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An adaptive scheme for chaotic synchronization in the presence of uncertain parameter and disturbances

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ABSTRACT

Recently, several schemes have been proposed in the literature to synchronize chaotic systems. However, in most of these approaches, the presence of uncertain parameters and external disturbances were not considered. Motivated by the above consideration, this paper proposes an adaptive methodology to synchronize any chaotic system with unified chaotic systems, even if bounded disturbances are present. The proposed controller is composed of both variable proportional and adaptive control actions for guaranteeing the convergence of the residual synchronization error to zero in the presence of disturbances. Two possible modifications are considered: 1) only adaptive control action is implemented to overcome the well-known assumption of prior knowledge of upper bounds to compensate for the disturbances, and 2) the control gain of the proportional part is saturated, when the residual synchronization error has, practically, been removed. Lyapunov theory, in combination with Barbalat's Lemma, is used to design the proposed controller. Experimental simulations are provided to show the effectiveness of the proposed controller and its advantages, when compared with a recent work in the literature.

1. Introduction

Encouraged by the discovery of the chaotic dynamics by Lorenz in 1963 [1], chaos synchronization has been studied over the last 30 years by several researchers [2–12]. This is due to its potential applications in numerous engineering problems ranging from living system applications [5] to non-living system applications [6]. For instance, chaos synchronization has been applied in electrical [7,8], biological [9], chemical [10], secure communication [11], and finance systems [12].

After Lorenz' model, several other chaotic systems such as Chua [13,14], Rössler [15,16], Lü [17,18], Chen [19,20], Liu [21,22], finance [23], unified [24], etc. have been proposed, and a great number of techniques, such as linear, nonlinear, passivity based, adaptive, backstepping, and sliding control, among others, have been introduced to achieve its synchronization. See, for instance, [25–29] and the references therein.

Despite the large number of existing techniques, few papers have been devoted to the chaotic synchronization in the presence of uncertain parameters and bounded disturbances. In most of these studies [29–36], the main particularity is that the control law is not smooth, since the control law depends on a sign function which is discontinuous [30–34,36]. Also, the parameter adaptation law is not robust, because it lacks a leakage term [30,31–34,36]. Moreover, the control law uses the time derivative of the synchronization error [29], the master system parameters [29] and disturbances [35], and it ensures that the synchronization error is only uniformly bounded in the presence of uncertain parameters and disturbances [36].

For instance, in [29], a chatter free sliding mode control strategy for chaos control and synchronization was introduced. Basically, by using a particular class of dynamic sliding mode surface, the discontinuous sign function was transferred to the first derivative of the control input. Hence, the chattering phenomenon was alleviated. However, the price paid is the time derivative of the synchronization error used in the control law. The time-differentiation of signals may not be desirable, mainly in the presence of high frequency disturbances. In [31], a robust adaptive sliding mode controller was proposed. The control input was based on a discontinuous feedback where the parameters were adapted by laws without leakage terms. A remarkable feature of this approach is that disturbances are assumed to have unknown bounds. In [33], robust adaptive modified function projective synchronization between different hyperchaotic systems was studied. This strategy was based on a discontinuous control input and integral parameter adaptive laws without any leakage modification.

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Unfortunately, in spite of the relevance of the above mentioned works [29–36], they have some limitations. It is well known that the discontinuous feedback control raises theoretical and practical issues [37]. From the theoretical point of view, standard existence and uniqueness of solutions of differential equations are, in general, not applicable [38]. Furthermore, the validity of the Lyapunov analysis will have to be examined in a framework that does not require the right-hand side of the state equation to be locally Lipschitz, in the state variables, and piecewise continuous with respect to time [37–38]. Practical issues are associated with the imperfections of switching devices and delays leading to chattering, which result, for example, in low control accuracy and high heat losses in electrical power circuits [37]. On the other hand, it has been known since the early 1980s that nonrobust adaptive laws may suffer from parameter drift phenomenon [39], that is, the parameters drift to infinity with time. It is often due to the "pure" integral action of the adaptive law. Several leakage modifications to counteract this have been proposed since then [39,40]. Finally, it is a basic rule in chaos synchronization based cryptography that the details of the encryption algorithm are always known by the attacker [41]. Hence, the use of master system parameters in the control law is controversial.

On the other hand; recently, a large number of studies have focused on synchronization of unified chaotic systems for applications in secure communications [42]. The unified chaotic system is a three dimensional system that has a broad spectrum of chaotic behavior, which is associated with a scalar parameter used in its model. As the unified system can display hyperchaos [43] and the master system parameter can be used as a modulator element in chaotic parameter modulation schemes [11], it seems adequate for chaotic communication applications, and, hence, this chaotic system will be employed to show the proposed design methodology.

Motivated by the aforementioned facts, in this paper we propose a new scheme to synchronize any chaotic systems with unified chaotic systems. Depending on the choice of some design parameters, the proposed controller can be used: 1) to synchronize any chaotic system with unified chaotic systems without a well-known assumption of prior knowledge of an upper bound on the disturbance; 2) to ensure that the synchronization error converges to a desire arbitrary neighborhood of the origin. The proposed controller has advantageous features, when compared with previous works [29–36], since it is smooth, it does not require the master parameter for implementation purposes, and it uses a e-modification robust adaptive law [39] for adjusting the unknown parameter. Hence, it does not present chattering or parameter drift. The design methodology is based on Lyapunov theory and Barbalat's Lemma [39] and ensures the convergence of the synchronization error to zero, even in the presence of uncertain system parameter and bounded disturbances.

This paper is organized as follows. We begin with the problem formulation in Section 2, where the class of chaotic models, principal assumptions, synchronization problem, and synchronization error equation are established. Section 3 presents the main result of the paper, an adaptive controller which ensures the convergence of the synchronization error to zero, even in the presence of bounded disturbances, and their stability and convergence properties are established. Three examples are provided to validate the theoretical results and to illustrate the application of the proposed strategy in Section 4. Finally, the main conclusions of the work are presented in Section 5.

2. Problem formulation

Consider the problem of control chaotic systems described by the following differential equation

$$\dot{x}_s = A(\beta)x_s + f_s(x_s) + d_s(x_s,t) + u \tag{1}$$

where $x_s \in \Re^3$ is the state of the slave system, $u \in \Re^3$ is the control input, $f_s(.)$ is a known map, $d_s(.)$ is an unknown disturbance, β is a known parameter,

$$A(\beta) = \begin{bmatrix} -25\beta-10 & 25\beta+10 & 0 \\ 28-35\beta & 29\beta-1 & 0 \\ 0 & 0 & -\frac{8+\beta}{3} \end{bmatrix} \tag{2}$$

and

$$f_s(x_s) = \begin{bmatrix} 0 \\ -x_{s1}x_{s3} \\ x_{s1}x_{s2} \end{bmatrix} \tag{3}$$

We assume that the following can be established.

Assumption 1. : The right-hand side of (1) is piecewise continuous with respect to time and locally Lipschitzian with respect to x_s, such that (1) has a unique solution globally in time for any given initial condition.

Assumption 2. : On the region $\Re^3 \times [0,\infty)$

$$\|d_s(x_s,t)\| \le d_{s0} \tag{4}$$

where d_{s0} is a positive constant, such that $d_{s0} < \overline{d}_s$ and $\overline{d}_s$ is a known constant.

Remark 1. : Assumption 2 is usual in synchronization of chaotic systems.

Remark 2. : In the case that $\beta=0$, $\beta=0.8$, and $\beta=1$, system (1) becomes the Lorenz, Lü, and Chen systems, respectively, when perturbations are not present. However, any chaotic system of the form $\dot{x}_s = F(x_s) + \overline{d}_s(x_s,t) + u$ can be posed in the form (1), where $d_s(x_s,t) = F(x_s) + \overline{d}_s(x_s,t) - A(\beta) - f_s(x_s)$.

Remark 3. : It should be noted that the conditions of existence and uniqueness of solutions of (1), introduced in Assumption 1, are a prerequisite to the Lyapunov-type arguments to be used in the stability analysis. Basically, it is necessary to show that the state trajectories do not escape to infinity in finite time [38].

We consider the master system as

$$\dot{x}_m = A(\alpha)x_m + f_m(x_m) + d_m(t,x_m) \tag{5}$$

where $x_m \in \Re^3$, α is a unknown parameter, $d_m(.)$ is an unknown disturbance,

$$A(\alpha) = \begin{bmatrix} -25\alpha-10 & 25\alpha+10 & 0 \\ 28-35\alpha & 29\alpha-1 & 0 \\ 0 & 0 & -\frac{8+\alpha}{3} \end{bmatrix} \tag{6}$$

and

$$f_m(x_m) = \begin{bmatrix} 0 \\ -x_{m1}x_{m3} \\ x_{m1}x_{m2} \end{bmatrix} \tag{7}$$

Assumption 3. : The parameter α is upper bounded by a known positive constant $\overline{\alpha}$, such that $\overline{\alpha} > \alpha$.

Assumption 4. : On the region $\Re^3 \times [0,\infty]$

$$\|d_m(x_m,t)\| \le d_{m0} \tag{8}$$

where d_{m0} is a positive constant, such that $d_{m0} < \overline{d}_{ms}$ and $\overline{d}_{ms}$ is a known constant.