Adaptive fuzzy output-feedback control for a class of nonlinear switched systems with unmodeled dynamics

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In this paper, an adaptive fuzzy output tracking control approach is proposed for a class of uncertain nonlinear switched systems with unmeasured states, unknown nonlinear functions, unmodeled dynamics, and dynamical disturbances. In the control design, fuzzy logic systems are used to approximate the unknown switched nonlinear functions and a switched fuzzy state observer is designed for estimating the unmeasured states. The problem of unmodeled dynamics and dynamic disturbance are solved by introduce a modified dynamical signals and nonlinear damping terms into the backstepping design procedure. Under the framework of backstepping control design, fuzzy adaptive control and average dwell time method, an adaptive fuzzy output-feedback tracking control approach is developed. It is proved that the proposed control approach can guarantee that all the signals in the closed-loop switched system are bounded and the tracking error remains an adjustable neighborhood of the origin. A numerical example is provided to illustrate the effectiveness of the proposed approach.

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1. Introduction

In recent years, with the help of fuzzy logic systems and backstepping technique many adaptive backstepping control design approaches has been developed to deal with the problem of uncertain nonlinear systems in strict-feedback from [1–8]. Adaptive backstepping control design approaches are developed in [1–3] for single-input and single-output (SISO) nonlinear systems, and [4,5] for multiple-input and multiple-output (MIMO) nonlinear systems. The authors in [6–8] proposed an adaptive fuzzy output tracking control approach for SISO or MIMO nonlinear systems with immeasurable states. The developed adaptive backstepping control design approaches not only can solve the control design problem of uncertain nonlinear systems which do not satisfy the matching conditions, but also can give the methodology of systematically constructing both feedback control laws and associated Lyapunov functions. Therefore, it generally utilized to design the controller for a large class of uncertain nonlinear systems.

It should be mentioned that the aforementioned control approaches in [1–8] did not consider the problem of unmodeled dynamics and dynamic disturbances. In practice, unmodeled dynamics and dynamic disturbances are frequently appears in many practical nonlinear systems, and they often degrade system control performance and even results in instability of the control system. Therefore, the control design for nonlinear systems with unmodeled dynamics and dynamic disturbances is very important in control theory and practical applications. In past decades, base on the adaptive backstepping control design technique, some design methods have been proposed for several classes of nonlinear systems with unmodeled dynamics and dynamic disturbances [9–15]. The authors in [9–11] proposed some adaptive backstepping design procedures for nonlinear system with unmodeled dynamics and dynamic disturbances. These approaches introduced nonlinear damping terms and dynamic signal to counteract the uncertain nonlinear functions and dominate the dynamic disturbances, respectively. In [12], an adaptive output feedback control approach is developed by combined backstepping technique with small-gain theory to construct a global adaptive output feedback controller to ensure robustness with respect to the unmodeled dynamics and nonlinear disturbances. In [13–15], the dynamical signal techniques combined with changing supply function methods are incorporated into the backstepping recursive design technique. However, these control approaches in [9–15] are used to control non-switched nonlinear systems and cannot be applied to switched nonlinear systems.

Switched systems are dynamical systems consisting of a collected of continuous-time subsystems and a switching rule that orchestrates the switching among them. In the last decades, switched systems have attracted more and more attention due to their significance in the modeling of many engineering applications, such as chemical processes, robot manipulators and power systems. So far, many
remarkable achievements have been achieved on the controller design and stability analysis on switched systems, see, e.g., [17–20] and the references therein. Recently, some backstepping control design methods have been proposed for several classes of switched nonlinear systems [21–23]. Three state feedback control approaches in [21–23] have investigated based on the common Lyapunov function method for a class of switched nonlinear systems under arbitrary switchings. [24] has proposed an adaptive neural network control for a class of switched nonlinear systems with completed unknown nonlinear functions, but this approach needs construct switching law, that is the switching in [24] is not arbitrary. In [25], a novel adaptive neural network control method has been presented for a class of switched nonlinear systems with switching jumps and uncertainties in both system models and switching signals, the condition about switching signal’s dwell-time property need to be satisfied in [25]. However, the aforementioned controlled systems do not consider the problems of unmodeled dynamic and dynamic disturbances.

Motivated by the above observations, an adaptive fuzzy output tracking control approach is presented for a class of SISO switched nonlinear systems with completed unknown nonlinear functions, unmeasured states, unmodeled dynamics, and dynamical disturbances. It is shown that the proposed adaptive control method can guarantee that all the signals in the closed-loop systems are bounded, and the tracking error converges to a small neighborhood of the origin.

Compared with the previous literature, the main contributions of this paper are as follows: (i) by employing fuzzy logic systems to approximate unknown nonlinear functions, and through the introduction a modified dynamical signals to solve the problem of unmodeled dynamics and dynamic disturbance, the adaptive fuzzy control method has been developed, this is different from the existing works, where the unmodeled dynamics is neglected in [24,25]; (ii) by designing a fuzzy state observer, the proposed adaptive control method can cancel the restrictive condition in [22–25] that all the states of the controlled nonlinear systems are available for measurement; and (iii) by incorporating the dynamical signal into the average dwell time method, the proposed adaptive control method can solve the problem of switched systems, and has the robustness to the unmodeled dynamics.

2. Problem formulations and preliminaries

2.1. System descriptions and assumptions

Consider the following uncertain nonlinear switched systems with unmodeled dynamics and dynamical disturbances:

\[
\begin{align*}
\dot{z} &= q_{\alpha}(z, y) = x_2 + f_{\alpha(1,1)}(x_1) + A_{\alpha(1,1)}(z, y) \\
x_1 &= x_{i+1} + f_{\alpha(1,1)}(x_0) + A_{\alpha(1,1)}(z, y), i = 2, \ldots, n - 1 \\
x_n &= u + f_{\alpha(1,1)}(x_0) + A_{\alpha(1,1)}(z, y) \\
y &= x_t
\end{align*}
\]

where \((x_1, \ldots, x_n) \in \mathbb{R}^n, u \in \text{Rand} \ y \in \mathbb{R} \) are the state vector, the input and output of the system, respectively. \(\alpha(t) : [0, \infty) \rightarrow \mathbb{Z} = \{1, 2, \ldots, N\} \) is a piecewise constant function called switching signal (or law), which takes values in the compact set \(\mathbb{Z} \). When \(t \in [t_i, t_{i+1}), \alpha(t) = k \), then we say the \(k\)th subsystem is active and the remaining subsystems are inactive. \(f_{\alpha}(x)(k \in \mathbb{Z}, j = 1, \ldots, n) \) is an unknown smooth function. \(A_{\alpha}(z) \) are the nonlinear dynamic disturbances, \(z \in \mathbb{R}^n \) is unmodeled dynamics. Throughout this paper, it is assumed that \(A_{\alpha}(z) \) is not measured.

The control objective of this paper is to design adaptive fuzzy controller for switched system with unmodeled dynamics (1), under a class of switching signals with average dwell time such that: (i) all signals in the closed-loop system are remain bounded and (ii) the observer error and the tracking error \(x_1 = y - y_1, (y, \dot{y}) \) is the given reference signal and all the derivative of \(y_1 \) are bounded can converge to a small neighborhood of the origin.

To facilitate the control system design, we need the following assumptions and lemmas:

**Assumption 1.** [9–11] The unmodeled dynamics is exponentially-input-to-state practically stable (exp-IcPs); i.e., the systems \(z = q_{\alpha}(z, y) \) has exp-IcPs Lyapunov function \(V_\alpha(z) \), which satisfies

\[
\begin{align*}
\alpha_1 \ |z| &\leq V_\alpha(z) \leq \alpha_2 (|z|) \\
\dot{V}_\alpha(z) &\leq -\alpha_0 V_\alpha(z) + \gamma(y) + d_0
\end{align*}
\]

where \(\alpha_1, \alpha_2, \gamma, \alpha_0 \), and \(d_0 \) are of class \(\kappa_{\infty}\)-functions, \(\alpha_0 \) and \(d_0 \) are known positive constants.

Based on concept of input-to-state practically stable, a dynamical signal was introduced by [16] as

\[
\tau = -\tau_0 + \gamma(y_1) + d_0
\]

where \(\gamma(y_1) = x_2^T \gamma(x_2) \), \(\gamma(y_1) \) is a known smooth function and \(\tau_0 \in (0, \alpha_0) \) is a known positive constant.

Following [10], Ref. [11] gave a modified dynamical signal as

\[
\tau = \begin{cases} 
-\tau_0 + x_1^T \gamma(x_1) + d_0, & \text{if } m(t) \geq 0 \\
0, & \text{if } m(t) < 0
\end{cases}
\]

where \(m(t) = -\tau_0 + x_1^T \gamma(x_1) + d_0 \).

**Lemma 1.** [9,11] If \(V_\alpha(z) \) is an exp-IcPs Lyapunov function for a control system \(z = q_{\alpha}(z, y) \), i.e., Eqs. (2) and (3) hold, then for any constants \(\tau_0 \in (0, \alpha_0) \), initial condition \(z_0 = z(0) \), and \(r > 0 \), for any function \(\gamma(y) \), there exist a finite \(T_0 = T_0(\tau_0, r, \alpha_0, \gamma_0(\xi_1)) \), a non-negative function \(D(t, \xi) \) defined for all \(t \geq \tau_0 + T_0 \) and a dynamical signal described by (5) such that \(D(t, \xi) = 0 \) for all \(t \geq \tau_0 + T_0 \) and \(V_\alpha(z) \leq r(t) + D(t, \xi) \)

for all \(t \geq \tau_0 \) where the solutions are defined. In this paper, we adopt the modified dynamical signal [10].

**Assumption 2.** [10,11] There exists the unknown positive constants \(\eta_{k,il} \) and \(f_{k,il} \). The dynamical disturbances \(A_{\alpha}(1 \leq i \leq n) \) are bounded by a polynomial-type nonlinearities and \(z \), such that

\[
\begin{align*}
|\Delta_{k,il}| &\leq \sum_{l=1}^{h_i} \eta_{k,il} |z|^l + \frac{f_{k,il}}{y} |y|^l
\end{align*}
\]

where \(h_i \) is a known integer, defined as \(h_i = \max \{h_i \} \).

**Lemma 2.** [26,27] A switched nonlinear system (1) is called to have a switching signal \(\alpha(t) \) with average dwell time \(T_\circ \) if there exist two positive numbers \(N_\circ \) and \(T_\circ \) such that

\[
N_\circ(T, t) \leq N_\circ(T, t) \leq T_\circ, \forall T \geq 0
\]

where \(N_\circ(T, t) \) is the number of switches occurring in the interval \([t, T]\).

2.2. Fuzzy logic systems

We introduce the fuzzy logic systems [10,13,28]. A fuzzy logic system (FLS) consists of four parts: the knowledge base, the fuzzifier, the fuzzy inference engine working on fuzzy rules, and the defuzzifier. The knowledge base for FLS comprises a collection of fuzzy if-then rules of the following form:

\[
R^i : \text{If } x_1 \text{ is } F^i_1 \text{ and } x_2 \text{ is } F^i_2 \text{ and ... and } x_n \text{ is } F^i_n
\]

Then \(y = C^i, \ i = 1, 2, \ldots, N \).