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Distributed reference model based containment control of second-order multi-agent systems ☆

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ABSTRACT

This paper addresses the distributed containment control problem in a group of agents governed by second-order sampled-data dynamics with directed network topologies. Based on the assumption on the communication topology between agents, a distributed reference model based containment controller is adopted to drive the followers to the convex hull spanned by leaders. By studying the performance of the reference models connected in a distributed network and discussing the limiting behavior of agents by treating the effects of error states of reference models as disturbances in a tracking problem, it can be proved that the containment control problem can be solved by the proposed controller with appropriate sample period and control parameters. Finally, numerical examples are used to illustrate our theoretical results.

1. Introduction

There has been considerable interest in the area of distributed cooperative control of multiple dynamic agents in recent years. Applications of distributed cooperative control are vast including target assignment for unmanned air vehicles [1], averaging in communication networks [2] and distributed sensor networks [3].

A fundamental problem in cooperative control of multi-agent systems is consensus, in which the agents, or alternatively, the nodes of a network exchange their local state information with their neighbors with the goal of reaching a certain criteria of common interest. One of the most important aspects in consensus problems is designing distributed protocols which guarantee the agreement of agents. Building on the seminal work [4], a large number of results concerning distributed protocols in groups of agents have appeared in the past few years [4–12], which can be classified into leader-following consensus and leaderless consensus. The multiple leader concept was introduced in [13] where the goal of driving the followers to the convex hulls spanned by the leaders was described as the containment control problem. Reference [14] further studied the containment problems of first-order agents, which provided necessary and sufficient conditions on network topologies which guarantee that the containment can be solved. The containment of double-integrator kinematics was first investigated in [15], where distributed algorithms were proposed for groups with stationary and dynamic leaders. Reference [16] provided several necessary and sufficient conditions for containment control of networks of first-order and second-order agents. The authors in [17] investigated the containment control problem in networks of rigid bodies by using both the one-hop and two-hop neighbors information. Among these works, a common feature is that the states of an agent can be obtained directly by its neighbors.

Our paper contributes to the growing literature on containment control problems in multi-agent systems with multiple leaders; we focus on the case where the error states of an agent and its neighbors can hardly be used directly in the controller design. We assume that the agents communicate over a network with directed interaction topology, and we also assume that the agents are governed by second-order sampled-data dynamics. Considering that the error states of an agent and its neighbors can hardly be obtained by an agent, we explore a reference model based control scheme which can be decomposed to the interaction

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among reference models and the tracking between an agent and its own reference model. Our work is related to [9] and our previous work [6], both of which focus on the containment of continuous-time multi-agent systems by using the reference model based method. However, in view of characteristics of sampled-data systems, it is not possible to decouple the effects of the sample period from the system performance. Our work is also related to the observer-based containment problems [18]. The difference is that the control parameters of containment controllers in [18], which are obtained by solving several LMIs, affect both the performance of the observer and the agent, while in the reference model based method, each agent tracks its own reference model and the parameter design only exists in the tracking part. More specifically, the analysis of this paper involves studying the performance of the reference models connected in a distributed network and discussing the limiting behavior of agents by treating the effects of error states of reference models as disturbances in a tracking problem under which each agent tracks a certain reference model of its own. By adding several constraints on the sample period as well as control parameters, we show that the containment control problem can be solved effectively with the proposed control method.

The reminder of this paper is organized as follows. In Section 2, the preliminaries and notation will be introduced. In Section 3, the distributed model based containment controller will be introduced and the containment control problem will be investigated. The illustrative examples will be provided in Section 4. Section 5 will give a brief conclusion of this paper.

2. Problem formulation

In this section, we formulate the problem of interest and describe the distributed containment controller that we propose. We also provide several lemmas and assumptions.

Let $\mathcal{G}=(\mathcal{V},\mathcal{E},\mathcal{A})$ be a directed graph, where $\mathcal{V}=\{1,\dots,N\}$ is the nonempty set of nodes, $\mathcal{E}\subseteq\mathcal{V}\times\mathcal{V}$ is the edge set and $\mathcal{A}=[a_{ij}]\in\mathbb{R}^{N\times N}$ represents the nonnegative adjacency matrix. An edge of $\mathcal{G}$ is represented by a pair of distinct nodes $(j,i)\in\mathcal{E}$, where node i can obtain information from node j. Denote $\mathcal{N}_i=\{j\in\mathcal{V}|(j,i)\in\mathcal{E}\}$ as the set of neighbors of node i. A directed path in a graph is a sequence $k_0,k_1,\dots,k_f$ of different nodes such that (k_{j-1},k_j) is an edge for $j=1,2,\dots,f$, where $f\in\mathbb{Z}^+$. The adjacency matrix $\mathcal{A}$ is defined such that a_{ij} is the nonnegative weight of edge (j,i). In this paper, we assume $a_{ij}=1$ if $(j,i)\in\mathcal{E}$ and 0 otherwise and $a_{ii}=0$ for all $i\in\{1,\dots,n\}$. The in-degree of node i is defined as $\deg_{in}(i)=\sum_{j=1}^N a_{ij}$. The diagonal in-degree matrix is a $N\times N$ matrix defined as $\mathcal{D}=\mathrm{diag}\{\deg_{in}(i)\}$. It can be seen that the Laplacian matrix $\mathcal{L}=[l_{ij}]\in\mathbb{R}^{N\times N}$ can be represented as $\mathcal{L}=\mathcal{D}-\mathcal{A}$.

We use $A\otimes B$ to represent the Kronecker product of the matrices A and B. $\lambda_i(M)$ denotes the i-th eigenvalue of matrix M. I_n represents the identity matrix with dimension n. $\mathbf{0}_{N\times M}$ represents the $N\times M$ matrix with all elements equal to zero.

We first introduce the following definition.

Definition 2.1 (*Rockafellar [19]*). Let $\mathcal{C}$ be a set in a real vector space $V\subseteq\mathbb{R}^p$. The set $\mathcal{C}$ is said to be *convex* if for any $x,y\in\mathcal{C}$ and any $\alpha\in[0,1]$, we have $(1-\alpha)x+\alpha y\in\mathcal{C}$. The *convex hull* of a set of points $\mathcal{X}=\{x_1,\dots,x_q\}$ in V, denoted $\mathbf{co}\,\mathcal{X}$, is the minimal convex set which contains all points in $\mathcal{X}$. In particular, $\mathbf{co}\,\mathcal{X}=\{\sum_{i=1}^q\theta_i x_i|x_i\in\mathcal{X},\theta_i\ge 0,i=1,\dots,q,\sum_{i=1}^q\theta_i=1\}$.

The objective of containment control problems is to design effective controllers to solve the distributed containment control problem, which can be defined as follows:

Definition 2.2 (*Li et al. [18]*). The containment control problem is said to be solved for system (1) if there exists a distributed controller such that the states of all the followers converge to the convex hull formed by the states of the leaders for arbitrary initial conditions.

In this paper, we consider a group of N agents governed by second-order dynamics. Moreover, we assume that there are M followers and $N-M$ leaders; denote the index set of the followers and the leaders as $\mathcal{F}=\{1,\dots,M\}$ and $\mathcal{R}=\{M+1,\dots,N\}$, respectively.

The agent dynamics is described by

$$\begin{aligned}x_i(k+1)&=x_i(k)+\epsilon v_i(k)+\frac{\epsilon^2}{2}u_i(k),\\ v_i(k+1)&=v_i(t)+\epsilon u_i(k),\quad i\in\mathcal{R}\cup\mathcal{F},\end{aligned}\tag{1}$$

where $x_i(t)\in\mathbf{R}$, $v_i(t)\in\mathbf{R}$ and $u_i(t)\in\mathbf{R}$ are the position-like state, the velocity-like state and the input of agent i, respectively.

Next we introduce the distributed reference model based containment controller. For the followers, the controller is described by

$$\begin{aligned}\hat{x}_i(k+1)&=\hat{x}_i(k)-\epsilon\sum_{j\in\mathcal{F}\cup\mathcal{R}}a_{ij}[\hat{x}_i(k)-\hat{x}_j(k)]+\epsilon\hat{v}_i(k+1),\\ \hat{v}_i(k+1)&=\hat{v}_i(k)-\epsilon\sum_{j\in\mathcal{F}\cup\mathcal{R}}a_{ij}[\hat{v}_i(k)-\hat{v}_j(k)],\\ u_i(k)&=-\alpha[x_i(k)-\hat{x}_i(k)]-\beta[v_i(k)-\hat{v}_i(k)],\quad i\in\mathcal{F},\end{aligned}\tag{2}$$

where $\hat{x}_i(k)\in\mathbf{R}$, $\hat{v}_i(k)\in\mathbf{R}$, are the states of reference model of agent i, and α, β are the control parameters. For the leaders, the controller is designed as

$$\begin{aligned}\hat{x}_i(k+1)&=\hat{x}_i(k)+\epsilon\hat{v}_i(k+1),\\ \hat{v}_i(k+1)&=\hat{v}_i(k),\\ u_i(k)&=-\alpha[x_i(k)-\hat{x}_i(k)]-\beta[v_i(k)-\hat{v}_i(k)],\quad i\in\mathcal{R}.\end{aligned}\tag{3}$$

The following assumptions are useful in this paper.

Assumption 2.1. Suppose that, for each follower, there exists at least one leader that has a directed path to that follower.

Assumption 2.2. Suppose that the sampling period ϵ satisfies $0<\epsilon<1/\max_i\deg_{in}(i)$.

3. Containment control under the distributed reference model based containment controller

In this section, we discuss the containment control problem under the distributed reference model based containment controller. We provide several constrains on the sample period and control parameters. Moreover, we show that the containment control problem can be solved by the proposed controller.

Define $\eta_i(k)=[x_i(k),v_i(k),\hat{x}_i(k),\hat{v}_i(k)]^T$ and $\eta(k)=[\eta_1^T(k),\dots,\eta_N^T(k)]^T$. Then, it can be seen that multi-agent system (1) under (2)–(3) can be written as follows:

$$\eta_i(k+1)=A\eta_i(k)+\sum_{j\in\mathcal{R}\cup\mathcal{F}}l_{ij}B\eta_j(k),\quad i\in\mathcal{F}\tag{4}$$

and

$$\eta_i(k+1)=A\eta_i(k),\quad i\in\mathcal{R}\tag{5}$$

where

$$A=\begin{pmatrix}\Lambda_1 & \Lambda_2-\Lambda_1\\ \mathbf{0}_{2\times2} & \Lambda_2\end{pmatrix},\quad B=\begin{pmatrix}\mathbf{0}_{2\times2} & \mathbf{0}_{2\times2}\\ \mathbf{0}_{2\times2} & -\epsilon\Lambda_2\end{pmatrix},$$

and

$$\Lambda_1=\begin{pmatrix}1-\frac{\epsilon^2}{2}\alpha & \epsilon-\frac{\epsilon^2}{2}\beta\\ -\epsilon\alpha & 1-\epsilon\beta\end{pmatrix},\quad \Lambda_2=\begin{pmatrix}1 & \epsilon\\ 0 & 1\end{pmatrix}.$$