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Dynamic behavior of nonautonomous cellular neural networks with time-varying delays

Shujun Long *, Hongheng Li, Yongxin Zhang

College of Mathematics and Information Science, Leshan Normal University, Leshan 614004, PR China

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ABSTRACT

In this paper, we investigate the dynamic behavior of nonautonomous cellular neural networks with time-varying delays. We firstly develop a differential inequality which plays an important role in study of boundedness, attracting set and stability of differential systems and improves some early Halanay-type inequalities. Based on the new inequality, the boundedness, attracting set, exponential stability and existence of periodic solution of the considered neural networks are obtained. Our results improve the early results in the literature. One example is given to illustrate the correctness and superiority of our conclusion.

1. Introduction

Recently, due to the wide application in solving pattern recognition, associative memories, signal processing and optimization problems, the dynamic behaviors of autonomous neural networks are popular with many researchers [1,3–5,13,15,23,25,27]. Actually, the nonautonomous phenomenon often occurs in many realistic systems. Especially, when we consider a long-term dynamic behavior of a system, the parameters of the system frequently change along with time for the environmental disturbances. For example, in order to analyze and simulate the dynamic behaviors of the human brain, the authors in [4,5] introduced the cellular neural networks. However, the human brain has usually been in periodic oscillatory of chaos state, hence it is of prime importance to investigate stability, periodic oscillation, and chaos phenomenon of cellular neural networks. Therefore, it is a vital topic to study the neural networks with variable coefficients. In fact, there are many results on the dynamic behaviors of nonautonomous neural networks [6,7,9,10,12,14,16–21,28–34].

In 2008, the authors in [10] considered the existence and stability of periodic solution of nonautonomous bidirectional associative memory neural networks with delay, and obtained the conditions in matrix function inequality. But it is difficult to be dealt with by using Matlab LMI Control Toolbox. In [9,12,18–21,30], the authors investigated the dynamic behaviors of varying kinds of nonautonomous neural networks by constructing some kinds of Lyapunov–Krasovskii functional, and obtained some new results which require that the determinant conditions are all true for the variation of the time variable t. Besides, this method must require additional constraint conditions on delay functions such as differentiability. In addition, it is well known that differential inequalities are very important tools for investigating the dynamic behaviors of differential dynamical systems by the Lyapunov–Krasovskii functional method [2,8,14,16,17,22,24,26,28]. For example, by establishing new differential inequalities, the authors in [16,17] investigated the exponential stability of two kinds of nonautonomous neural networks with impulses and time-varying delays, and got some new results which no longer require that determinant conditions are all true for the variation of the time variable t or the differentiability of delay functions. However, the new inequalities in [16,17] cannot deal with the boundedness and attracting set of nonautonomous differential systems.

Motivated by the above analysis, we investigate the dynamic behavior of nonautonomous cellular neural networks with time-varying delays. We outline the main contribution of this paper as follows:

- A new differential inequality is given which plays an important role in the study of boundedness, attracting set and stability of differential systems and improves Halanay-type inequalities in [2,8,22,24,26].
- The sufficient conditions are obtained for ensuring the boundedness, attracting set, exponential stability and existence of periodic solution of the considered neural networks.

* Corresponding author.
E-mail address: longer207@sina.com (S. Long).

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- The obtained results no longer require that determinant conditions are all true for the variation of the time variable t or the differentiability of delay functions (we highlight this improvement by Remarks 2 and 3).

Meanwhile, one example is given to illustrate the superiority of our conclusion.

2. Model description and preliminaries

First, we introduce several notations and recall some basic definitions.

Let R^n be the space of n-dimensional real column vectors, $\mathcal{N} \triangleq \{1,2,\dots,n\}$, $R_+ \triangleq [0,+\infty)$, and $L^1(R_+,R_+)$ denote the family of all continuous functions $h: R_+ \to R_+$ satisfying $\int_0^{+\infty} h(t)\,dt < \infty$.

$C[X,Y]$ denotes the space of continuous mappings from the topological space X to the topological space Y. In particular, let $\mathcal{C} \triangleq C[[-\tau,0],R^n]$ denote the family of all bounded continuous R^n-valued functions φ defined on $[-\tau,0]$ with the norm $\|\varphi\| = \sup_{-\tau\le\theta\le 0}|\varphi(\theta)|$.

For any $\varphi \in \mathcal{C}$, we define

$$\varphi_t(s)=\varphi(t+s),\quad s\in[-\tau,0],\quad t\ge 0,\qquad \|\varphi_t\|_r=\sup_{-\tau\le s\le 0}|\varphi(t+s)|_r,$$

where $\quad |\varphi(t+s)|_r=\left[\sum_{i=1}^{n}|\varphi_i(t+s)|^r\right]^{1/r}.$

and $D^+\varphi(t)$ denotes the upper-right-hand derivative of $\varphi(t)$ at time t.

In this paper, we consider the following nonautonomous cellular neural network model with time-varying delays:

$$\begin{cases} \dfrac{dx_i(t)}{dt} = -c_i(t)x_i(t)+\displaystyle\sum_{l=1}^{m}\sum_{j=1}^{n}a_{ijl}(t)f_{ijl}(x_j(t-\tau_{ijl}(t)))+I_i(t), & t\ge 0,\\ x_i(s)=\phi_i(s), \quad -\tau\le s\le 0, \quad i\in\mathcal{N}, \end{cases} \tag{1}$$

where x_i corresponds to the state of the ith unit at time t; $f_{ijl}(x_j(t))$ denotes the output of the jth unit at time t; the integer n corresponds to the number of units in a neural networks and the integer m corresponds to the number of neural axons, that is, signals that emit from the ith unit have m pathways to the jth unit; $a_{ijl}(t)$ denotes the strength of the jth unit on the ith unit at time $t-\tau_{ijl}(t)$; $I_i(t)$ denotes the external bias on the ith unit at time t; $c_i(t)$ represents the rate with which the ith unit will reset its potential to the resting state in isolation when disconnected from the network and external input; and $\tau_{ijl}(t)$ corresponds to the transmission delay of the ith unit along the l axon of the jth unit at time t and there exists a constant τ such that $0\le\tau_{ijl}(t)\le\tau$. $\phi(s)=(\phi_1(s),\phi_2(s),\dots,\phi_n(s))^T\in C[[-\tau,0],R^n]$ is the initial function vector.

Definition 1. System (1) is said to be uniformly bounded if, for any constant $\delta>0$, there is a $B=B(\delta)>0$ such that, for all $t_0\in R_+$, $\phi\in\mathcal{C}$ and $\|\phi\|_r<\delta$, one has $\|x(t,t_0,\phi)\|<B$ for all $t\ge t_0$.

Definition 2. System (1) is said to be uniformly ultimately bounded if there is a $B>0$ such that, for any $\delta>0$, there exists a $T=T(\delta)>0$ such that $\|x(t,t_0,\phi)\|_r<B$ for $t\ge t_0+T$ and for all $t_0\in R_+$, $\phi\in\mathcal{C}$ and $\|\phi\|_r<\delta$.

Definition 3. System (1) is said to be globally exponentially stable, if there are constants $\lambda>0$ and $M\ge 1$ such that

$$\|x_t-y_t\|_r\le M\|\phi-\psi\|_r e^{-\lambda t}$$

for all $t\ge 0$, in which $\phi\in\mathcal{C}$ and $\psi\in\mathcal{C}$ are the initial functions of solutions $x(t)$ and $y(t)$, respectively.

Definition 4. The set $S\subset\mathcal{C}$ is called a global attracting set of (1) if, for any initial value $\phi\in\mathcal{C}$, the solution $x(t,t_0,\phi)$ converges to S as $t\to+\infty$. That is,

$$\mathrm{dist}(x(t,t_0,\phi),S)\to 0 \quad \text{as} \quad t\to+\infty,$$

where $\mathrm{dist}(\phi,S)=\inf_{\psi\in S}\|\phi-\psi\|_r$.

Consider the following equations:

$$\frac{dv_i(t)}{dt}=g_i(t,v_t),\quad i\in\mathcal{N}, \tag{2}$$

where $g_i(t,\varphi): R_+\times\mathcal{C}\to R$ is continuous with respect to (t,φ) and satisfies the Lipschitz condition with respect to $\varphi(i\in\mathcal{N})$.

Lemma 1 (*Li and Wen [11]*). *For system (2), let $g_i(t+\omega,\varphi)=g_i(t,\varphi)$ and the solutions be uniformly bounded and uniformly ultimately bounded. Then system (2) has a ω-periodic solution if, for any constant $\delta>0$, there is a constant $B=B(\delta)>0$ such that, for all φ with $\|\varphi\|_r<\delta$, we have $|g_i(t,\varphi)|<B$ for all $t\in[-\tau,0](i\in\mathcal{N})$.*

3. Main results

We all know that differential inequalities are the main tools for studying the continuous differential systems. In this section, we firstly introduce a new differential inequality to improve some Halanay-type inequalities in the early literature. Then, based on the new inequality, we investigate the dynamic behavior of nonautonomous cellular neural networks with time-varying delays, and derive out the sufficient conditions for ensuring the boundedness, attracting set, exponential stability and existence of periodic solution of the considered system.

Theorem 1. *Let $t_0<b\le+\infty$ and $u\in C[[t_0,b),R_+]$ be a solution of the following delay differential inequality:*

$$\begin{cases} D^+u(t)\le(-a+\alpha(t))u(t)+(b+\beta(t))[u(t)]_\tau+\gamma(t), & t\in[t_0,b),\\ u(t_0+s)\in\mathcal{C}, \quad s\in[-\tau,0], \end{cases} \tag{3}$$

where $\alpha(t)\ge 0$, $\beta(t)\ge 0$ for any $t\in[t_0,b)$, a, b are nonnegative constants and $a>b$, and there exists nonnegative constant J such that $0\le\gamma(t)\le J$ for any $t\in[t_0,b)$, then we have

$$u(t)\le ke^{-\lambda(t-t_0)}e^{\int_{t_0}^{t}\theta(s)\,ds}+\frac{Je^h}{a-b-r},\quad t\in[t_0,b), \tag{4}$$

provided that the initial condition satisfies

$$u(t)\le ke^{-\lambda(t-t_0)},\quad t\in[t_0-\tau,t_0], \tag{5}$$

where $\lambda>0$ satisfies

$$\lambda-a+be^{\lambda\tau}<0 \tag{6}$$

and $\theta(s)=\alpha(s)+\beta(s)e^{\lambda\tau}$ satisfies that there exist constants $r\in[0,\lambda)$ and $h\ge 0$ such that

$$\int_v^t\theta(s)\,ds\le r(t-v)+h \tag{7}$$

for any $t,\ v\in[t_0,b)$.

Proof. From $a>b$, by using continuity, we know there exists a constant $\lambda>0$ satisfying (6).

In order to get (4), we firstly prove

$$u(t)\le ke^{-\lambda(t-t_0)}e^{\int_{t_0}^{t}\theta(s)\,ds}+\int_{t_0}^{t}\gamma(v)e^{\int_v^t p(s)\,ds}\,dv,\quad t\in[t_0,b) \tag{8}$$

is true, where $p(s)=-a+b+\alpha(s)+\beta(s)$.