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Brief Papers

Observer-based cluster consensus control of high-order multi-agent systems

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ABSTRACT

The cluster consensus problem of high-order multi-agent systems is considered in this note with an observer-based control scheme. Sufficient condition for cluster consensus is presented in terms of easily checkable algebraic topology criterion. It is found that intra-cluster balanced topologies with antisymmetric (i.e., bidirectional and opposite signed) inter-cluster links promote cluster consensus, irrelevant of the magnitudes of intra-cluster coupling weights. The effectiveness of the theoretical results is illustrated by a numerical example.

1. Introduction

Multi-agent systems (MAS) have potential applications in varieties of areas, e.g., data fusion of sensor networks, task cooperation of robots, synchronization of distributed oscillators, and formation maneuver of unmanned vehicles. As the most fundamental research topic for MAS coordination, the consensus problem has been investigated intensively in the last decade, and many remarkable contributions have been made [1–4].

While most of the existing works are concerned with complete consensus cases (i.e., all the agents achieve a common state), in many real-world circumstances, agents in a network may work in groups to accomplish different but coordinated tasks. Examples include obstacle avoidance of animal herds, team hunting of predators, cooperative searching of autonomous vehicles for multiple objects, and task allocation over the network between groups. Related results also help to understand the opinion forming in social networks. As an extension of complete consensus, cluster consensus can be applied in lots of scenarios, thus it has been studied in depth recently [5–18].

This paper focuses on the cluster consensus control of high-order MAS, and addresses the two following problems:

(1) Existing literatures on cluster consensus only considered the state feedback control approach. However, the state information of agents may not be directly available to the neighbors in many applications. In this paper, an observer-based cluster consensus control scheme is presented.
(2) It was found in [10] and [11] that directed acyclic interaction topologies promote cluster consensus regardless of the magnitudes of the coupling strengths among the agents. Compared with the acyclically partitionable digraphs, topologies with cyclic couplings are more general, yet more complicated. Recently, [16] and [17] proved that when the intra-cluster coupling weights are stronger than certain lower bound, cluster consensus can be achieved. As for what kinds of non-acyclically partitionable topologies guarantee cluster consensus regardless of the magnitude of inter-agent coupling strengths, no results have been found in the literature up to date. By exploring the properties of block antisymmetric matrices, we show that intra-cluster balanced topologies with antisymmetric (i.e., bidirectional and opposite signed) inter-cluster links have such feature.

Throughout this paper, $\mathbb{C}^+$ represents the open right-half complex plane. $\sigma(A)$ denotes the set of all the eigenvalues of the square matrix A. Other notations are standard.

2. Preliminaries and problem formulation

The communication network of a system with N agents is modeled as a directed graph denoted by $\mathcal{G}=(\mathcal{V},\mathcal{E},\mathcal{A})$, where

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$\mathcal{V}=\{1,2,\ldots,N\}$ is the set of nodes representing agents, and $\mathcal{E}\subseteq\mathcal{V}\times\mathcal{V}$ is the set of edges representing the information flow. Suppose the agents can be partitioned into p clusters $\{\mathcal{V}_1,\mathcal{V}_2,\ldots,\mathcal{V}_p\}$. The relationships among agents within the same cluster are always cooperative, and that between agents from different clusters could be cooperative or antagonistic. Correspondingly, the inter-cluster couplings are either positive or negative, while the intra-cluster ones remain positive. The neighbor set of agent i is denoted by $\mathcal{N}_i$. The adjacent matrix $\mathcal{A}=[a_{ij}]\in\mathbb{R}^{N\times N}$ is defined as $a_{ij}\neq 0$ if $(j,i)\in\mathcal{E}$, otherwise $a_{ij}=0$. The Laplacian matrix $L=[l_{ij}]\in\mathbb{R}^{N\times N}$ for graph $\mathcal{G}$ is defined as $l_{ii}=\sum_{j=1}^{N}a_{ij}$ and $l_{ij}=-a_{ij}$, $i\neq j$, and can be presented in a block form corresponding to p clusters:

$$L=\begin{bmatrix} L_{11} & L_{12} & \cdots & L_{1p} \\ L_{21} & L_{22} & \cdots & L_{2p} \\ \vdots & \vdots & \ddots & \vdots \\ L_{p1} & L_{p2} & \cdots & L_{pp}\end{bmatrix}. \tag{1}$$

One commonly used assumption on the topology for inter-cluster desynchronization [6–12,16,17] is given below.

Assumption 1. Each row sum of L_{ij} is zero for $i,j\in\{1,2,\ldots,p\}$.

Assume that the state information of a agent is not available to its neighbors, and can only be measured indirectly by the relative output. The dynamics of the agents are modeled as

$$\dot{x}_i(t)=Ax_i(t)+Bu_i(t),\quad y_i(t)=Cx_i(t),\quad i=1,2,\ldots,N, \tag{2}$$

where $x_i\in\mathbb{R}^n$ and $y_i(t)\in\mathbb{R}^q$ are the state and output of agent i, $u_i\in\mathbb{R}^m$ is the input, $A\in\mathbb{R}^{n\times n}$, $B\in\mathbb{R}^{n\times m}$, $C\in\mathbb{R}^{q\times n}$.

Definition 1. The MAS (2) is said to achieve cluster consensus if for any initial states, $\lim_{t\to+\infty}\|x_i(t)-x_j(t)\|=0$ when agents i and j are in the same cluster, $i,j\in\{1,2,\ldots,N\}$.

3. Cluster consensus of MAS via observer-based control

Suppose the local information available for agent i is

$$z_i(t)=\sum_{j\in\mathcal{N}_i}a_{ij}\left(y_j(t)-y_i(t)\right)+d_i(y_{\hat{i}}(t)-y_i(t)), \tag{3}$$

where $\hat{i}$ denotes the cluster that agent i belongs to ($\hat{i}\in\{1,2,\ldots,p\}$), $y_{\hat{i}}(t)$ is the reference output for cluster $\hat{i}$ designed as $y_{\hat{i}}(t)=Cs_{\hat{i}}(t)$, $d_i>0$ if agent i is pinned and $d_i=0$ otherwise, $s_{\hat{i}}(t)$ is the reference state for pinning control in cluster $\hat{i}$ satisfying

$$\dot{s}_{\hat{i}}(t)=As_{\hat{i}}(t),\quad \lim_{t\to+\infty}\left\|s_{\hat{i}}(t)-s_{\hat{j}}(t)\right\|\neq 0,\ \text{when } \hat{i}\neq\hat{j}. \tag{4}$$

Based on relative output measurements, an observer-type cluster consensus protocol is designed as

$$\dot{\xi}_i(t)=(A+BK_o)\xi_i(t)+F\left(\sum_{j\in\mathcal{N}_i}a_{ij}C\left(\xi_j(t)-\xi_i(t)\right)-d_iC\xi_i(t)-z_i(t)\right),$$
$$u_i(t)=K_o\xi_i(t), \tag{5}$$

where $\xi_i(t)\in\mathbb{R}^n$ is the protocol state, $K_o\in\mathbb{R}^{m\times n}$ and $F\in\mathbb{R}^{n\times q}$ are the gain matrices to be determined.

Theorem 1. *For the MAS (2) under Assumption 1, if (A,B) is stabilizable, (A,C) is detectable, and $\sigma(L+D)\subseteq\mathbb{C}^+$, where $D=\mathrm{diag}\{d_1,d_2,\ldots,d_N\}$, there exist gain matrices K_o and F such that p-cluster consensus can be achieved with the observer-type protocol (5).*

Proof. From (4) we know that the reference state inputs $s_{\hat{i}}(t)$ $(\hat{i}=1,2,\ldots,p)$ are all special solutions to $\dot{s}(t)=As(t)$. Suppose $s_{\hat{i}}(t)=e^{At}s_{\hat{i}}(0)$ and $s_{\hat{j}}(t)=e^{At}s_{\hat{j}}(0)$, together with Assumption 1, we have

$$\sum_{j\in\mathcal{N}_i}a_{ij}\left(s_{\hat{j}}(t)-s_{\hat{i}}(t)\right)=\sum_{j\in\mathcal{N}_i}a_{ij}e^{At}\left(s_{\hat{j}}(0)-s_{\hat{i}}(0)\right)=0. \tag{6}$$

Let $\delta(t)=\left[\epsilon^T(t),\eta^T(t)\right]^T$ with $\epsilon(t)=[\epsilon_1^T(t),\epsilon_2^T(t),\ldots,\epsilon_N^T(t)]^T$, $\epsilon_i(t)=x_i(t)-s_{\hat{i}}(t)$, $\eta(t)=\xi(t)-\epsilon(t)$, and $\xi(t)=\left[\xi_1^T(t),\xi_2^T(t),\ldots,\xi_N^T(t)\right]^T$. Then, with (6), after a sequence of derivations, we have

$$\dot{\delta}(t)=\begin{bmatrix} I_N\otimes(A+BK_o) & I_N\otimes BK_o \\ \mathbf{0} & I_N\otimes A-(L+D)\otimes FC\end{bmatrix}\delta(t). \tag{7}$$

By the separation principle, the stability problem of (7) is equivalent to those of the following systems:

$$\dot{\epsilon}(t)=[I_N\otimes(A+BK_o)]\epsilon(t), \tag{8}$$

$$\dot{\eta}(t)=[I_N\otimes A-(L+D)\otimes FC]\eta(t). \tag{9}$$

Since (A,B) is stabilizable, there exists K_o such that $A+BK_o$ is Hurwitz, i.e., system (8) can be stabilized. Here by conducting a linear transformation to system (9) with $\hat{\eta}(t)=(U\otimes I_n)\eta(t)$, we have

$$\dot{\hat{\eta}}(t)=(I_N\otimes A-\Lambda\otimes FC)\hat{\eta}(t), \tag{10}$$

where U is a unitary matrix satisfies $U(L+D)U^{-1}=\Lambda$, and Λ is an upper triangular matrix with diagonal entries $\lambda_1,\lambda_2,\ldots,\lambda_N$ being the eigenvalues of $(L+D)$.

With the fact that Λ is upper triangular, the stability problem of system (10) can be further transformed into those of the subsystems

$$\dot{\hat{\eta}}_i(t)=(A-\lambda_iFC)\hat{\eta}_i(t),\quad i=1,2,\ldots,N. \tag{11}$$

With (A,C) detectable, there exists $P>0$ such that the following Riccati inequality holds:

$$PA+AP^T<C^TC.$$

Here we take $\alpha>1/(2\cdot\min\{\mathrm{Re}(\sigma(L+D))\})$, $F=\alpha P^{-1}C^T$, and construct the Lyapunov functional candidate as

$$V_i(t)=\hat{\eta}_i^TP\hat{\eta}_i,\quad i=1,2,\ldots,N.$$

Differentiating $V_i(t)$ along the trajectory of (11) yields

$$\begin{aligned}\dot{V}_i(t)&=\hat{\eta}_i^T(t)\left[(A-\lambda_iFC)^HP+P(A-\lambda_iFC)\right]\hat{\eta}_i(t)\\&=\hat{\eta}_i^T(t)\left[PA+A^TP-2\alpha\mathrm{Re}(\lambda_i)C^TC\right]\hat{\eta}_i(t)\\&<\hat{\eta}_i^T(t)\left[C^TC-2\alpha\mathrm{Re}(\lambda_i)C^TC\right]\hat{\eta}_i(t)<0.\end{aligned}$$

Thus F can asymptotically stabilize system (9). With the observer-type cluster consensus protocol (5) using feedback gain matrices K_o and F, $\epsilon(t)$ and $\eta(t)$ asymptotically converge to zeros, which is equivalent to $x_i(t)\to s_{\hat{i}}(t)$ and $\xi_i(t)\to 0$, as $t\to+\infty$. Thus p-cluster consensus is achieved.

Remark 1. Theorem 1 extends the work of [11] in two aspects. Firstly, the state feedback cluster control scheme applied in the literature is a special case of the observer-based approach with C as an identity matrix. Secondly, it can be proved that the topological requirements given in [10] and [11] are special cases that guarantee $\sigma(L+D)\subseteq\mathbb{C}^+$. Although the system topology is assumed to be fixed in the paper, the result in Theorem 1 can be extended to the switching topology case using a similar proof procedure applied in [11].

Remark 2. In many scenarios, the system model may be subject to some uncertainty, i.e., the dynamics of the agent is $\dot{x}_i(t)=(A+M\Delta N)x_i(t)+Bu_i(t)$, where M and N are known matrices of appropriate dimensions, and Δ is bounded uncertain matrix.