polynomial estimator design via the classical Kalman filtering theory [12]. Later, the result was successfully extended to polynomial filter for stochastic bilinear systems [16] and polynomial extended Kalman filter [17]. When the state-space model was unknown, the fixed-point, fixed-interval and fixed-lag smoothers from uncertain observations were presented based on the covariance information of the processes in [18]. Recently, this method was applied to the study of multi-sensor information fusion quadratic filter for linear systems with uncertain observations [19]. However, these works have a limitation that the Non-Gaussian noise polynomial estimator was not investigated.

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http://dx.doi.org/10.1016/j.neucom.2015.10.015
0925-2312/

In this paper, we will investigate the non-Gaussian noise quadratic estimation problem for linear discrete-time time-varying systems. The linear recursive estimator, the non-Gaussian noise quadratic deconvolution filter and the non-Gaussian noise quadratic fixed-lag smoother are proposed. The stability of the non-Gaussian noise quadratic estimator is also discussed. Although the deconvolution estimation has been well studied, the non-Gaussian noise quadratic estimation is still difficult since the stochastic characteristics analysis problem for the second-order polynomial extended system involves the Kronecker product. To solve this problem, some Kronecker algebra rules constituting a powerful tool in treating vector polynomials are adopted in this paper. The main contribution of this paper can be summarized as follows: (i) it extends the polynomial filtering methodology to the input noise estimation of the linear discrete-time time-varying systems and (ii) it develops a recursive Kalman-like input noise quadratic estimator with more accurately.

The remainder of this paper is arranged as follows. The linear discrete-time non-Gaussian systems and the least mean-squared error second-order polynomial estimation problem are introduced in Section 2. In Section 3, the linear recursive estimator is developed by using Kalman filtering theory. In Section 4, the non-Gaussian noise quadratic deconvolution filter and the non-Gaussian noise quadratic fixed-lag smoother are derived by calculating the extended Riccati difference equation. The stability analysis of the non-Gaussian noise quadratic estimator is proposed in Section 5. And an example is provided to prove the effect of the presented estimator in Section 6. Finally, the conclusions are proposed in Section 7.

2. System model and problem formulation

We consider the following class of linear discrete-time systems:

$$x(k+1)=A(k)x(k)+F(k)N(k),\quad x(0)=\bar{x} \tag{1}$$

$$y(k)=C(k)x(k)+G(k)N(k) \tag{2}$$

$$z(k)=L(k)N(k) \tag{3}$$

where $x(k)\in R^n$ is the state, $y(k)\in R^m$ is the measurement output, $z(k)\in R^q$ is the signal to be estimated, and the noise $N(k)\in R^r$ forms a sequence of non-Gaussian random vector variables, with all moments up to the fourth order finite and known:

$$E(N(k))=0,\quad E(N^{[i]}(k))=\Psi_{N,i},\quad i=2,3,4. \tag{4}$$

Moreover, without loss of generality, we assume that

$$st^{-1}\Psi_{N,2}=E(N(k)\cdot N^T(k)) \tag{5}$$

The sequence $\{N(k)\}$ forms, with the initial state random vector $\bar{x}$, a family of independent random variables. Also, the initial state $\bar{x}$ is endowed with statical moments, namely

$$E(\bar{x})=0 \tag{6}$$

$$E(\bar{x}^{[i]})=\Psi_{x,i},\quad i=2,3,4. \tag{7}$$

The non-Gaussian noise quadratic estimation problem for the system model (1)–(3) can be stated as

Problem 1. Given an integer $d\geq 0$ and the observation sequence $\{\{y(s)\}_{s=0}^{k+d}\}$, find a least mean-squared error second-order polynomial estimator $\hat{z}(k|k+d)$ of $z(k)$.

Note that the above estimation problem includes two cases, i.e. $d=0$ and $d>0$ which correspond to the cases of non-Gaussian noise quadratic filtering estimate and non-Gaussian noise quadratic fixed-lag smoothing estimate, respectively.

3. The linear recursive estimator

Let us find the linear recursive estimator for system (1)–(3). By using the classical Kalman filtering theory, we state the linear deconvolution filtering of $z(k)$ in the following lemma.

Lemma 1. *Consider the system (1)–(3) under the assumptions (4)–(7). Then the linear deconvolution filter $\hat{z}(k|k)$ of $z(k)$ is given by*

$$\hat{z}(k|k)=L(k)(st^{-1}\Psi_{N,2})G^T(k)R_{\tilde{y}}^{-1}(k)\tilde{y}(k) \tag{8}$$

where

$$\tilde{y}(k)=y(k)-C(k)\hat{x}(k|k-1) \tag{9}$$

$$\hat{x}(k+1|k)=A(k)\hat{x}(k|k-1)+K_0(k)\tilde{y}(k) \tag{10}$$

$$K_0(k)=\Big(A(k)P_0(k)C^T(k)+F(k)(st^{-1}\Psi_{N,2})G^T(k)\Big)R_{\tilde{y}}^{-1}(k) \tag{11}$$

$$R_{\tilde{y}}(k)=C(k)P_0(k)C^T(k)+G(k)(st^{-1}\Psi_{N,2})G^T(k) \tag{12}$$

$$P_0(k+1)=A(k)P_0(k)A^T(k)+F(k)(st^{-1}\Psi_{N,2})F^T(k)-K_0(k)R_{\tilde{y}}(k)K_0^T(k),\quad P_0(0)=st^{-1}\Psi_{x,2} \tag{13}$$

Furthermore, we present the linear fixed-lag smoother of $z(k)$ in the following lemma.

Lemma 2. *Consider system (1)–(3) under the assumptions (4)–(7). Then, for a given integer $d>0$, a linear fixed-lag smoother $\hat{z}(k|k+d)$ of $z(k)$ is given by*

$$\hat{z}(k|k+d)=\hat{z}(k|k)+L(k)\sum_{j=1}^{d}\Gamma_{k+j}^{k}C^T(k+j)R_{\tilde{y}}^{-1}(k+j)\tilde{y}(k+j) \tag{14}$$

where

$$\Gamma_{k+j+1}^{k}=\Gamma_{k+j}^{k}[A(k+j)-K_0(k+j)C(k+j)]^T,\quad j=1,2,\ldots,d-1 \tag{15}$$

with

$$\Gamma_{k+1}^{k}=(st^{-1}\Psi_{N,2})F^T(k)-(st^{-1}\Psi_{N,2})G^T(k)K_0^T(k) \tag{16}$$

Besides, $\hat{z}(k|k)$, $\tilde{y}(k+j)$, $K_0(k+j)$ and $R_{\tilde{y}}(k+j)$ are computed by (8), (9), (11) and (12), respectively.

4. The quadratic recursive estimator

4.1. The extended state-space model

In order to obtain the estimator for system (1)–(3), let us define the following extended vector:

$$X_e(k)=\begin{bmatrix} x(k) \\ x^{[2]}(k) \end{bmatrix} \tag{17}$$

According to Eqs. (1), (4) and (5), we have

$$\begin{aligned} x^{[2]}(k+1)&=x(k+1)\otimes x(k+1)=[A(k)x(k)+F(k)N(k)]\\ &\quad\otimes[A(k)x(k)+F(k)N(k)]=A^{[2]}(k)x^{[2]}(k)+(A(k)x(k))\\ &\quad\otimes(F(k)N(k))+(F(k)N(k))\\ &\quad\otimes(A(k)x(k))+F^{[2]}(k)N^{[2]}(k)=A^{[2]}(k)x^{[2]}(k)+d(k)+f(k) \end{aligned} \tag{18}$$