Neurocomputing 168 (2015) 896–907

Contents lists available at ScienceDirect

Neurocomputing

journal homepage: www.elsevier.com/locate/neucom

New delay-dependent stability criteria for neutral-type neural networks with mixed random time-varying delays ☆

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ARTICLE INFO

Article history:
Received 19 January 2015
Received in revised form
8 April 2015
Accepted 9 May 2015
Communicated by Y. Liu
Available online 19 May 2015

Keywords:
Neutral-type neural networks
Stability analysis
Terms-Quadratic convex combination
Mixed random time-varying delays
Linear matrix inequality

ABSTRACT

This study is concerned with the problem of stability analysis of neutral-type neural networks with mixed random time-varying delays. Firstly, by using a novel and resultful mathematical approach and considering the sufficient information of neuron activation functions, improved delay-dependent stability results are formulated in terms of linear matrix inequalities (LMIs). Secondly, in order to obtain less conservative delay-dependent stability criteria, an augmented novel Lyapunov–Krasovskii functional (LKF) that contains triple and quadruple-integral terms is constructed. Moreover, our derivation makes full use of the idea of second-order convex combination and the property of quadratic convex function, which plays a key role in reducing further the conservatism of conditions. Finally, four numerical examples are presented to illustrate the effectiveness and advantages of the theoretical results.

1. Introduction

In the past decade, neural networks has been the central focus of intensive research activities for their extensive applications in many practical systems, such as adaptive neural network control [1], associative memories [2], signal processing [3], circuit design [4], stochastic system [5], and other scientific areas [48–50].

It is well-known that time delay is frequently encountered in various neural networks owing to neural processing and signal transmission. However, time delay is often one of the main sources of poor performance or oscillations for these systems [6–39]. Therefore, the problem of stability analysis for delayed neural networks has attracted increasing attention in the recent. Besides, in order to obtain more less conservative stability criteria, numerous important and interesting methods have been also proposed in [6–20,26–29,34–39]. For example, free-weighting matrix technique and a convex optimization approach in [6,7,35,36], delay-fraction technique in [17,37], constructing new LKF containing triple-integral terms in [7,28,19,37], novel activation function conditions [19,20]. In [26], the authors derived some less conservative stability criteria by considering some useful terms and using free-weighting matrix technique. By considering the relationship between the time-varying delay and its lower and upper bound, the results obtained in [26] were improved in [27]. By constructing a new LKF and using free-weighting matrix method, some more lessconservative criteria than those obtained in [28] were proposed in [29]. Furthermore, based on the stability analysis, some other problems of neural networks have also been discussed, such as synchronization problems in [8], passivity analysis in [12,13,18], Markovian jumping neural networks [13,14], finite-time boundedness of state estimation in [19].

Moreover, it is worth pointing out that the most stability criteria for neural networks were obtained based only on the deterministic time-delay case or the information of variation range of the time delay. As a matter of fact, time delay in a stochastic fashion exists usually in a large amount of many industrial and engineering systems [10,11,44–46], such as chemical, biological, and networked control systems. Its probabilistic characteristic can be obtained by statistical methods, for instance, poisson distribution and normal distribution. Besides, it often appears in many real neural networks that some values of time delay are very large, but the probabilities of this time delay occurring in practice are very small. In this case, if only the constant time delay or the variation

☆This work was supported by National Basic Research Program of China (2010CB732501), National Natural Science Foundation of China (61273015), The National Defense Scientific Research Project (9140A27040213DZ02001), The Program for New Century Excellent Talents in University (NCET-10-0097).

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http://dx.doi.org/10.1016/j.neucom.2015.05.035
0925-2312/

range of time delay is employed to obtain the stability criteria, the conservatism of results [12–15] are more larger to a certain extent. Thus, improved stability conditions and better methods for neural networks with random delays have been proposed. In [10], by constructing a LKF and using some analysis techniques, delay-dependent robust exponential state estimation of Markovian jumping fuzzy NNs with mixed random time-varying delay is investigated. By employing the discrete-time Jensen inequality and the probability distribution of the time delays, less conservative results are established to guarantee that the stochastic neural networks are globally exponential stability in [11]. The controller synthesis problem of a non-linear networked controlled system subject to delays in the measurement and actuation channels is considered in [45]. Based on the construction of appropriate LKF and some inequality techniques, delay-dependent stability criteria of Markovian jumping genetic regulatory networks with mode-dependent probabilistic time-varying delays are obtained in terms of LMIs in [46]. By choosing the piecewise LKF together with LMIs technique and average dwell time approach, the exponential H_∞ filtering for switched neural networks with random time-varying delays are designed in [47].

On the other hand, the existing neural networks models in many cases cannot characterize the properties of a neural reaction process precisely because of the complicated dynamic properties of the neural cells in the real world. Moreover, it is indispensable to have some information about the derivative of the past state in the systems to characterize the dynamics of such complex neural reactions. Therefore, some researchers have begun introducing the neutral delay into neural networks and some results on the stability analysis for neural networks of neutral type have been reported in [21–25]. Many valid methods have been proposed in these results to reduce the conservatism of the stability criteria. In [21], the authors have studied the problem of stability analysis of neutral type neural networks with both discrete and unbounded distributed delays by employing an LMI approach. Based on the topological degree theory, Lyapunov method and LMI approach, the global asymptotic stability of neural networks of neutral type with leakage, discrete and distributed delays has been investigated in [22]. Some improved delay-dependent stability results have been established by using a delay-partitioning approach and new type of LKF in [23,24]. In addition, robust stability analysis for neutral-type neural networks with time-varying delays and Markovian jumping parameters has been also conducted by introducing the idea of a delay-decomposition technique in [25]. However, to the best of our knowledge, the problem of stability analysis of neutral-type neural networks with mixed random time-varying delays has not been fully studied yet, which are important in both theories and applications. The problem is interesting but also challenging, which motivated this study for us.

Based on the above-mentioned discussion, in this paper, we investigate firstly neutral type neural networks with mixed random time-varying delays by constructing an augmented LKF containing triple and quadruple-integral terms, such as $\int_{t-h}^{t}\int_{s_1}^{t}\int_{s_2}^{t}\eta_2^T(t,s)\{E_3Q_1E_3^T\}\eta_2(t,s)\,ds\,ds_1\,ds_2$ $\int_{t-h}^{t}\int_{s_1}^{t}\int_{s_2}^{t}\int_{s_3}^{t}\eta_2^T(t,s)\{E_3Q_2E_3^T\}\eta_2(t,s)\,ds\,ds_1\,ds_2\,ds_3$, $\int_{-\tau_1}^{0}\int_{\theta}^{0}\int_{t+\lambda}^{t}\eta_2^T(t,s)\{E_3Z_7E_3^T\}\eta_2(t,s)\,ds\,d\lambda\,d\theta$ and $\int_{-\tau_2}^{0}\int_{\theta}^{0}\int_{t+\lambda}^{t}\eta_2^T(t,s)\{E_3Z_8E_3^T\}\eta_2(t,s)\,ds\,d\lambda\,d\theta$, which play a key role in obtaining more less conservative stability conditions. Besides, a novel handling method is given to establish the relationship among $\int_{t-\tau_1}^{t}\dot{x}(s)Z_5\dot{x}(s)\,ds$, $\int_{t-\tau_1}^{t}x^T(s)\,ds$ and $x(t-\tau_1)$ and $\int_{t-\tau_2}^{t}\dot{x}(s)Z_6\dot{x}(s)\,ds$, $\int_{t-\tau_2}^{t}x^T(s)\,ds$ and $x(t-\tau_2)$, which may reduce further the conservatism of stability criteria. Furthermore, compared with previous results by using the first-order convex combination property, our derivation makes full use of the idea of second-order convex combination and the property of quadratic convex function given in the form of Lemmas 1 and 2 without employing Jensen's inequality and delay-partitioning approach. Finally, four numerical examples are given to illustrate the effectiveness and the advantage of the proposed main results.

Notation: Notations used in this paper are fairly standard: $\mathbb{R}^n$ denotes the n-dimensional Euclidean space, $\mathbb{R}^{n\times m}$ is the set of all $n\times m$ dimensional matrices; I denotes the identity matrix of appropriate dimensions, T stands for matrix transposition, the notation $X>0$ (respectively $X\geq 0$), for $X\in\mathbb{R}^{n\times n}$ means that the matrix is real symmetric positive definite (respectively, positive semi-definite); $diag\{r_1, r_2, \ldots, r_n\}$ denotes block diagonal matrix with diagonal elements r_i, $i=1,2,\ldots,n$, the symbol $*$ represents the elements below the main diagonal of a symmetric matrix, $\langle M\rangle_s$ is defined as $\langle M\rangle_s=\frac{1}{2}(M+M^T)$.

2. Preliminaries

Consider the following neural networks of neutral-type with mixed random time-varying delays:

$$\dot{y}(t)=-Ay(t)+Bg(y(t))+Cg(y(t-h(t)))+D\dot{y}(t-\tau(t))+I, \tag{1}$$

where $y(t)=[y_1(t),\ldots,y_n(t)]^T\in\mathbb{R}^n$ is the neural state vector, $g(y(t))=[g(y_1(t)),\ldots,g(y_n(t))]^T\in\mathbb{R}^n$ is the neuron activation function; $I=[I_1,\ldots,I_n]^T\in\mathbb{R}^n$ is an external constant input vector, $A=diag\{a_1,\ldots,a_n\}>0$, $B\in\mathbb{R}^{n\times n}$ is the interconnection weight matrix, $C\in\mathbb{R}^{n\times n}$ and $D\in\mathbb{R}^{n\times n}$ are the delayed interconnection weight matrices.

Besides, $\tau(t)\geq 0$ denotes the time-varying delay and is assumed to satisfy $0\leq\tau(t)\leq\tau_2$. To ensure the existence of a solution to (1), it is assumed that the time-varying delay $h(t)$ is differential function that satisfies $0\leq h(t)\leq h$ and $\dot{h}(t)\leq h_D$, and $\tau(t)$ has a bounded derivative.

In practice, there exists a constant τ_1, $0\leq\tau_1\leq\tau_2$, such that $\tau(t)$ takes values in $[0,\tau_1]$ and $(\tau_1,\tau_2]$ with certain probability. Therefore $\tau(t)$ is a random variable which takes values in the intervals $[0,\tau_1]$ and $(\tau_1,\tau_2]$. Here, the probability distribution of $\tau(t)$ is assumed to be

$$Prob\{0\leq\tau(t)\leq\tau_1\}=\alpha_0,\quad Prob\{\tau_1<\tau(t)\leq\tau_2\}=1-\alpha_0. \tag{2}$$

Then define a random variable as follows:

$$\alpha(t)=\begin{cases}1, & 0\leq\tau(t)\leq\tau_1,\\ 0, & \tau_1<\tau(t)\leq\tau_2.\end{cases} \tag{3}$$

From (2) and (3), we can obtain easily

$$\begin{aligned}Prob\{\alpha(t)=1\}&=Prob\{0\leq\tau(t)\leq\tau_1\}=\mathbb{E}\{\alpha(t)\}=\alpha_0,\\ Prob\{\alpha(t)=0\}&=Prob\{\tau_1<\tau(t)\leq\tau_2\}=1-\mathbb{E}\{\alpha(t)\}=1-\alpha_0,\end{aligned} \tag{4}$$

where $0\leq\alpha_0\leq 1$ is a constant and $\mathbb{E}\{\alpha(t)\}$ is the expectation of $\alpha(t)$, and we have

$$\mathbb{E}\{\alpha(t)-\alpha_0\}=0,\quad \mathbb{E}\{(\alpha(t)-\alpha_0)^2\}=\alpha_0(1-\alpha_0). \tag{5}$$

We introduce two time-varying delays $\tau_1(t)$ and $\tau_2(t)$ such that

$$\tau_1(t)=\begin{cases}\tau(t), & 0\leq\tau_1(t)\leq\tau_1,\\ 0 & \text{else},\end{cases}\qquad \tau_2(t)=\begin{cases}\tau(t), & \tau_1<\tau_2(t)\leq\tau_2,\\ 0 & \text{else},\end{cases}$$
$$\dot{\tau}_1(t)\leq\mu_1,\quad \dot{\tau}_2(t)\leq\mu_2. \tag{6}$$

Then, the system (1) can be rewritten as

$$\begin{aligned}\dot{y}(t)=&-Ay(t)+Bg(y(t))+Cg(y(t-h(t)))+\alpha(t)D\dot{y}(t-\tau_1(t))\\ &+(1-\alpha(t))D\dot{y}(t-\tau_2(t))+I,\end{aligned} \tag{7}$$

$$\begin{aligned}\dot{y}(t)=&-Ay(t)+Bg(y(t))+Cg(y(t-h(t)))+\alpha_0D\dot{y}(t-\tau_1(t))\\ &+(\alpha(t)-\alpha_0)D\dot{y}(t-\tau_1(t))\\ &+(1-\alpha_0)D\dot{y}(t-\tau_2(t))-(\alpha(t)-\alpha_0)D\dot{y}(t-\tau_2(t))+I,\end{aligned} \tag{8}$$