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Brief Papers

Finite-time synchronization of coupled networks with one single time-varying delay coupling

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ABSTRACT

This paper studies finite-time synchronization of hybrid coupled networks. There is only one transmittal delay in the delayed coupling. The fact is that in the signal transmission process, the time delay affects only the variable that is being transmitted from one system to another, so assume that there is only one single time-varying delay coupling is more consistent with the reality. At the same time, the internal delay and coupling delay are time-varying and different. Based on the Lyapunov stability theory, a feedback controller is designed for achieving synchronization between two coupled networks with time-varying delays in finite time. Finally, simulation examples are given to illustrate the theoretical analysis.

1. Introduction

Chaos synchronization [1,2] has attracted considerable attention due to its theoretical importance and practical applications in various fields such as secure communication and automatic control. Since the concept for constructing synchronization of coupled chaotic systems was proposed in 1990 [3], various types of control method have been used in the synchronization problems of chaotic systems, such as feedback control [4], back-stepping control [5], adaptive control [6,7], and pinning control [8].

Although there have been many literatures to discuss the synchronization of coupled chaotic systems, most of the existing synchronization algorithms are asymptotic synchronization algorithms [9-11]. An increased interest has been devoted to study finite-time control for synchronization of chaotic systems [12-14]. Compared to the asymptotically convergent algorithms, the finite-time convergence algorithms demonstrate not only faster convergence rates, but also better disturbance rejection properties and robustness against uncertainties [15-17]. Therefore, a number of researchers have studied the finite-time control for chaotic systems and obtained some interesting results. In [18], finite-time stability and stabilization of time-delay systems are considered. Yang et al. [19] focuses on the problem of finite-time synchronization of complex networks with nonidentical discontinuous nodes. In [20], the paper proposes an approach of finite-time synchronization to identify the topological structure and unknown parameters simultaneously for under general complex networks.

Considering time delay is ubiquitous in nature [21,22], it is important to study the effect of time delay in synchronization of coupled systems. In practice, time delay involves two parts. One is internal delay because the delay occurs inside the systems, the other is caused by the exchange of information between systems, called coupling delay. Despite some literatures to be found on synchronization of coupled networks with time delay, many of them have been focused the case in which all of the time delay are the same and the coupling term is given by $D(x_j(t-\tau(t))-x_i(t-\tau(t)))$ [23-25]. Of course, it is unreasonable. Firstly, internal delay and coupling delay may not be the same because of different occurrence mechanism. Secondly, in the signal transmission process, the time delay affects only the variable that is being transmitted from one system to another. In other words, if there is a connection from node j to node i. The information received by node i is with time delay. Therefore, the coupling is inevitably recognized as $D(x_j(t-\tau(t))-x_i(t))$. Refs. [26,27] propose coupled networks with one single time-varying delay coupling and illustrate that this model is more consistent with the reality. Moreover, this general model includes delayed Hopfield neural networks, delayed cellular neural networks and some famous chaotic systems, such as Lorenz system and Chua's systems [6,28].

Our main contribution in this paper is to address finite-time synchronization between two coupled networks in a general model. In this model, we not only consider different internal delay and coupling delay, but also study the coupled systems with one single time-varying delay coupling. Motivated by the above discussions,

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the model is more realistic and the complexity increases. On this basis to consider the finite time control for the two coupled chaotic systems will be more challenging and more application value. Based on the Lyapunov stability theory, a feedback controller is designed to guarantee the synchronization in finite time. At the same time, we also discuss that the synchronization manifold of coupled networks can be reached in finite time by using this method. Two examples are given to illustrate our theoretical results.

The rest of the paper is organized as follows. In Section 2, some preliminaries are briefly given. In Section 3, main results are obtained for finite-time synchronization based on Lyapunov functional and feedback control. In Section 4, simulation results aiming at substantiating the theoretical analysis are presented. This paper is concluded in Section 5.

Notations: R^n denotes the n-dimensional Euclidean space, $R^{n\times m}$ is the set of $n\times m$ real matrices. The superscript T denotes matrix transposition and the notation $A\ge B$ (respectively $A>B$) where A and B are symmetric matrices, means that $A-B$ is positive semi-definite (respectively positive definite). $\|x\|=(x^Tx)^{1/2}$, where $x\in R^n$. sign$(\cdot)$ stands for the sign function.

2. Preliminaries and problem formulation

Consider the following time delay coupled network consisting of N nodes, in which each node is an n-dimensional dynamical systems:

$$\dot{x}_i(t)=-Cx_i(t)+Af(x_i(t))+Bf(x_i(t-\tau_1(t)))+\sum_{j=1,j\neq i}^{N}G_{ij}D(x_j(t)-x_i(t))$$
$$+\sum_{j=1,j\neq i}^{N}G_{ij}D_\tau(x_j(t-\tau_2(t))-x_i(t)),\quad i=1,\dots,N \tag{1}$$

where $x_i(t)=(x_{i1}(t),\dots,x_{in}(t))^T\in R^n$ is the state variable of the ith node; C, A and B are constant matrices; $f(x_i(t))=(f_1(x_{i1}(t)),\dots,f_n(x_{in}(t)))$ is the activation function of the neurons; $\tau_1(t)$ and $\tau_2(t)$ are internal delay and coupling delay, respectively; $D=diag\{d_1,\dots,d_n\}$ and $D_\tau=diag\{d_{\tau1},\dots,d_{\tau n}\}$ are the positive semi-definite inner coupling matrices between the connected nodes i and j at time t and $t-\tau_2(t)$, respectively; $G=(G_{ij})_{N\times N}$ is the configuration matrix that is irreducible and satisfies the following conditions:

$$G_{ij}=G_{ji}\ge 0,\quad i\neq j,\quad G_{ii}=-\sum_{j=1,j\neq i}^{N}G_{ij} \tag{2}$$

$G_{ij}>0$ if there is a connection between node i and node j and $G_{ij}=0$ otherwise. The degree of node i is equal to $\sum_{j=1,j\neq i}^{N}G_{ij}$.

We refer to model (1) as the drive complex dynamical network, and the response complex network can be rewritten as follows:

$$\dot{y}_i(t)=-Cy_i(t)+Af(y_i(t))+Bf(y_i(t-\tau_1(t)))+\sum_{j=1,j\neq i}^{N}G_{ij}D(y_j(t)-y_i(t))$$
$$+\sum_{j=1,j\neq i}^{N}G_{ij}D_\tau(y_j(t-\tau_2(t))-y_i(t))+u_i(t),\quad i=1,\dots,N \tag{3}$$

where $y_i(t)=(y_{i1}(t),\dots,y_{in}(t))^T\in R^n$ denotes the response state vector associated with the ith node.

Assume $C([-\tau,0],R^n)$ be a Banach space of continuous functions mapping the interval $[-\tau,0]$ into R^n with the norm $\|\phi\|=\sup_{-\tau\le\theta\le 0}\|\phi(\theta)\|$, where τ is the upper bound of $\tau_1(t)$ and $\tau_2(t)$. For the functional differential equations (1) and (2), their initial conditions are given by $x_i(t)=\phi_i(t)\in C([-\tau,0],R^n)$ and $y_i(t)=\varphi_i(t)\in C([-\tau,0],R^n)$. We always assume that Eq. (1) has a unique solution with respect to initial condition, the same as Eq. (2).

Define the synchronization errors $e_i(t)=y_i(t)-x_i(t)$, $i=1,\dots,N$. Based on condition (2), we have the following error dynamical system:

$$\dot{e}_i(t)=-Ce_i(t)+A(f(y_i(t))-f(x_i(t)))+B(f(y_i(t-\tau_1(t)))-f(x_i(t-\tau_1(t))))$$
$$+\sum_{j=1}^{N}G_{ij}De_j(t)+\sum_{j=1}^{N}G_{ij}D_\tau e_j(t-\tau_2(t))-G_{ii}D_\tau(e_i(t-\tau_2(t))$$
$$-e_i(t))+u_i(t),\quad i=1,\dots,N \tag{4}$$

For starting simplification, one has the following fundamental assumptions.

Assumption 1. There exists constant $L>0$, for any $x,y\in R^n$, such that

$$\|f(y)-f(x)\|^2\le L\|y-x\|^2$$

Assumption 2. $0\le\dot{\tau}_1(t)\le h_1<1$, $0\le\dot{\tau}_2(t)\le h_2<1$, where h_1 and h_2 are constants.

To end this section, we introduce the following lemmas which is useful in deriving sufficient conditions of finite-time synchronization.

Lemma 1 (*Bhat and Bernstein [29]*). *Consider system $\dot{x}=f(x)$, $f(0)=0, x\in R^n$, where $f(\cdot):R^n\to R^n$ is a continuous vector function. Suppose there exists a C^1 positive definite and proper function $V: R^n\to R$ and real numbers $\mu>0$ and $\eta\in(0,1)$ such that $\dot{V}+\mu V^\eta$ is negative semi-definite. Then the origin is a globally finite-time stable equilibrium of system $\dot{x}=f(x)$. Moreover, the settling time $T\le V^{1-\eta}(0)/(1-\eta)\mu$.*

Lemma 2 (*Bovd et al. [30]*). *Given any real matrices A, B, Σ of appropriate dimensions and a scalar $s>0$, such that $0<\Sigma=\Sigma^T$. Then the following inequality holds:*

$$A^TB+B^TA\le sA^T\Sigma A+s^{-1}B^T\Sigma^{-1}B$$

Lemma 3. *Given any real matrices $G=(G_{ij})_{N\times N}$, $A=(a_{ij})_{n\times N}$, $B=(b_{ij})_{n\times N}$ and diagonal matrices $\Gamma=diag\{\gamma_1,\dots,\gamma_n\}$, $G'=diag\{G_{11},\dots,G_{NN}\}$. Denote $a_i,\tilde{a}_j^T$ are the ith column vector and jth row vector of A, respectively; b_i, $\tilde{b}_j^T$ are the ith column vector and jth row vector of B, respectively. Then the following equations hold:*

$$\sum_{i=1}^{N}a_i^T\sum_{j=1}^{N}G_{ij}\Gamma b_j=\sum_{i=1}^{n}\tilde{a}_i^T\gamma_iG\tilde{b}_i \tag{5}$$

$$\sum_{i=1}^{N}a_i^TG_{ii}\Gamma b_i=\sum_{i=1}^{n}\tilde{a}_i^T\gamma_iG'\tilde{b}_i \tag{6}$$

Proof. First, discuss the left side of Eq. (5):

$$\sum_{i=1}^{N}a_i^T\sum_{j=1}^{N}G_{ij}\Gamma b_j=\sum_{i=1}^{N}(a_{1i},\dots,a_{ni})\sum_{j=1}^{N}G_{ij}\Gamma\begin{pmatrix}b_{1j}\\ \cdots\\ b_{nj}\end{pmatrix}$$
$$=\sum_{i=1}^{N}(a_{1i},\dots,a_{ni})\begin{pmatrix}G_{i1}r_1b_{11}+\cdots+G_{iN}r_1b_{1N}\\ \cdots\\ G_{i1}r_nb_{n1}+\cdots+G_{iN}r_nb_{nN}\end{pmatrix}$$
$$=\sum_{i=1}^{n}\sum_{i'=1}^{N}\sum_{j'=1}^{N}G_{i'j'}r_ia_{ii'}b_{ij'} \tag{7}$$

Then, we have

$$\sum_{i=1}^{n}\tilde{a}_i^T\gamma_iG\tilde{b}_i=\sum_{i=1}^{N}(a_{i1},\dots,a_{iN})\gamma_iG\begin{pmatrix}b_{i1}\\ \cdots\\ b_{iN}\end{pmatrix}$$
$$=\sum_{i=1}^{N}(a_{i1},\dots,a_{iN})\begin{pmatrix}G_{11}r_ib_{i1}+\cdots+G_{1N}r_ib_{iN}\\ \cdots\\ G_{n1}r_ib_{i1}+\cdots+G_{nN}r_ib_{iN}\end{pmatrix}$$