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New stability condition for discrete-time fully coupled neural networks with multivalued neurons ☆

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ABSTRACT

This paper discusses the stability condition for discrete-time multi-valued recurrent neural networks (MVNRNNs) in asynchronous update mode. In the existing research literature, an MVNRNN in asynchronous update mode has been found convergent if its weight matrix is Hermitian with nonnegative diagonal entries. However, our finding has been that the weight matrix with zero diagonal entries cannot guarantee the network stability. Furthermore, the new stability condition and proof is offered to allow diagonal entries to be complex-valued, which extends previous theoretical result. Simulation results are used to illustrate the theory.

1. Introduction

The multi-valued neuron (MVN) was first introduced in [7], and its theory was further extended in [5,8]. Neural networks with MVN (MVNNNs) adopt a complex-valued activation function, which maps complex-valued inputs into outputs on the unit circle in the complex domain.

An MVN activation function is different to other neural networks' function. Firstly, function output is sensitive to input's argument, which lies on $[0, 2\pi)$. Thus we should consider the imaginary part and real part all together. Such situation never happens in real-valued neural networks (NNs), or those complex-valued NNs which deal with imaginary part and real part separately. Secondly, many activation functions are monotonously increasing or decreasing, and changing the output from its maximum to minimum involves time. However, MVN can switch its output state quickly, by simply multiplying a complex number to change its argument. It has been shown that the functionality of an MVN is higher than the functionality of a sigmoidal neuron [6]. For example, a multilayer neural network based on MVN outperforms a classical multilayer feed-forward network and several kernel-based NNs with faster learning speed and fewer neurons [4]. Some successful recent applications of MVNNNs have been reported in [1–4].

Dynamical analysis is of primary importance for emulation of stability and of fixed points of recurrent neural networks (RNNs). Comparing dynamical analysis work with real-valued analysis, the work in complex-valued domain is more difficult because we only get partial theoretical support from current mathematical theories. For example, it seems impossible to study the dynamics of continuous-time complex-valued MVNRNNs directly, because MVN activation function is not holomorphic. Fortunately, the requirement for holomorphism can sometimes be eliminated when we deal with discrete-time RNNs. On the other hand, due to the strong component correlation between real and imaginary parts, the decrease or increase in either imaginary or real part usually means nothing. In real-valued domain, it is common to observe, calculate, or analyze the trajectory of RNNs simply based on the network outputs' decrease or increase. However, in complex-valued domain, we need to investigate the trajectory movement on the whole complex plane.

Stability and complete stability are different concepts. If a network is stable, stable fixed point(s) or periodic solution (s) may exist. However, if a network is completely stable, all trajectories will be convergent, which means that no periodic

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solutions exist. In most cases, we hope RNNs to be completely stable.

RNNs dynamics greatly depends on the update model used. Like other RNNs, MVNRNNs use two update models: synchronous update mode and asynchronous update mode. The stability condition in synchronous update mode can been found in [14]. Here, we focus on MVNRNNs' dynamics in asynchronous update mode. In the seminal paper [9], the stability condition has established that a discrete-time MVNRNN is convergent if its weight matrix is Hermitian with nonnegative diagonal entries. Applications related to this property focus on associative memory design [5,6,9,11]. In [12], through analyzing the same energy function used in [9], the authors prove that the energy function for each of the stored patterns will also take the minimum values. Although much successful work has been based on [9], an obscure flaw exists in its original proof. According to our most recent research, an MVNRNN may not be completely stable if its weight matrix is Hermitian with zero diagonal entries [15].

Another interesting topic is to study MVN networks with non-Hermitian matrices. In [13], a threshold complex-valued neural networks associative memory is proposed for information retrieval. The test results show that MVN networks with small asymmetry in weight matrix can be stable and function as well as Hermitian one. However, the analytical proof of stability of such MVN networks is still missing.

In this paper, we deal with MVN networks with non-Hermitian weights, and present a revised stability condition based on [9,15], which extends previous results by allowing MVNRNNs to be completely stable with complex-valued diagonal entries. Regarding asymmetric MVN networks in asynchronous update mode, to the best of our knowledge, no theoretical stability result has been reported. Therefore, our work also presents a novel research approach to study the stability of MVN networks with non-Hermitian matrices.

The rest of this paper is organized as follows. The architecture of MVNRNNs is described in Section 2. Section 3 is the theoretical analysis. Simulations are presented in Section 4. Conclusions are given in Section 5.

2. Multi-valued recurrent neural networks

The MVN model is based on the activation function defined as complex-signum operation (see Fig. 1). For a specified number of values K, called the resolution factor, and an arbitrary complex

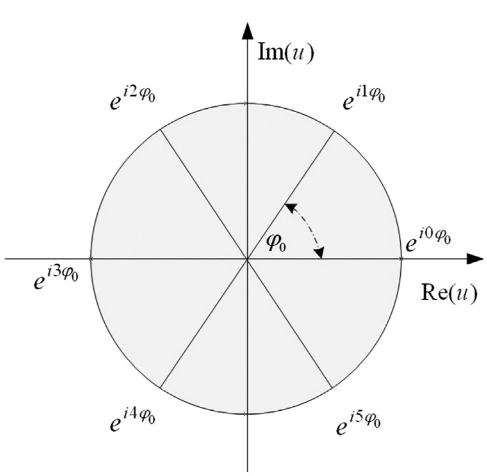

Fig. 1. Complex signum function CSIGN (case shown for $K=6$).

number u, the complex-signum function is defined as follows:

$$CSIGN(u) \triangleq \begin{cases} z^0, & 0 \le \arg(u) < \varphi_0 \\ z^1, & \varphi_0 \le \arg(u) < 2\varphi_0 \\ \vdots & \\ z^{K-1}, & (K-1)\varphi_0 \le \arg(u) < K\varphi_0 \end{cases},$$

where φ_0 is a phase quantum delimited by $K: \varphi_0 = 2\pi/K$, and z is the corresponding Kth root of unity: $z = e^{i\varphi_0}$. Then, the output state of each neuron is represented by a complex number from the set $\{z^0, z^1, \ldots, z^{K-1}\}$. Thus the network state $s(k)$ at k-th iteration number, is a complex-valued vector of n components $s(k) = [s_1(k), s_2(k), \ldots, s_n(k)]^T$. For simplicity, in this paper, we use $\sigma(\cdot)$ instead of $CSIGN(\cdot)$.

Each input $I_m(k+1)$ of the mth neuron is dependent upon the network state $s(k)$ through synaptic weights w_{ij}:

$$I_m(k+1) = \sum_{j=1}^{n} w_{mj} s_j(k) + h_m,$$

where h_m is a bias, $W = (w_{ij})_{n\times n}$ is a complex-valued matrix, each of its elements w_{mj} denotes the synaptic weights and represents the strength of the synaptic connection from neuron m to neuron j. Here, we set $h_m = 0$ for simplicity. The output of the m-th neuron is centered within respective K sectors shown in Fig. 1, and we have

$$s_m(k+1) = \sigma(I_m(k+1) \cdot z^{1/2}) \tag{1}$$

where $z^{1/2} = e^{i(\varphi_0/2)}$.

3. New stability condition for MVNRNNs

First, we provide preliminaries used in the following to establish the theory.

For any $c \in \mathbb{C}$, we denote

$$c^* = (\overline{c})^T,$$

where $\overline{c}$ is the conjugate of c.

Definition 1. A vector $s^\dagger$ is called an equilibrium point (fixed point) of network (1), if each element $s_m^\dagger$ in $s^\dagger$ satisfies

$$s_m^\dagger = \sigma\left(z^{1/2} \cdot \sum_{j=1}^{n} w_{mj} s_j^\dagger \right).$$

Clearly, for $s^\dagger$, it holds that

$$s^\dagger = \sigma(WQs^\dagger),$$

where $Q = diag(z^{1/2}, z^{1/2}, \ldots, z^{1/2})$.

Denote by Ω the set of equilibrium points of the network (1).

Definition 2. The network (1) is said to be completely convergent (completely stable), if each trajectory $s(k)$ satisfies

$$dist(s(k), \Omega) \triangleq \min_{x^* \in \Omega} \| s(k) - s^\dagger \| \to 0$$

as $k \to +\infty$.

Theorem 1. *For a complex-valued matrix W, if W can be presented as $W = W' + D$, where W' is a Hermitian matrix with zero diagonal entries ($w'_{ii} = 0$ and $w'_{ij} = w_{ij} = w^*_{ji}$), D is a diagonal matrix with diagonal elements $d_{ii} = w_{ii} \neq 0$ and $\arg(d_{ii}) \in [0, \varphi_0/2) \cup (2\pi - \varphi_0/2, 2\pi)$ for all $i,j \in \{1,2,\ldots,n\}$, then the network (1) is completely convergent.*