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Image segmentation based on weak fuzzy partition entropy

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ABSTRACT

Fuzzy partition entropy-based method is an effective way for image segmentation. In this paper a segmentation method based on a weak fuzzy partition is presented. Firstly, we propose a method to construct generalized fuzzy complement, and moreover construct a generalized fuzzy complement operator which has a nice property for parameter optimization in real application. Then, a one-dimensional (1D) weak fuzzy partition, a two-dimensional (2D) weak fuzzy partition being obtained by a Cartesian product of two 1D fuzzy partitions, are defined using the proposed generalized fuzzy complement. With these concepts, a weak fuzzy partition entropy-based image segmentation method is proposed. The method is described in the 1D and 2D cases by modeling the 1D and 2D histograms. The 2D approach allows us to ensure a spatial regularity of the fuzzy classification. Finally, a nested optimization method is developed, based on an improved uniformity measure, to search for the optimal threshold in the image segmentation method. Empirical results show that the proposed weak fuzzy partition entropy-based method is capable of achieving better segmentation results than several state-of-the-art methods that are based on or not based on fuzzy entropy. The proposed 2D weak fuzzy partition entropy-based method is especially effective for noisy images.

1. Introduction

Image segmentation, which is referred to dividing images into several meaningful areas, is the most basic and hardest part of image processing. Since Pal and King [1] firstly introduced fuzzy membership and its expression in gray image processing, fuzzy theory-based segmentation methods have been widely studied and are considered effective due to their excellent description ability for the fuzzy uncertainty of images [2,3].

Entropy measures the spontaneous dispersal of energy-how much energy is spread out in a process, or how widely spread out it becomes-as a function of temperature. Fuzziness, a feature of imperfect information, results from the lack of a crisp distinction between the elements belonging and not belonging to a set (i.e. the boundaries of a set under consideration are not sharply defined). Fuzzy entropy is firstly mentioned by Zadeh [4] and often used as a measure of fuzziness [5]. De Luca and Termini simulated Shannon entropy in information theory and gave an expression of fuzzy entropy on a fuzzy set [5]. Pal and King tried to apply fuzzy entropy for image threshold segmentation [1]. Since then, studying fuzzy entropy and applying it to image segmentation have become a hot research topic. Researchers studied on the construction of fuzzy membership [6], the expression of the threshold segmentation methods based on fuzzy entropy [7].

Usually, the fuzzy entropy-based segmentation methods segment an image according to the principle that membership value equal to 0.5 serves as the segmentation line, which is not applicable to some images and makes it necessary for the generalized fuzzy entropy-based segmentation methods to be developed.

Classical fuzzy entropy uses the basic union, intersection and complement operators defined by Zadeh [4], i.e. $c(\tau) = 1 - \tau (\tau \in [0,1])$. A notable feature of this complement operator is that its fixed point locates at 0.5, which means that the fuzzy set $A = [0.5]$ (i.e. $\mu_A(x) \equiv 0.5$ for all x in the universal set) has the most fuzziness. This also means that an image is segmented according to the principle that the membership degree equal to 0.5 serves as segmentation line. In 1982, Pal [8] pointed out the limitation of using 0.5 as the classification boundary. In 2000, Jawahas et al. [9] showed an example that the object regions were completely separated from the background according to the principle that the membership degree equal to 0.1 served as the segmentation line for the fuzzy c-means clustering algorithm, whereas it was not able to obtain satisfactory results according to the principle that the membership degree equal to 0.5 served as the segmentation

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line. Zenzo et al. [10] also found that it obtained bad segmented results for the images with strong background noise based on the basic fuzzy entropy methods with the principle that $A=[0.5]$ had the most fuzziness. They changed the conditions of fuzzy entropy with fuzzy set $A=[\alpha]$ $(\alpha \in (0,1))$ which had the most fuzziness. Using a mapping, they constructed a generalized fuzzy entropy formula based on the fuzzy entropy formula and applied it to image segmentation and denoising, and obtained good process results. Unfortunately, Zenzo's definition did not involve the generalized complement operator of a fuzzy set. That was to say, the fourth condition in the definition of fuzzy entropy was abandoned, resulting into incapability of connecting naturally with the classical fuzzy entropy. Based on the definition of fuzzy entropy and generalized complement operator, Fan et al. [11] presented a complete definition of generalized fuzzy entropy which was entirely corresponding to the conditions of the traditional fuzzy entropy, and gave an image segmentation method based on it. The method segmented an image based on the principle that $A=[\alpha]$ $(\alpha \in (0,1))$ had the most fuzziness, where the fuzzy complement operator played an important role.

As the generalizations of Zadeh's standard complement, many researchers have proposed various generalized fuzzy complement operators, of which there are two classic generalized fuzzy complement operators proposed by Sugeno [12] and Yager [13]. The unique fixed point α of the two generalized fuzzy complement operators can be any point in the interval $(0,1)$. However, the parameters corresponding to the unique fixed point α have large ranges. The parameter λ in Sugeno's complement operator takes value in $(-1,+\infty)$. The parameter w in Yager's complement operator takes value in $(0,+\infty)$. The wide ranges make it difficult to determine λ and w in actual applications. Therefore a new generalized fuzzy complement operator is constructed and used in image segmentation in this paper.

As another approach to fuzzy image segmentation, fuzzy partition entropy-based methods have also been proposed. Fuzzy partition means that a sample space is parted into several groups by some fuzzy sets. A well-known algorithm proposed by Cheng et al. [14] employed the concept of fuzzy c-partition and the maximum fuzzy partition entropy principle to 1D image segmentation. However, the entropy function used in [14] could not reflect the information of an image due to the assumption that the object and the background followed the same probability distribution, thus bad segmented results might be obtained in some cases. In 2002, Jin et al. [15] defined a novel fuzzy partition entropy using conditional probability and conditional entropy and applied it into 2D image segmentation, which successfully overcame the disadvantage that the probability of the object and the background were both equal to 0.5 and obtained good results. Zhao et al. [16] also presented an entropy function by the fuzzy c-partition (FP) and the probability partition (PP) which was used to measure the compatibility between the FP and the PP for 1D three-level thresholding. But this algorithm was also based on a necessary condition of the entropy function arriving at a maximum, where the probabilities of dark, gray and white parts were all equal to 1/3. Tao et al. [17] showed a 1D three-level thresholding method through maximizing the fuzzy entropy of the fuzzy region based on conditional probability and conditional entropy, and got better performance than Zhao's method.

The above mentioned fuzzy partition methods proposed by Cheng et al. [14], Jin et al. [15], Zhao et al. [16] and Tao et al. [17], are all based on the classical fuzzy partition concept. Actually classical fuzzy partition is a special case of weak fuzzy partition [18]. In this paper, we propose 1D and 2D weak fuzzy partition entropies using the new generalized fuzzy complement operator and apply them into image segmentation. In addition, we consider using an image segmentation quality evaluation criterion to determine the fixed point α in the generalized fuzzy complement operator and using an optimization algorithm, quantum genetic algorithm (QGA) [19], to reduce the computation time.

Usually image segmentation evaluation criterion is used to evaluate the effectiveness of a segmentation method. There are two common evaluation methods, supervised evaluation and unsupervised evaluation [20]. In supervised evaluation a segmented image is compared against a manually-segmented or pre-processed reference image, which requires user assistance and thus is usually infeasible in many real-time vision applications, so unsupervised methods are necessary. Unsupervised evaluation enables the objective comparison of both different segmentation methods and different parameterizations of a single method, without requiring human visual comparisons or comparison with a manually-segmented or pre-processed reference image. Uniformity measure is one of the most common methods for unsupervised evaluation [20]. However, for the evaluation of noisy image segmentation it is found by experiment that uniformity measure is quite different from the visual effects. In this paper, a new uniformity measure is proposed for the evaluation of noisy image segmentation and also used as the fitness function for parameter optimization in the proposed segmentation methods.

The rest of the paper is organized as follows. Section 2 describes fuzzy set and generalized fuzzy complement, and a new generalized fuzzy complement is defined. Section 3 discusses weak fuzzy partition and weak fuzzy partition entropy, shows 1D and 2D weak fuzzy partitions based on generalized fuzzy complement and their corresponding weak fuzzy partition entropies. In Section 4 a new image quality evaluation function is proposed, which is more close to human visual system for the evaluation of noisy image segmentation. Section 5 devotes to applying the two proposed weak fuzzy partition entropies to image segmentation. Section 6 displays and discusses the results. Finally conclusion is drawn in Section 7.

2. Fuzzy sets and fuzzy complement

In this section, a brief recall of some fuzzy theory is given and a method to construct generalized fuzzy complement is proposed. A new generalized fuzzy complement is also defined by the new method.

2.1. Fuzzy set theory and generalized fuzzy complement

Let $X=\{x_1,x_2,\ldots,x_n\}$ express a finite universe. $\mathbf{F}(X)$ expresses the set of all fuzzy sets on the universal set X. $\mathbf{P}(X)$ expresses the set of all crisp sets on the universal set X. $\mu_{\tilde{A}}(x)\in[0,1]$ is the membership function of $\tilde{A}\in\mathbf{F}(X)$. The value $\mu_{\tilde{A}}(x)$ indicates the degree of the element x belonging to $\tilde{A}$. Generally, a fuzzy set $\tilde{A}$ is defined as a collection of ordered pairs and can be expressed by the following notations:

$$\tilde{A}=\{(\mu_{\tilde{A}}(x_i),x_i)\,|\,i=1,2,\ldots,n\}=\mu_{\tilde{A}}(x_1)/x_1+\mu_{\tilde{A}}(x_2)/x_2+\cdots+\mu_{\tilde{A}}(x_n)/x_n$$
$$=\sum\nolimits_{i=1}^{n}\mu_{\tilde{A}}(x_i)/x_i. \tag{1}$$

For two fuzzy sets $\tilde{A}$ and $\tilde{B}$, the membership functions of $\tilde{A}\cap\tilde{B}$, $\tilde{A}\cup\tilde{B}$ are defined as $\mu_{\tilde{A}\cap\tilde{B}}(x)=\min(\mu_{\tilde{A}}(x),\mu_{\tilde{B}}(x))$, $\mu_{\tilde{A}\cup\tilde{B}}(x)=\max(\mu_{\tilde{A}}(x),\mu_{\tilde{B}}(x)),\forall x\in X$. $\tilde{A}^c$ is used to express the standard complement of $\tilde{A}$, that is $\mu_{\tilde{A}^c}(x)=1-\mu_{\tilde{A}}(x)$, $\forall x\in X$.

Definition 1. A function $c_\alpha:[0,1]\to[0,1]$ is called as a *generalized complement operator*, if c_α satisfies [12]: (C1) $c_\alpha(0)=1$ and $c_\alpha(1)=0$; (C2) for all $a,b\in[0,1]$, if $a\le b$ then $c_\alpha(a)\ge c_\alpha(b)$; (C3) c_α is a continuous function; (C4) for every point $\tau\in[0,1]$, $c_\alpha(c_\alpha(\tau))=\tau$.