Neurocomputing 166 (2015) 8–13

Contents lists available at ScienceDirect

Neurocomputing

journal homepage: www.elsevier.com/locate/neucom

Brief Papers

Consensus tracking for multi-agent systems with directed graph via distributed adaptive protocol

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ARTICLE INFO

Article history:
Received 22 June 2014
Received in revised form
29 March 2015
Accepted 9 April 2015
Communicated by: Rongni Yang
Available online 30 April 2015

Keywords:
Multi-agent systems
Directed graph
Adaptive control
Consensus tracking

ABSTRACT

The consensus tracking problem is investigated for multi-agent systems with directed graph. To avoid using any global information, a novel adaptive protocol is proposed based only on the relative state information. A monotonically increasing function for each agent is inserted into the protocol to provide extra freedom for design. By using matrix theory and appropriate Lyapunov techniques, it is shown that the consensus tracking can be achieved in a fully distributed fashion if agents' dynamics are stabilizable and the topological graph contains a directed spanning tree with the leader as the root node. A simulation example shows the effectiveness of the design method.

1. Introduction

Consensus, as a typical collective behavior in the area of coordinative control, has drawn significant research interest recently, due to its broad applications such as biological systems, satellite formation, sensor network [1], etc. A critical problem is how to design appropriate protocols and requires all the agents to converge to a common value [2,3]. Due to the spatial scattering of the agents and limited sensing capability of sensors, consensus protocols for multi-agent systems should be fully distributed, which means that the protocols do not need any global information, only depend on the local state or output information of each agent and its neighbors.

It is well-known that the agent dynamics and the interaction topological graphs are two key factors to achieve consensus. Based on the consideration of two factors, some efforts have been made recently. Theoretical results on consensus with multi-agent systems were presented including first-order integrator [4,5], second-order integrator [6,7], high-order integrator [8] and general linear dynamics [9,10]. Note that research on consensus problem with more practical nonlinear dynamics is still ongoing today. Recently, consensus by state feedback for general linear dynamics with the Lipschitz nonlinearity was studied in [11], and these results were generalized to the case with observer-based protocols [12]. Synchronization problem for neural networks with time delays was also investigated in [13,14].

It is worth noting that a prominent feature in the aforementioned works is that eigenvalue information of the Laplacian matrix associated with the communication graphs is required to be known a prior for the consensus protocol design. As indicated in [15], the eigenvalues are global information in the sense that each agent has to know the entire communication graphs to compute them. Therefore, the protocols designed in this manner cannot be implemented by each agent in a fully distributed fashion. To mitigate this shortcoming, an adaptive control presents a good approach. Recently, two types of distributed adaptive coupling weights, namely vertex-based strategy and edge-based strategy, were proposed in [16,17] to achieve synchronization of complex networks without utilizing global information on network topology, and this effective adaptive scheme was generalized to second-order consensus of multi-agent systems with nonlinear dynamics in [18] and high-order finite-time consensus with unknown nonlinear dynamics in [19]. To reach consensus for the general linear model, the static (or dynamical) adaptive protocol was constructed in [20,21] (or [22]) based on the relative states (or outputs) of neighbors. For the linear and Lipschitz nonlinear cases, distributed adaptive protocols and Lipschitz distributed adaptive protocols are designed in [23], under which leader-following consensus is reached over jointly connected topology. An adaptive control method was also applicable to other coordinative control problems, e.g. formation control of nonholonomic mobile robots [24], satellite formation flying [25], to name just a few. However, all these works [16–25] are based on bidirectional information exchange, modeled by undirected graph or leader-follower graph where the sub-graph among the followers is undirected.

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http://dx.doi.org/10.1016/j.neucom.2015.04.057
0925-2312/

Indeed, a critical and standard assumption that underlies the available literatures on adaptive consensus is that the network topological graph is undirected. As argued in [26], networks operating under adverse environments may be prone to packet losses or node failure, which may render symmetric exchange protocols and bidirectional communication infeasible. Thus, the undirected graphs imply more stringent restriction on the communication protocols in practice. As we all know, the directed graphs are quite general since they include the undirected graphs as a special case. Thus, the adaptive protocol on the symmetric graphs may be invalid when asymmetric graphs are encountered. So, it's necessary and urgent to extend communication network from the bidirectional case to the unidirectional case. However, this extension is nontrivial and introduces significant technical challenges. The main difficulty lies in that the Laplacian matrix of directed graph is asymmetric, which makes the construction of adaptive protocols and the selection of Lyapunov function very intractable [27,28].

Motivated by the above analysis, this paper considers the consensus tracking problem for multi-agent systems over directed graph (not necessarily symmetric and balanced). A novel node-based adaptive protocol is proposed here by assigning a time-varying coupling weight to each node (i.e. each agent), and this protocol, independent of any global information, relies only on the relative states of neighboring agents, and therefore is fully distributed. The main contributions of this paper are threefold. First, aim at rendering that our protocol is fully distributed and is also unidirectional information exchange, a monotonically increasing function for each agent is inserted into our protocol to provide extra freedom for design, this idea is inspired by adding surplus variables to tackle quantized consensus on digraph [26]. Second, motivated by the Lyapunov techniques in [29], an integral Lyapunov function is artfully constructed to prove the stability of the tracking error system. At last, compared with the existing adaptive protocols on digraphs [28], the merits of our protocol are as follows: the feedback gain matrices can be determined by solving a simple LMI, and its solution existence can be guaranteed by system controllability. Thence, the solvability of consensus tracking problem can be ensure; duo to avoiding canonical decomposition, the complexity of protocol design is reduced, and then our protocol is easy to design and implement; finally, the convergence analysis is not too intricate due to elaborately constructed Lyapuny function.

The remainder of the paper is organized as follows. Section 2 provides some preliminaries and problem formulation. In Section 3, the main results for the consensus tracking over digraph are presented. Simulation results are given showing the effectiveness of our proposed protocol in Section 4, and conclusions and possibilities of future work comprise Section 5.

2. Preliminaries and problem formulation

2.1. Preliminaries

Let $\mathbb{R}^{n\times n}$ be the set of $n\times n$ real matrices. T means the transpose for matrix. I_p represents the identity matrix of dimension p. Denote by $\mathbf{1}$ a column vector with all entries equal to one. The matrix inequality $A > B$ means $A-B$ is positive definite. $\lambda_{\max}(P)$ (or $\lambda_{\min}(P)$) denotes the maximal (or minimal) eigenvalue of the symmetric matrix P. For a vector x, let $\|x\|$ denote its 2-norm. $\otimes$ denotes Kronecker product.

Let $\mathcal{G}=(\mathcal{V},\mathcal{E},\mathcal{A})$ be a weighed digraph with the set of nodes $\mathcal{V}=\{v_1,v_2,\cdots,v_N\}$ and the set of edges $\mathcal{E}\subseteq\mathcal{V}\times\mathcal{V}$, and the adjacency matrix $\mathcal{A}=[a_{ij}]\in\mathbb{R}^{N\times N}$ where $a_{ij}>0$ if $(v_j,v_i)\in\mathcal{E}$, and $a_{ij}=0$ otherwise. Denote by $\mathcal{L}=[l_{ij}]\in\mathbb{R}^{N\times N}$ the Laplacian matrix of $\mathcal{G}$, where $l_{ij}=-a_{ij}$, $i\neq j$, and $l_{ii}=\sum_{j=1,j\neq i}^{N}a_{ij}$.

Lemma 1. *(Young's Inequality) [30]. If a and b are nonnegative real numbers, p and q are positive real numbers such that* $(1/p)+(1/q)=1$, *then* $a\cdot b\leq(a^p/p)+(b^q/q)$.

2.2. Problem formulation

Consider a collection of $N+1$ agents, where one agent indexed by 0 is the leader and the rest labeled by $1,\cdots,N$ are followers. The dynamics of the leader can be described as

$$\dot{x}_0=Ax_0 \tag{1}$$

and the dynamics of the followers are given by

$$\dot{x}_i=Ax_i+Bu_i,\quad i=1,\cdots,N. \tag{2}$$

where the state x_0, $x_i\in\mathbb{R}^n$ and the control input $u_i\in\mathbb{R}^p$, $i=1,\cdots,N.A\in\mathbb{R}^{n\times n}$, $B\in\mathbb{R}^{n\times p}$are constant matrices.

Consider a directed graph $\overline{\mathcal{G}}$ consisting of N followers and one leader. The information exchange between N followers is represented by a directed graph $\mathcal{G}$, which is a subgraph of $\overline{\mathcal{G}}$. The adjacency matrix between the leader and followers is the diagonal matrix $\mathcal{B}=diag(b_1,b_2,\cdots,b_N)$, where $b_i>0$ if the follower i has access to the leader's state, and $b_i=0$ otherwise. In most cases, the leader's state information cannot be available to all followers but to only a part of followers. Denote $\mathcal{H}=\mathcal{L}+\mathcal{B}$. A useful property associated with $\mathcal{H}$ is given in the following lemma.

Lemma 2. *[31]. If the directed graph $\overline{\mathcal{G}}$ contains a spanning tree with the root node being the leader agent, then $\mathcal{H}$ is invertible. Moreover, denote* $(\theta_1\cdots,\theta_N)^T=\mathcal{H}^{-1}\mathbf{1}_N$ *and* $\Theta=\mathrm{diag}\left\{\frac{1}{\theta_1},\cdots,\frac{1}{\theta_N}\right\}$, *then* Θ *and* $\Theta\mathcal{H}+\mathcal{H}^T\Theta$ *are positive definite matrices.*

The objective of this paper is to solve the following consensus tracking problem for systems (1) and (2).

Definition 2. (Consensus tracking problem). Design distributed control laws $u_i=u_i(e_i)$, $i=1,\cdots,N$, where e_i denotes the local relative state information between agent i and its neighboring agents, and it is given by

$$e_i\triangleq\sum_{j=1}^{N}a_{ij}(x_i-x_j)+b_i(x_i-x_0),\quad i=1,\cdots,N \tag{3}$$

such that for any initial conditions $x_i(0)$, the following equality

$$\lim_{t\to\infty}\|x_i(t)-x_0(t)\|=0,\quad i=1,\cdots,N$$

holds

To facilitate our analysis, we need the following mild assumptions.

Assumption 1. (A,B) is stabilizable.

Remark 1. Assumption 1 is equivalent to that linear matrix inequality (LMI) (6) has a feasible positive definite solution P [9].

The existing work on adaptive protocol which can be only applied to the undirected graphs or leader-follower graphs where the subgraphs among the followers are undirected [16–18,20–22], and this restriction is relaxed to the following case.

Assumption 2. The graph $\overline{\mathcal{G}}$ contains a directed spanning tree with the leader as the root node.

3. Main results

In this section, our objective is to find a distributed adaptive protocol for systems (1) and (2) over digraph to solve the consensus tracking problem. Based on the relative state information (3), a novel distributed adaptive protocol is proposed as