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Improved exponential stability criteria for time-varying delayed neural networks ☆

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ABSTRACT

This paper is concerned with the exponential stability for neural networks with mixed time-varying delays. By using a more general delay-partitioning approach, an augmented Lyapunov functional that contains some information about neuron activation function is constructed. In order to derive less conservative results, an adjustable parameter is introduced to divide the range of the activation function into two unequal subintervals. Moreover, the application of combination of integral inequalities further reduces the conservativeness of the obtained exponential stability conditions. Numerical examples illustrate the advantages of the proposed conditions when compared with other results from the literatures.

1. Introduction

During the past several decades, a great deal of attention has been focused on the study of neural networks. This is due to the fact that neural networks are a class of important mathematic models which are widely used in many areas, such as pattern recognition, signal processing, associative memories and other scientific areas [1]. It is well-known that time delay is frequently encountered in various neural networks systems owing to neural processing and signal transmission. What's more, time delay is also an important source of instability, oscillation or poor performance for these systems. Because there exists a distribution of propagation delays in neural networks, the distributed delays should be incorporated in the model. It is often the case that the neural network models possess both discrete and distributed delays. Therefore, the problem of stability analysis of neural networks with discrete and distributed delays has become interesting and received increasing attention [2–8]. Many important control problems such as passivity/dissipativity analysis [9], synchronization/asynchronous problems [10,11] and state estimation problems [12] are also discussed in neural networks systems [13–21].

For the stability analysis of neural networks systems with time delays, one of the major goals is to seek the maximum delay that can guarantee the stability of the systems. In order to obtain some more less conservative stability conditions, many effective methods have been utilized, such as free-weighting matrices method [22], delay-partitioning approach [23], convex combination techniques [24]. Meanwhile, the conservativeness of stability criteria can also be reduced by constructing a Lyapunov functional with tripe-integral terms [25] or dividing the boundary interval of activation function into two equal subintervals [26–28]. A large number of stability criteria of time-varying delayed neural networks have been obtained by applying the above methods. In [27], an exponential stability criterion is proposed by constructing an augmented Lyapunov-functional, while the discrete time-varying delay $d(t)$ is required to be differential. In [28,29], some less conservative stability criteria are derived by defining an augmented Lyapunov functional and using a convex combination approach. In [30], a new delay-dependent stability criterion is established by constructing a Lyapunov functional with tripe-integral terms and dividing the discrete delay interval into multiple subintervals. Recently, some improved delay-dependent stability criteria have been provided by using some improved delay-partitioning methods and making good use of the information of neuron

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activation functions [31,32]. It should be point out that some improved stability criteria for time-delay neural networks have been obtained, but the obtained upper bounds of delays by using the above criteria are far from the desired values. Therefore, it is worth further improving the proposed methods. Besides, many applications require fast speed of convergence, and exponential stability can offer faster speed of convergence than asymptotic stability.

Motivated by the aforementioned discussion, in this paper, we will study the problem of the exponential stability for neural networks with discrete and distributed time-varying delays. The main purpose of this paper is to establish several less conservative exponential stability criteria. By using Lyapunov stability theory, sufficient conditions are established to ensure the exponential stability of delayed neural networks. Three numerical examples are provided to illustrate the advantages of the proposed criteria in this paper. The main novelty of our work lies in three folds: (1) a more general partitioning method for range of activation function is adopted by introducing an adjustable parameter ρ; (2) the augmented vector $\xi(t)$ contains the following terms $\int_{t-r(t)}^{t} g(z(s))\,ds$, $\int_{t-\delta_1(t)}^{t-r(t)} g(z(s))\,ds$, $\int_{t-r}^{t-\delta_1(t)} g(z(s))\,ds$, $\int_{t-r(t)}^{t}\int_{t-r(t)}^{u} g(z(s))\,ds\,du$, $\int_{t-\delta_1(t)}^{t-r(t)}\int_{t-\delta_1(t)}^{u} g(z(s))\,ds\,du$, and $\int_{t-r}^{t-\delta_1(t)}\int_{t-r}^{u} g(z(s))\,ds\,du$, which makes more information on activation function to be used; (3) the relationships between $z(t)$, $z(t-d(t))$ and $\frac{2}{d(t)}\int_{t-d(t)}^{t} z(s)\,ds$, $z(t-d(t))$, $z(t-\alpha d)$ and $\frac{2}{(\alpha d-d(t))}\int_{t-\alpha d}^{t-d(t)} z(s)\,ds$, $z(t-\alpha d)$, $z(t-d)$ and $\frac{2}{(1-\alpha)d}\int_{t-d}^{t-\alpha d} z(s)\,ds$ are considered sufficiently, which are helpful in adopting convex combination techniques.

The rest of the current paper is organized as follows. Section 2 introduces the problem formulation and some preliminaries. Section 3 presents some stability criterion obtained for time delay systems. Section 4 gives some numerical examples to demonstrate the effectiveness of our main result. Finally, Section 5 draws the conclusion.

Notation: Throughout this paper, $\mathbb{R}^n$ denotes the n-dimensional Euclidean space; I denotes the identity matrix of appropriate dimensions. A^T represents the transpose of A; $X>0(<0)$ means X is a symmetric positive (negative) definite matrix; $diag\{r_1, r_2, \ldots, r_n\}$ denotes block diagonal matrix with diagonal elements $r_i, i=1,2,\ldots,n$; the symbol $*$ represents the elements below the main diagonal of a symmetric matrix; $\Psi_{i,j}$ presents the element in the ith row and jth column of Ψ.

2. Preliminaries

Consider the following neural networks with discrete and distributed time-varying delays:

$$\begin{cases} \dot{u}(t) = -W_0 u(t) + W_1 f(u(t)) + W_2 f(u(t-d(t))) + W_3 \int_{t-r(t)}^{t} f(u(s))ds + J, \\ u(t) = \varphi(t), t \in [-h, 0]. \end{cases} \tag{1}$$

where $u(t) = [u_1(t), \ldots, u_n(t)]^T \in \mathbb{R}^n$ is the neuron state vector, $f(u(\cdot)) = [f_1(u_1(\cdot)), f_2(u_2(\cdot)), \ldots, f_n(u_n(\cdot))]^T \in \mathbb{R}^n$ is the neuron activation function, $J = [J_1, \ldots, J_n]^T \in \mathbb{R}^n$ is an external constant input vector, $W_0 = \text{diag}\{W_{01}, W_{02}, \ldots, W_{0n}\} > 0$ represents the self-feedback term with $W_{0i} > 0$, $i = 1, 2, \ldots, n$, $W_1 \in \mathbb{R}^n$ is the interconnection weight matrices, W_2 and $W_3 \in \mathbb{R}^n$ are the delayed interconnection weight matrices, $d(t)$ and $r(t)$ are the time-varying delays satisfying $0 \le d(t) \le d, \dot{d}(t) \le d_D < 1$ and $0 \le r(t) \le r$, where $\max\{d, r\} = h$.

Assumption 1. For the constants γ_i^-, γ_i^+, the bounded activation function $f_i(\cdot)$ in (1) satisfies the following condition:

$$\gamma_i^- \le \frac{f_i(x) - f_i(y)}{x-y} \le \gamma_i^+, \quad \forall x, y \in R, \quad x \ne y, \quad i = 1, 2, \ldots, n. \tag{2}$$

It is clear that under the above assumption, system (1) has one equilibrium point denoted as $u^* = [u_1^*, \ldots, u_n^*]^T$. For convenience, we firstly shift the equilibrium point u^* to the origin by letting $z(t) = u(t) - u^*$, $g(z(t)) = f(u(t)) - f(u^*)$, then the system (1) can be converted to

$$\dot{z}(t) = -W_0 z(t) + W_1 g(z(t)) + W_2 g(z(t-d(t))) + W_3 \int_{t-r(t)}^{t} g(z(s))\,ds, \tag{3}$$

where $g(z(\cdot)) = [g(z(\cdot)), g(z(\cdot)), \ldots, g(z(\cdot))]^T$. It is easy to check that the function $g_i(\cdot)$ satisfies $g_i(0) = 0$, and

$$\gamma_i^- \le \frac{g_i(z)}{z} \le \gamma_i^+, \quad \forall \alpha \in R, \quad x \ne 0, \ i = 1, 2, \ldots, n. \tag{4}$$

Under Assumption 1, the following inequality is true for symmetric positive definite diagonal matrix G:

$$z^T(t)\Gamma G \Gamma z(t) \ge g^T(z(t)) G g(z(t)). \tag{5}$$

Assumption 2. The functions $g_i(\cdot)$ $(i = 1, 2, \ldots, n)$ have the same structure form. For instance, $g_1(z(\cdot)) = \beta_1(|z(\cdot)+1| - |z(\cdot)-1|)$, $g_2(z(\cdot)) = \beta_2(|z(\cdot)+1| - |z(\cdot)-1|), \ldots, g_n(z(\cdot)) = \beta_n(|z(\cdot)+1| - |z(\cdot)-1|)$, where $\beta_i (i = 1, 2, \ldots, n)$ are some constants.

In this paper, we will attempt to formulate some exponential stability conditions of system (3). The following definition and lemmas are useful in deriving the main results.

Definition 1 (*Tian et al. [33]*). The equilibrium point $z^* = 0$ of system (3) is said to be exponentially stable, if there exist scalars $k > 0$ and $\beta > 0$ such that

$$\| z(t) \| \le \beta e^{-kt} \sup_{-h \le s \le 0} \| z(s) \|, \quad \forall t > 0.$$