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Dissipativity analysis of neural networks with time-varying delays ☆

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ABSTRACT

This paper focuses on the problem of delay-dependent dissipativity analysis for a class of neural networks with time-varying delays. A free-matrix-based inequality method is developed by introducing a set of slack variables, which can be optimized via existing convex optimization algorithms. Then, by employing Lyapunov functional approach, sufficient conditions are derived to guarantee that the considered neural networks are strictly $(\mathcal{Q},\mathcal{S},\mathcal{R})$-$\gamma$-dissipative. The conditions are presented in terms of linear matrix inequalities and can be readily checked and solved. Numerical examples are finally provided to demonstrate the effectiveness and advantages of the proposed new design techniques.

1. Introduction

During the last decades, neural networks have received considerable attentions owing to their successful applications in a variety of areas, such as image processing, associative memory, pattern recognition, and optimization problem [1–3]. In the implementation of artificial neural networks, time delays cannot be avoided as a result of the inherent communication time between neurons and the finite switching speed of amplifiers, which might cause oscillation, even destabilization of neural networks. Therefore, many efforts have been made to the stability analysis of neural networks [4–31]. Both delay-independent [4–9] and delay-dependent [11–31] conditions have been developed. The delay-dependent conditions, which include the size information of time-delay, are usually less conservative than delay-independent ones, especially for neural networks with small delays. Thus, more attentions have been paid to delay-dependent conditions.

In recent years, the topic of dissipativity has attracted a great deal of attention as dissipativity is an important property of physical systems, which is closely related with the intuitive phenomenon of loss or dissipation of energy. Generally, dissipativity tells more than stability [37]. In [38], the problem of reliable dissipative control of stochastic hybrid systems has been addressed, where two kinds of controllers have been designed to guarantee the stochastic hybrid system to be strictly dissipative. In [39], the dissipativity analysis of singular systems with time-delay has been investigated. In addition, the dissipativity problem has been addressed for continuous-time neural networks [40] and discrete-time neural networks [41], respectively. Recently, the problem of robust dissipativity has been investigated for neural networks with time-delay and randomly occurring uncertainties in [48], where some dissipativity conditions have been established by using Jensen inequality. However, it was pointed in [36] that the results derived by Jensen inequality are very conservative. In [49], improved dissipativity conditions have been obtained via a Wirtinger-based inequality. However, there is room for further investigation.

In this paper, we investigate the problem of dissipativity analysis for neural networks with a time-varying delay. A free-matrix-based inequality is firstly proposed. It is theoretically proved that the proposed inequality encompasses some existing ones as special cases. By utilizing the new inequality, less conservative dissipativity conditions are established. Numerical examples are given to demonstrate the effectiveness and the improvement of the proposed approach.

Notation: Throughout this paper, the superscripts '-1' and 'T' stand for the inverse and transpose of a matrix, respectively; $\mathbb{R}^n$ denotes the n-dimensional Euclidean space; $\mathbb{R}^{n\times m}$ is the set of all $n \times$

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m real matrices; $P>0$ means that the matrix P is symmetric and positive definite; diag$\{\cdots\}$ denotes a block-diagonal matrix; I and 0 represent the identity matrix and a zero matrix, respectively; the symmetric terms in a symmetric matrix are denoted by $*$; and $\mathrm{Sym}\{X\}=X+X^T$. Matrices, if it not explicitly stated, are assumed to have compatible dimensions.

2. Preliminaries

Consider the following uncertain neural network:

$$\begin{cases} \dot{x}(t)=-Ax(t)+W_0 g(x(t))+W_1 g(x(t-\tau(t)))+u(t) \\ y(t)=g(x(t)) \end{cases} \tag{1}$$

where $x(t)=[x_1(t)\ x_2(t)\ \cdots\ x_n(t)]^T$ and $g(x(t))=[g_1(x_1(t))\ g_2\ (x_2(t))\ \cdots\ g_n(x_n(t))]^T$ are the neuron state vector and the neuron activation function, respectively; $u(t)$ and $y(t)$ are the input and the output of the neural network; $A=\mathrm{diag}\{a_1,a_2,\ldots,a_n\}$ is a positive diagonal matrix, W_0 and W_1 are known connection weight matrices; the delay, $\tau(t)$, is a time-varying function satisfying

$$0\le\tau(t)\le\overline{\tau},\quad \dot{\tau}(t)\le\mu \tag{2}$$

where $\overline{\tau}$ and μ are known constants.

Assumption 1. The function $g_i(\cdot)$ in (1) are continuous and bounded, and satisfy

$$k_i^-\le\frac{g_i(\alpha_1)-g_i(\alpha_2)}{\alpha_1-\alpha_2}\le k_i^+,\quad i=1,2,\ldots,n \tag{3}$$

where $g_i(0)=0$, $\alpha_1,\alpha_2\in\mathbb{R}$, $\alpha_1\ne\alpha_2$, and k_i^- and k_i^+ are known real scalars.

To introduce the property of dissipativity, let us define an energy supply function as follows:

$$G(u,y,t^*)=\langle y,\mathcal{Q}y\rangle_{t^*}+2\langle y,\mathcal{S}u\rangle_{t^*}+\langle u,\mathcal{R}u\rangle_{t^*},\quad \forall t^*\ge 0 \tag{4}$$

where $\mathcal{Q}$, $\mathcal{S}$ and $\mathcal{R}$ are real matrices with $\mathcal{Q}$, $\mathcal{R}$ symmetric, and $\langle a,b\rangle_{t^*}=\int_0^{t^*}a^T b\,dt$. Without loss of generality, it is assumed that $\mathcal{Q}\le 0$ and denoted that $-\mathcal{Q}=\mathcal{Q}_-^T\mathcal{Q}_-$ for some $\mathcal{Q}_-$.

In the sequel, we introduce the following definition and lemmas, which are indispensable to derive our main results.

Definition 1. Neural network (1) is said to be strictly $(\mathcal{Q},\mathcal{S},\mathcal{R})$-$\gamma$-dissipative if, for some scalar $\gamma>0$, the following inequality:

$$G(u,y,t^*)\ge\gamma\langle u,u\rangle_{t^*},\quad \forall t^*\ge 0 \tag{5}$$

holds under zero initial condition.

Lemma 1 (*Jensen inequality [32]*). *For $R>0$, and a vector function $x:[\alpha,\beta]\to\mathbb{R}^n$ such that the integrations concerned are well defined, the following inequality holds:*

$$\int_\alpha^\beta x^T(s)\,ds R\int_\alpha^\beta x(s)\,ds\le(\beta-\alpha)\int_\alpha^\beta x^T(s)Rx(s)\,ds \tag{6}$$

Lemma 2 (*[33]*). *Given positive integers m and n, a scalar $\beta\in(0,1)$, a given $R>0$, and two matrices H_1, $H_2\in\mathbb{R}^{n\times m}$, define, for all vector $\xi\in\mathbb{R}^m$, the function $\Theta(\beta,R)$ described by*

$$\Theta(\beta,R)=\frac{1}{\beta}\xi^T H_1^T RH_1\xi+\frac{1}{1-\beta}\xi^T H_2^T RH_2\xi \tag{7}$$

then, if there exists a matrix $X\in\mathbb{R}^{n\times n}$ such that $\begin{bmatrix} R & X\\ X^T & R\end{bmatrix}>0$, the following inequality holds:

$$\min_{\beta\in(0,1)}\Theta(\beta,R)\ge\begin{bmatrix}H_1\xi\\ H_2\xi\end{bmatrix}^T\begin{bmatrix}R & X\\ X^T & R\end{bmatrix}\begin{bmatrix}H_1\xi\\ H_2\xi\end{bmatrix} \tag{8}$$

Lemma 3. *Let x be a differentiable signal in $[\alpha,\beta]\to\mathbb{R}^n$, for symmetric matrices $R\in\mathbb{R}^{n\times n}$, X, $Z\in\mathbb{R}^{3n\times 3n}$, and any matrices $Y\in\mathbb{R}^{3n\times 3n}$, N_1, $N_2\in\mathbb{R}^{3n\times n}$ satisfied*

$$\Phi=\begin{bmatrix}X & Y & N_1\\ * & Z & N_2\\ * & * & R\end{bmatrix}\ge 0$$

the following inequality holds:

$$-\int_\alpha^\beta\dot{x}^T(s)R\dot{x}(s)\,ds\le\varpi^T\hat{\Omega}\varpi \tag{9}$$

where

$$\hat{\Omega}=(\beta-\alpha)(X+\frac{1}{3}Z)+\mathrm{Sym}\{N_1G_1+N_2G_2\}$$

$$G_1=\left[\overline{e}_1^T-\overline{e}_2^T\right]^T$$

$$G_2=\left[\overline{e}_1^T+\overline{e}_2^T-2\overline{e}_3^T\right]^T$$

$$\overline{e}_1=[I\ 0\ 0],\quad \overline{e}_2=[0\ I\ 0],\quad \overline{e}_3=[0\ 0\ I]$$

$$\varpi=\left[x^T(\beta)\ x^T(\alpha)\ \frac{1}{\beta-\alpha}\int_\alpha^\beta x^T(s)\,ds\right]^T$$

Proof. Define

$$f(s)=\frac{2s-\beta-\alpha}{\beta-\alpha}$$

$$\zeta(s)=\left[\varpi^T\ f(s)\varpi^T\ \dot{x}^T(s)\right]^T$$

it is easy to see that

$$\zeta^T(s)\Phi\zeta(s)\ge 0 \tag{10}$$

Integrating the left side of (10) from α to β yields

$$\begin{aligned}\int_\alpha^\beta\zeta^T(s)\Phi\zeta(s)\,ds=&(\beta-\alpha)\varpi^T X\varpi+\frac{(\beta-\alpha)}{3}\varpi^T Z\varpi\\ &+2\varpi^T N_1[\overline{e}_1-\overline{e}_2]\varpi\\ &+2\varpi^T N_2[\overline{e}_1+\overline{e}_2-2\overline{e}_3]\varpi\\ &+\int_\alpha^\beta\dot{x}^T(s)R\dot{x}(s)\,ds\end{aligned} \tag{11}$$

To sum up, one can get

$$\begin{aligned}-\int_\alpha^\beta\dot{x}^T(s)R\dot{x}(s)\,ds\le&(\beta-\alpha)\varpi^T X\varpi+\frac{(\beta-\alpha)}{3}\varpi^T Z\varpi\\ &+2\varpi^T N_1[\overline{e}_1-\overline{e}_2]\varpi\\ &+2\varpi^T N_2[\overline{e}_1+\overline{e}_2-2\overline{e}_3]\varpi\\ =&\varpi^T\hat{\Omega}\varpi\end{aligned}$$

This completes the proof.□

Remark 1. It is worth mentioning that the Wirtinger-based inequality proposed in [36], which is shown more tighter than the well-known Jessen inequality, is a spacial case of (9). By setting

$$N_1=\left[-\frac{1}{\beta-\alpha}R\ \frac{1}{\beta-\alpha}R\ 0\right]^T$$

$$N_2=\left[-\frac{3}{\beta-\alpha}R\ -\frac{3}{\beta-\alpha}R\ \frac{6}{\beta-\alpha}R\right]^T$$

$$X=N_1R^{-1}N_1^T,\quad Y=N_1R^{-1}N_2^T,\quad Z=N_2R^{-1}N_2^T$$

Lemma 3 reduces to Corollary 5 in [36].