Neurocomputing 166 (2015) 193–200

Contents lists available at ScienceDirect

Neurocomputing

journal homepage: www.elsevier.com/locate/neucom

H_∞ filter design for fuzzy systems with quantized measurements ☆

Shaosheng Zhou [a,*], Yanpeng Guan [b]

[a] School of Automation, Hangzhou Dianzi University, Hangzhou, Zhejiang 310018, PR China
[b] Department of Automation, Shanxi University, Taiyuan 030006, PR China

ARTICLE INFO

Article history:
Received 14 January 2015
Received in revised form
25 March 2015
Accepted 8 April 2015
Communicated by Hongyi Li
Available online 20 April 2015

Keywords:
Basis-dependent Lyapunov function
H_∞ filtering
Quantization
T–S model

ABSTRACT

This paper addresses the problem of H_∞ filtering for a class of discrete time T–S model based fuzzy systems with quantized measurements. The measurement signal is quantized by a logarithmic quantizer. By using basis dependent Lyapunov function, a sufficient condition is provided to ensure the stability of the filtering error system and to guarantee a given H_∞ performance level. Based on this analysis result of the filtering error system, an explicit filter design approach is proposed and the basis dependent filtering algorithm is further improved by using a kind of relaxed technique. Two numerical examples are also presented to show that the filtering algorithm based on the basis dependent Lyapunov function has less conservative than the one based on the basis independent Lyapunov function.

1. Introduction

Recently, signal quantization has attracted great attention. It usually arises in a digital communication channel, which is an important component of distributed or networked-based dynamical system. It is essential that signals should be quantized before being transmitted through a communication channel with limited bandwidth. As a result, various quantizers have been designed to achieve different estimation or control objectives. For quadratically stabilizing linear system with quantized feedback, a kind of static logarithmic quantizer is constructed in [1], where logarithmic quantizers are shown to be the coarsest ones to solve the problem. The authors in [2] generalize this result to a few quantized feedback design problems by using a sector bound approach (see also [3]). To study the feedback stabilization problem of linear systems, [4] provides a dynamic quantizer, which can be adjusted by updating the parameter of the quantizer at discrete instants of time. However, it is inevitable that there will be data losses resulting from signal quantization, which may deteriorate the performance of the systems or even cause the systems instability. Hence, it is necessary to consider the quantization effect when signal quantization emerges in a system [18,21].

As to the H_∞ filtering problem, it is concerned with the design of an estimator guaranteeing that the L_2-gain from the noise signal to the estimation error is within a prescribed level. Compared with a Kalman filter, one advantage of a H_∞ filter is that no statistical assumption on the noise signal is needed [5]. In the last decade, the H_∞ filtering problem of T–S fuzzy systems has been widely investigated (see, for example, [6–11,19,22,24], and the references therein). A basis dependent Lyapunov function approach was proposed to study the H_∞ filter design problem for a class of discrete time T–S fuzzy systems in [6], and it is shown that the results obtained by this approach are less conservative than those obtained by the single Lyapunov function approach. By a descriptor representation approach, a gain scheduled fuzzy filter is developed for T–S model based systems with linear fractional uncertainties in [23]. The authors in [20] introduce a fuzzy-filtering method, which can get a better noise-attenuation performance when frequency ranges of noises are known beforehand. For a typical H_∞ filter, it is assumed that the measurement output of the system can be transmitted directly to the filter without information loss. However, in practice, the filter may be geographically separated from the plant, and the measurement signals are required to be quantized before being transmitted through communication channels [12]. In this case, data losses occur as a result of signal quantization, and the H_∞ filtering problem under such situation is worthy of investigation.

Inspired by the results mentioned above, this paper will study the problem of H_∞ filter design for a class of discrete time fuzzy systems, in which the quantization effect will be considered. It is also worth mentioning that the well-known logarithmic quantizer will be considered and that the basis dependent Lyapunov approach will be used to develop a fuzzy filter.

The paper is organized as follows. The H_∞ filtering problem with quantized measurement is formulated in Section 2. The analysis of stability with H_∞ performance for the filtering error system is given in

☆This work was supported in part by 973 Program no. 2012CB821204, by the National Natural Science Foundation of PR China under Grants 61273093 and 61333009, also by the Natural Science Foundation of Zhejiang Province under Grant LZ12F03001.

* Corresponding author.
E-mail address: sszhou65@163.com (S. Zhou).

http://dx.doi.org/10.1016/j.neucom.2015.04.016
0925-2312/

Section 3. The result will then be employed in Section 4 to design a H_∞ filter. An example is given to illustrate the effectiveness of the approach in Section 5, and the paper is concluded in Section 6.

Notation: Throughout this paper, a real symmetric matrix $P>0\ (\geq 0)$ denotes P being a positive definite (or positive semidefinite) matrix, and $A>(\geq)B$ means $A-B>(\geq)0$. I is an identity matrix with appropriate dimension. The superscript τ represents the transpose. The symbol $*$ is used as an ellipsis for terms that are induced by matrix symmetry. The notation $l_2[0,\infty)$ represents the space of square summable infinite vector sequences with the usual norm $\|\cdot\|_2$. A sequence $\omega=\{\omega_k\}\in l_2[0,\infty)$ if $\|\omega\|_2=\sqrt{\sum_{k=1}^{\infty}\omega_k^\tau\omega_k}<\infty$.

2. Problem formulation

Consider the discrete-time T–S model based nonlinear system described by [6]

Plant Rule i: IF ξ_{1k} is F_1^i and ξ_{2k} is F_2^i and $\cdots$ and ξ_{rk} is F_r^i, THEN

$$x_{k+1}=A_ix_k+B_i\omega_k$$
$$y_k=C_ix_k+D_i\omega_k$$
$$z_k=L_ix_k,\quad i=1,\dots,s$$

where F_j^i is a fuzzy set and s is the number of IF–THEN rules; $x_k\in R^n$, $\omega_k\in R^p$, $z_k\in R^q$, and $y_k\in R^m$ denote the state, noise signal, signal to be estimated, and the measurement output respectively; and A_i, B_i, C_i, D_i and L_i are system matrices with compatible dimensions. The symbols $\xi_{1k},\dots,\xi_{rk}$ denote the premise variables. The fuzzy basis functions are defined by

$$h(\xi_k):=(h_1(\xi_k),h_2(\xi_k),\dots,h_s(\xi_k))\in\Xi,\quad h_i(\xi_k)=\frac{\prod_{j=1}^r\mu_{ij}(\xi_{jk})}{\sum_{l=1}^s\prod_{j=1}^r\mu_{lj}(\xi_{jk})},\quad i=1,\dots,s$$

in which $\mu_{ij}(\xi_{jk})$ is the membership function of fuzzy set F_j^i, $\xi_k:=(\xi_{1k},\dots,\xi_{rk})$ and the set Ξ is defined by

$$\Xi:=\left\{h(\xi_k):=(h_1(\xi_k),h_2(\xi_k),\dots,h_s(\xi_k)):\sum_{i=1}^s h_i(\xi_k)=1,\ h_i(\xi_k)\geq 0,\ i=1,\dots,s\right\}. \tag{1}$$

In order to simplify the symbols, we will drop the argument ξ_k of the fuzzy basis function $h(\xi_k)$ as well as its elements $h_i(\xi_k)$ in the following development. The system under consideration can be compactly represented by Σ:

$$x_{k+1}=A(h)x_k+B(h)\omega_k \tag{2}$$

$$y_k=C(h)x_k+D(h)\omega_k \tag{3}$$

$$z_k=L(h)x_k \tag{4}$$

where

$$\begin{bmatrix}A(h)&B(h)\\C(h)&D(h)\end{bmatrix}=\sum_{i=1}^s h_i\begin{bmatrix}A_i&B_i\\C_i&D_i\end{bmatrix},\quad L(h)=\sum_{i=1}^s h_iL_i. \tag{5}$$

The filter we considered is in the form of

$$\hat{x}_{k+1}=A_f(h)\hat{x}_k+B_f(h)\overline{y}_k \tag{6}$$

$$\hat{z}_k=L_f(h)\hat{x}_k \tag{7}$$

$$\overline{y}_k=f(y_k) \tag{8}$$

where the basis dependent matrices $A_f(h)$, $B_f(h)$ and $L_f(h)$ are to be determined. $\hat{x}_k\in R^n$ is the filter state, $\overline{y}_k\in R^m$ is the input of the filter and $f(\cdot)=[f_1(\cdot)\ \cdots\ f_m(\cdot)]^\tau$ is a quantizer which is assumed to be symmetric, that is $f(-v)=-f(v)$. Here we employ the static logarithmic quantizer. For each $f_j(\cdot)$, the set of quantized levels is described by

$$\mathcal{U}_j=\{\pm u_i^{(j)},\ u_i^{(j)}=\rho_j^iu_0^{(j)},\ i=0,\pm1,\pm2,\cdots\}\cup\{0\},$$
$$0<\rho_j<1,\ u_0^{(j)}>0. \tag{9}$$

Each of the quantization level $u_i^{(j)}$ corresponds to a segment such that $f_j(\cdot)$ can map the whole segment to this quantization level. Furthermore, all the segments form a partition of R. That is, they are disjoint and exhaustive. For the logarithmic quantizer, the associated quantizer $f_j(\cdot)$ is defined as follows [1,2]:

$$f_j(v)=\begin{cases}u_i^{(j)} & \text{if } \dfrac{1}{1+\sigma_j}u_i^{(j)}<v\leq\dfrac{1}{1-\sigma_j}u_i^{(j)},\quad v>0\\ 0 & \text{if } v=0\\ -f_j(-v) & \text{if } v<0\end{cases} \tag{10}$$

where

$$\sigma_j=\frac{1-\rho_j}{1+\rho_j}. \tag{11}$$

According to [1,2], it is easily verified from (8)–(11) that the quantized measurement $\overline{y}_k$ can be expressed as

$$\overline{y}_k=f(y_k)=[f_1(y_{k1})\ \cdots\ f_m(y_{km})]^\tau=(I+\Lambda(k))y_k \tag{12}$$

where y_{kj} denotes the jth component of y_k, and

$$\Lambda(k)=\text{diag}\{\Lambda_1(k),\Lambda_2(k),\dots,\Lambda_m(k)\},\quad |\Lambda_j(k)|\leq\sigma_j,\ j=1,2,\dots,m. \tag{13}$$

Let

$$\overline{x}_k=\left[x_k^\tau\ \hat{x}_k^\tau\right]^\tau,\quad \tilde{z}_k=z_k-\hat{z}_k. \tag{14}$$

Then the filtering error system concluded from (2)–(8) and (13)–(14) can be described by $\tilde{\Sigma}$:

$$\overline{x}_{k+1}=\overline{\mathcal{A}}(h)\overline{x}_k+\overline{\mathcal{B}}(h)\omega_k \tag{15}$$

$$\tilde{z}_k=\mathcal{L}(h)\overline{x}_k \tag{16}$$

where

$$\overline{\mathcal{A}}(h)=\mathcal{A}(h)+\mathcal{B}_f^\tau(h)\Delta(k)\mathcal{C}(h),\quad \overline{\mathcal{B}}(h)=\mathcal{B}(h)+\mathcal{B}_f^\tau(h)\Delta(k)\mathcal{D}(h) \tag{17}$$

$$\mathcal{A}(h)=\begin{bmatrix}A(h)&0\\B_f(h)C(h)&A_f(h)\end{bmatrix},\quad \mathcal{B}(h)=\begin{bmatrix}B(h)\\B_f(h)D(h)\end{bmatrix},$$
$$\mathcal{C}(h)=\begin{bmatrix}\Lambda C(h)&0\end{bmatrix},\quad \mathcal{L}(h)=\begin{bmatrix}L(h)&-L_f(h)\end{bmatrix},\quad \mathcal{B}_f(h)=\begin{bmatrix}0&B_f^\tau(h)\end{bmatrix},$$
$$\mathcal{D}(h)=\Lambda D(h),\quad \Delta(k)=\Lambda(k)\Lambda^{-1},\quad \Lambda=\text{diag}\{\sigma_1,\sigma_2,\dots,\sigma_m\}. \tag{18}$$

The H_∞ filtering problem to be addressed in this paper is described as follows. For the T–S model nonlinear system with quantized signals, given a prescribed noise attenuation level γ, determine a stable filter in the form of (6)–(7) such that the following specifications are satisfied:

(I): The resulting filtering error system $\tilde{\Sigma}$ is asymptotically stable for any fuzzy basis function $h\in\Xi$ when $\omega_k\equiv0$.
(II): The l_2-gain from the noise signal ω_k to the estimation error $z_k-\hat{z}_k$ is less than γ, that is, under zero initial condition, $\sum_{k=0}^{\infty}(z_k-\hat{z}_k)^\tau(z_k-\hat{z}_k)<\gamma^2\sum_{k=0}^{\infty}\omega_k^\tau\omega_k$ for any nonzero $\omega:=\{\omega_k\}\in l_2[0,\infty)$.

Lemma 1 (Xie [13]). *Given appropriately dimensioned matrices Σ_1, Σ_2 and Σ_3, with $\Sigma_1^\tau=\Sigma_1$, then*

$$\Sigma_1+\Sigma_3\Delta(k)\Sigma_2+\Sigma_2^\tau\Delta^\tau(k)\Sigma_3^\tau<0 \tag{19}$$

holds for all $\Delta(k)$ satisfying $\Delta^\tau(k)\Delta(k)\leq I$ if and only if for some $\varepsilon>0$

$$\Sigma_1+\varepsilon\Sigma_3\Sigma_3^\tau+\varepsilon^{-1}\Sigma_2^\tau\Sigma_2<0. \tag{20}$$