Neurocomputing 168 (2015) 276–282

Contents lists available at ScienceDirect

Neurocomputing

journal homepage: www.elsevier.com/locate/neucom

Delay-dependent passivity analysis of impulsive neural networks with time-varying delays

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ARTICLE INFO

Article history:
Received 12 February 2015
Received in revised form
2 April 2015
Accepted 28 May 2015
Communicated by Y. Liu
Available online 6 June 2015

Keywords:
Passivity
Impulsive neural networks
Time-varying delays
Lyapunov–Krasovskii functional
Matrix inequality

ABSTRACT

This paper investigates delay-dependent passivity for a class of impulsive neural networks with bounded or unbounded time-varying delays. By applying Lyapunov–Krasovskii functional and matrix inequality approach, some new delay-dependent passivity conditions are proposed in terms of the full use of the conditions of neuron activation functions and involved time-varying delays. These passivity conditions are presented in accordance with matrix inequalities, which can be easily verified via standard numerical software. Meanwhile, the results derived here include the existing relative results on the passivity for neural networks without impulse effects as special cases and can also be extended to other neural networks with more complex impulse disturbance. Finally, two numerical examples with simulations are given to demonstrate the effectiveness of the proposed criteria.

1. Introduction

Neural networks have extensive applications in signal processing, pattern recognition, associative memory, fixed-point computation, and other areas. Therefore, the study of neural networks has received considerable attention during the past decades, and various issues of neural networks [1–11] have been investigated and many important results on the dynamical behaviors have been reported for neural networks. In reality, however, time-delays in neural networks can be caused by neural processing and signal transmission that may lead to oscillation and instability [2–4]. At the same time, impulsive effects widely exist in many dynamical systems involving population dynamics, automatic control, drug administration and so on. For example, in implementation of electronic networks in which state is subject to instantaneous perturbations and experiences abrupt change at certain moments, which may be caused by switching phenomenon, frequency change or other sudden noise, that is, impulsive effects [5–8] do exhibit. Therefore, it is necessary to consider the characteristics of neural networks with both impulsive effect and time-delay effect, such as stability, periodicity, and passivity.

On the other hand, the passivity theory originated from circuit theory plays an essentially important role in the analysis and design of linear and nonlinear systems, especially for high-order systems [12]. The essence of the passivity theory is that the passive properties of a system can keep the system internal stability [13]. Thus, the passivity theory provides a nice tool for analyzing the properties of a nonlinear system, such as stability, signal processing and synchronization, and the passivity analysis has received a lot of attention [14–25]. In fact, one remarkable feature of passivity is that the passive system utilizes the product of input and output as the energy provision and embodies the energy attenuation character. Meanwhile, in many engineering problems, the theory of dissipative systems which postulates the energy dissipated inside a dynamic system is less than the energy supplied from external source often links the stability problems. Passivity is part of a broader and a general theory of dissipativeness [22–24]. Passivity analysis is also a major method for studying stability of uncertain system [21]. At the same time, passivity analysis approach has been used in control problems [25–27]. In [22,24], the authors investigated the passivity for neural networks of neutral type with time-varying delays in the leakage term by constructing proper Lyapunov–Krasovskii functional. Vembarasan et al. [25] discussed the state estimation for delayed genetic regulatory networks (GRNs) based on passivity analysis approach. Ahn [28] proposed a new passive and exponential filter for Takagi–Sugeno fuzzy Hopfield neural networks. Song et al. [29] investigated the passivity for a class of discrete-time stochastic neural networks with time-varying delays. Yao et al. [30] considered the passive stability and synchronization of a new model of complex spatio-temporal switching network. In [31], Wu et al. analyzed the problem of delay-dependent passivity analysis for the singular Markov jump systems with time-delays. Wu and Zeng [32,33] studied the passivity for a general class of memristive neural

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http://dx.doi.org/10.1016/j.neucom.2015.05.098
0925-2312/

networks. Wang et al. [17] investigated input passivity and output passivity for a class of impulsive complex networks with time-varying delays. In [18], the authors studied passivity of delayed impulsive neural networks with polytopic type uncertainties and Markovian jump parameters. However, to the best of our knowledge, the passivity conditions for impulsive neural networks with time-varying delays have not been fully investigated, and there is still much room left for further investigation. This constitutes the motivation for the present research.

Motivated by the above discussions, the passivity conditions for impulsive Hopfield type neural networks with time-varying delays are obtained analytically in this paper. By utilizing Lyapunov–Krasovskii functional and matrix inequality approach, some delay-dependent sufficient conditions are derived to guarantee passivity of the discussed impulsive neural networks. It is worth mentioning that the passivity conditions of neural networks with impulsive effects here include the passivity criteria of neural networks without impulsive effects as special cases. Furthermore, our results about passivity analysis of impulsive neural networks cannot be achieved by using any existing ones. In some sense, it is easy to point out that the impulsive neural network models in this paper contain the models in [3,4,15,16,19,34] as special cases. In addition, we will give some corollaries as novel passivity conditions for neural networks without impulsive effects. These corollaries given in this paper are different from those existing results [19,15,34]. According to the proof of theorem, we can directly derive some new stability conditions, which can be easily verified than the existing ones in [2–4]. Finally, two numerical examples are provided to show the effectiveness of the proposed passivity conditions.

The rest of this paper is organized as follows. In Section 2, some preliminaries are given. The main results of the paper are presented and proved in Section 3. Two numerical examples with simulation are presented in Section 4 to illustrate the effectiveness of the proposed results. Finally, conclusions are drawn in Section 5.

2. Preliminaries

Throughout this paper, let $\mathbb{R}^n$ be the n-dimensional Euclidean space, let $\mathbb{R}$ denote the set of real numbers. For real symmetric matrices X and Y, the notation $X \geqslant Y$ ($X > Y$), respectively means that the matrix $X-Y$ is positive semidefinite (positive definite, respectively). The superscript 'T' represents the transposition and I is the $n \times n$ identity matrix. Let the set $N = 1, 2, \ldots$.

The impulsive neural networks model with time-varying delays is described by the following equation group for $i = 1, 2, \ldots, n$:

$$\begin{cases} \dot{x}_i(t) = -c_i x_i(t) + \sum_{j=1}^{n} a_{ij} f_j(x_j(t)) + \sum_{j=1}^{n} b_{ij} f_j(x_j(t-\tau_j(t))) + u_i(t), & t \neq t_k, \\ \Delta x_i(t_k) = E_{ik}(x(t_k^-)), & k \in N, \\ y_i(t) = f_i(x_i(t)), \end{cases} \tag{1}$$

where $x_i(t)$ corresponds to the i-th neuron at time t, $c_i > 0$ is the connection weight, a_{ij} and b_{ij} are connection weights related to the neurons without and with delays, respectively. Activation function $f_j(\cdot)$ shows us how the neurons respond to each other and satisfying $f_j(0) = 0$. $\tau_j(t)$ is the time-varying delay. $u_i(t)$ is an external input of the i-th neuron. The time sequence t_k satisfies $0 < t_0 < t_1 < \cdots < t_k < t_{k+1} < \cdots$ and $\lim_{k \to +\infty} t_k = +\infty$. At time instant t_k, jumps in the state variable $x_i(t)$ are denoted by $\Delta x_i(t_k) = x_i(t_k^+) - x_i(t_k^-)$, $x_i(t_k^+) = x_i(t_k)$ and $x_i(t_k^-) = \lim_{t \to t_k^-} x_i(t)$, $E_{ik}(\cdot)$ denotes the incremental change of the state variable at time t_k. $y_i(t) = f_i(x_i(t))$ is the i-output of neural network (1). For impulsive network (1), its initial conditions are given by $x_i(s) = \phi_i(s) \in C([-\tau, 0], \mathbb{R}^n)$, where $\tau = \max_{t \geqslant 0}\{\tau_j(t), j = 1, 2, \ldots, n\}$. If $\tau_j(t)$ are unbounded, then let $\tau = +\infty$. Let $\phi = (\phi_1, \phi_2, \ldots, \phi_n)^T = 0$.

Let $x(t) = (x_1(t), x_2(t), \ldots, x_n(t))T$, $C = diag(c_1, c_2, \ldots, c_n)$, $A = (a_{ij})_{n \times n}$, $B = (b_{ij})_{n \times n}$, $u(t) = (u_1(t), u_2(t), \ldots, u_n(t))T$, $y(t) = (y_1(t), y_2(t), \ldots, y_n(t))^T$, $f(\cdot) = (f_1(\cdot), f_2(\cdot), \ldots, f_n(\cdot))^T$, we can rewrite network (1) in the following form:

$$\begin{cases} \dot{x}(t) = -Cx(t) + Af(x)t)) + Bf(x(t-\tau(t))) + u(t), & t \neq t_k, \\ \Delta x(t_k) = E_k(x(t_k^-)), & k \in N, \\ y(t) = f(x(t)), \\ x(t) = \phi(t), & t \in [t_0 - \tau, t_0], \end{cases} \tag{2}$$

Throughout this paper, we assume that the feedback $f_i(\cdot)$ satisfies:

Assumption (*H*1): There exist positive constants $L_i (i = 1, 2, \ldots, n)$ such that

$$|f_i(\tilde{x}) - f_i(\tilde{y})| \leqslant L_i |\tilde{x} - \tilde{y}|$$

for any $\tilde{x}, \tilde{y} \in \mathbb{R}$, and denote $L = \max\{L_1, L_2, \ldots, L_n\}$.

Assumption (*H*2): There exist positive constants K_i $(i = 1, 2, \ldots, n)$ such that

$$0 \leqslant \frac{f_i(\tilde{x}) - f_i(\tilde{y})}{\tilde{x} - \tilde{y}} \leqslant K_i$$

for any $\tilde{x}$, $\tilde{y} \in \mathbb{R}$, $\tilde{x} \neq \tilde{y}$ and denote $K = diag(K_1, K_2, \ldots, K_n)$.

Remark 1. We can easily find $(H2) \subseteq (H1)$, from $(H2)$, one has

$$x^T(t)K^2x(t) - f^T(x(t))f(x(t)) \geqslant 0, \quad x^T(t)Kx(t) - f^T(x(t))x(t) \geqslant 0. \tag{3}$$

For network (1), we also need the following assumption:

Assumption (*H*3): For each j, time-delay functions $\tau_j(t)$: $[0, +\infty) \to [0, +\infty)$ are real valued continuous functions and satisfies

$$\sigma_j = \inf_{t \in [0, +\infty)} \{1 - \dot{\tau}_j(t)\} > 0,$$

let $\sigma = \min\{\sigma_1, \sigma_2, \ldots, \sigma_n\}$.

Definition 1 (*Brogliato et al.* [13]). Neural network (1) or (2) with input $u(t)$ and output $y(t)$ is said to be passive if there is a constant γ such that

$$2\int_0^T u^T(s)y(s)\,ds \geqslant -\gamma \int_0^T u^T(s)u(s)\,ds \tag{4}$$

for all $T \geqslant 0$ and for all solutions of neural network (1) with zero initial values.

Lemma 1. *For any* $\varepsilon > 0, a \in \mathbb{R}^n, b \in \mathbb{R}^n$, *the inequality* $2a^Tb \leqslant \varepsilon a^Ta + \varepsilon^{-1}b^Tb$ *holds.*

3. Main results

Theorem 1. *Assume that assumptions (H1) and (H3) hold, and* $x_i(t_k) = \lambda_{ik}x_i(t_k^-)$ *with* $\lambda_{ik}^2 \leqslant 1$, *then neural network (2) is passive if there exist a positive definite matrix* $P = (p_{ij})_{n \times n}$ *and a positive scalars* $\varepsilon > 0$ *with* $\gamma > 2\varepsilon$ *such that the following inequality holds:*

$$-(PC + CP) + 3\varepsilon^{-1}P^2 + \varepsilon L^2 A^T A + \frac{\varepsilon L^2}{\sigma} B^T B + \varepsilon^{-1} L^2 I \leqslant 0. \tag{5}$$