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Non-fragile synchronization of dynamical networks with randomly occurring nonlinearities and controller gain fluctuations

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ABSTRACT

This paper investigates the non-fragile synchronization problem of dynamical networks with randomly occurring nonlinearities and controller gain fluctuations. More precisely, these randomly occurring phenomena are modeled by stochastic variables satisfying the Bernoulli distribution. By utilizing the Lyapunov–Krasovskii functional method, sufficient synchronization criteria are first established in terms of linear matrix inequalities (LMIs). Based on the obtained results, a set of non-fragile synchronization controllers is further designed to ensure that the synchronization of the dynamical networks can be achieved. Finally, a numerical example is provided to illustrate the applicability and effectiveness of our theoretical results.

1. Introduction

In the past decade, dynamical networks have attracted rapidly increasing attention due to their broad applications in many fields of science and engineering [1–4]. Generally, dynamical networks consisted of numbers of interconnected nodes, which exhibit topological properties. Several typical examples of dynamical networks can be found in the real world, such as the Internet, biological neural networks, and social networks [5–7]. Since synchronized dynamical networks could display collective behaviors and can offer an insight into the nature of the nodes, the synchronization problem of dynamical networks has been extensively studied in recent years and becomes one of the most interesting and fundamental research topics. As a result, many effective synchronization techniques have been proposed in the literature [8–13].

It is worth mentioning that nonlinearities exist ubiquitously in most practical applications, which has been a challenging issue in system designs [14–17]. For the synchronization of dynamical networks, the presence of nonlinearities would affect the dynamical behaviors or even break the synchronization. Therefore, the effect of nonlinearities should be taken into account in the synchronization of dynamical networks. Note that the nonlinearities in dynamical networks can vary in a random fashion, which gives rise to the interesting topic on randomly occurring nonlinearities. By introducing the stochastic variables according to certain probability distributions, encouraging results of randomly occurring nonlinearities of dynamical networks have been reported in the remarkable papers. For example, in [18], the global synchronization problem for delayed complex networks with randomly occurred nonlinearities and multiple stochastic disturbances has been thoroughly studied. In [19], the randomly occurred nonlinearity is discussed in the synchronization problem for the “blinking” networks as an important phenomenon. In [20], the H_∞ synchronization control problem is investigated for a class of dynamical networks with randomly varying nonlinearities. Following this research line, the randomly occurring nonlinearities are also studied for other systems, such as sensor networks [21], Markov jump systems [22] and neural networks [23]. In addition, another important issue in the synchronization of dynamical networks is the influence of time delays, which should not be neglected in modeling the realistic networks [24,25].

On the other hand, for some dynamical networks that cannot be synchronized by themselves, effective controllers are needed to achieve the synchronization. In practice, it should be pointed out that there are limitations for the designed controllers, since the designed parameters of the controllers cannot be implemented exactly. The synchronization results are very sensitive to the variations of the

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controllers, the non-fragile synchronization controllers have been developed for dynamical networks to tolerate some uncertainties [26,27]. Very recently, some initial attentions have been focused on the non-fragile controllers with randomly occurring controller gain fluctuations, whose key idea is to minimize the cost of implementation of the controllers [28]. However, to the best of the authors' knowledge, the synchronization problem with randomly occurring nonlinearities and controller gain fluctuations has not yet been adequately studied, and the purpose of this paper is therefore to shorten such a gap.

Motivated by the above analysis, in this paper, the non-fragile synchronization problem of dynamical networks with randomly occurring nonlinearities and controller gain fluctuations is investigated. Compared with the existing results, a more general model of an array of N coupled dynamical networks with time-varying delays is proposed, which can generalize some existing models to some extent. In particular, the non-fragile synchronization controllers are designed, which is more practical in the applications. The randomly occurring nonlinearities and controller gain fluctuations are both considered by stochastic models obeying certain Bernoulli distributions. By utilizing the Lyapunov–Krasovskii functional method, delay-dependent criteria are first established to guarantee that the synchronization can be achieved. Then, a set of synchronization controllers is designed based on the derived results in terms of LMIs. The remainder of this paper is organized as follows. Section 2 provides some preliminaries on the dynamical networks with randomly occurring nonlinearities and controller gain fluctuations, and formulates the synchronization problem. In Section 3, main results on the synchronization criteria are given. Section 4 presents a numerical example to illustrate the effectiveness of the proposed strategy and the paper is concluded in Section 5.

Notation: The notation in the paper is standard. $\mathbb{R}^n$ denotes the n dimensional Euclidean space, $\mathbb{R}^{m\times n}$ represents the set of all $m \times n$ real matrices. I and 0 represent identity matrix and zero matrix with appropriate dimensions, respectively. The notation $P>0$ means P is real symmetric and positive definite, and the superscript "T" denotes matrix transposition. $A \otimes B$ denotes the Kronecker product of matrices A and B. Moreover, in symmetric block matrices, $*$ is used as an ellipsis for the terms that are introduced by symmetry and diag$\{\cdots\}$ denotes a block-diagonal matrix. Finally, if not explicitly stated, all matrices are assumed to have compatible dimensions.

2. Problem formulation and preliminaries

Consider the following class of dynamical networks with time-varying delays:

$$\dot{x}_i(t)=Ax_i(t)+A_dx_i(t-\tau(t))+\alpha(t)f(t,x_i(t))$$
$$+(1-\alpha(t))g(t,x_i(t))+\sum_{j=1}^{N}b_{ij}\Gamma x_j(t),$$
$$i=1,2,\ldots,N, \tag{1}$$

where $x_i(t)=(x_{i1}(t),x_{i2}(t),\ldots,x_{in}(t))^T\in\mathbb{R}^n$ denotes the state vector of the ith node, A and A_d are constant matrices, $f,g:\mathbb{R}\times\mathbb{R}^n\to\mathbb{R}^n$ are smooth nonlinear functions, $\tau(t)$ represents the time-varying delays satisfying $0\le\tau(t)\le\overline{\tau}$, $\overline{\tau}>0$, $\Gamma\in\mathbb{R}^{n\times n}$ is the inner coupling matrix and $B=(b_{ij})_{N\times N}\in\mathbb{R}^{N\times N}$ is the outer coupling matrix, which denotes the network topology. If there is a connection from node i to node j $(i\neq j)$, then the coupling $b_{ij}\neq 0$; otherwise, $b_{ij}=0$. Moreover, the diagonal elements b_{ii} are defined as $b_{ii}=-\sum_{j=1,j\neq i}^{N}b_{ij}$, $i=1,2,\ldots,N$. The initial conditions are $x_i(t)=\phi_i(t)$, $t\in[-\overline{\tau},0]$, with some given continuous functions $\phi_i:[-\overline{\tau},0]\to\mathbb{R}^n$. Without loss of generality, the initial conditions are chosen as constant functions on $[-\overline{\tau},0]$. The stochastic variable $\alpha(t)\in\mathbb{R}$ is a Bernoulli distributed sequence defined by [28–32]

$$\alpha(t)=\begin{cases}1, f(t,x_i(t)) & \text{happens},\\ 0, g(t,x_i(t)) & \text{happens},\end{cases}$$

with

$$\Pr\{\alpha(t)=1\}=\alpha,$$
$$\Pr\{\alpha(t)=0\}=1-\alpha,$$

where $\alpha\in[0,1]$ is a known constant.

Remark 1. By utilizing the Bernoulli distributed sequence, probability distribution of the randomly occurring nonlinearities is introduced, which can model more realistic dynamical behaviors for the dynamical networks. The Bernoulli distributed model can also be adopted to model the randomly occurring time delays, which can be found in the literature, see e.g., [33,34].

Assumption 1. The nonlinear functions satisfy

$$[f(t,x)-f(t,y)-F_1(x-y)]^T[f(t,x)-f(t,y)-F_2(x-y)]\le 0,$$
$$[g(t,x)-g(t,y)-G_1(x-y)]^T[g(t,x)-g(t,y)-G_2(x-y)]\le 0,$$

$\forall x,y\in\mathbb{R}^n$, where F_1, F_2, G_1 and G_2 are constant matrices of appropriate dimensions with $F_2-F_1\ge 0$ and $G_2-G_1\ge 0$.

Remark 2. The nonlinear functions adopted in this paper are more general and have been widely utilized to describe the nonlinear phenomena. It can be verified that this nonlinear model includes the common Lipchitz condition and norm-bounded condition as two special cases.

By using the drive-response concept, model (1) is referred to as the drive dynamical networks and the corresponding response dynamical networks are given as

$$\dot{y}_i(t)=Ay_i(t)+A_dy_i(t-\tau(t))+\alpha(t)f(t,y_i(t))$$
$$+(1-\alpha(t))g(t,y_i(t))+\sum_{j=1}^{N}b_{ij}\Gamma y_j(t)+u_i,$$
$$i=1,2,\ldots,N, \tag{2}$$

where $y_i(t)=(y_{i1}(t),y_{i2}(t),\ldots,y_{in}(t))^T\in\mathbb{R}^n$ represents the response state vector of the ith node and $u_i\in\mathbb{R}^n$ denotes the control input on the ith node.

The synchronization errors are defined as

$$e_i(t)=y_i(t)-x_i(t),\quad i=1,2,\ldots,N.$$

Then, the following error dynamical system can be obtained:

$$\dot{e}_i(t)=Ae_i(t)+A_de_i(t-\tau(t))+\alpha(t)(f(t,y_i(t))-f(t,x_i(t)))$$
$$(1-\alpha(t))(g(t,y_i(t))-g(t,x_i(t)))+\sum_{j=1}^{N}b_{ij}\Gamma e_j(t)$$
$$+u_i,\quad i=1,2,\ldots,N. \tag{3}$$

In order to achieve the synchronization, the following non-fragile state feedback controllers are designed:

$$u_i=(K_i+\beta(t)\Delta K_i)e_i(t),\quad i=1,2,\ldots,N, \tag{4}$$

where $K_i\in\mathbb{R}^{n\times n}$ is the feedback controller gain to be determined and the real-valued matrix ΔK_i denotes the controller gain fluctuation. Moreover, it is assumed that ΔK_i has the following structure:

$$\Delta K_i=H_i\Delta_i(t)E_i, \tag{5}$$

where $\Delta_i(t)\in\mathbb{R}^{k\times l}$, $i=1,2,\ldots,N$ is an unknown time-varying matrix satisfying

$$\Delta_i^T(t)\Delta_i(t)\le I,$$

and H_i, E_i are known constant matrices. The stochastic variable $\beta(t)\in\mathbb{R}$ is a Bernoulli distributed sequence, which describes the phenomena of randomly occurring controller gain fluctuations.