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Brief Papers

Consensus of a heterogeneous multi-agent system with input saturation

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ABSTRACT

In this paper, the consensus problem for a class of heterogeneous multi-agent systems with input saturation is investigated. The heterogeneous multi-agent system is composed of first-order agents and second-order agents. We consider that only a few first-order agents are restrained and their control input must be bounded. Firstly, a consensus control protocol under leaderless network is proposed. By applying the graph theory and LaSalle invariance principle, the protocol is proved feasible. Secondly, the consensus problem of heterogeneous multi-agent systems is discussed with a static leader and an active leader, respectively. Particularly, when the velocity of the active leader cannot be obtained in real time, a new neighbour-based consensus control protocol based on distributed observers and auxiliary systems is developed. Some sufficient conditions for consensus are established under fixed undirected connected topology. Finally, some examples were presented to illustrate the theoretical results.

1. Introduction

In recent years, distributed coordinated control of multi-agent systems has attracted great attention of researchers. Consensus problem, which is fundamental to the multi-agent systems, has been a hotspot in biology, physics, mathematics, computer science and control science. Consensus means that all the agents in the multi-agent systems reach an agreement on a common value by following some control rules and negotiating with their neighbours [1].

A large volume of research works on consensus has emerged in the past years. In [2], Olfati-Saber et al. studied the consensus problem of multi-agent systems with switching topologies and time delays in a continuous-time model. Some results were obtained to solve the average-consensus problem. Ren and Beard discussed the consensus problem of discrete-time and continuous-time by discrete-time and continuous-time Kalman filters in [3]. The work in [4] focused on consensus problem of the multi-agent systems with an active leader and interconnection topology. Based on a neighbor-based local controller and a neighbor-based state-estimation, each agent could follow the leader. The average-consensus problem with switching topology and time-delay in the directed networks was studied by Lin et al. Some sufficient conditions were obtained by using Lyapunov–Krasovskii function and LMI [5]. The finite-time consensus problem for continuous nonlinear multi-agent systems was discussed by Shang in [6]. The schemes proposed in [7] can deal with consensus problems with discrete-time networks in uncertain communication environments. In [8], consensus problems of first-order multi-agent systems with multiple time delays were investigated. By the generalized Nyquist criterion and the frequency-domain analysis approach, some less conservative sufficient condition were presented. In [9], the author studied the mean square consensus of linear multi-agent systems with communication noises. There were many interesting agent-related works, such as [10] and [11], involved sampled information and adaptive approach, respectively. An adaptive control approach was proposed to deal with the multiple manipulators consensus problem by Cheng. In [12], Yu and Ren considered the second-order multi-agent systems, and designed a consensus control protocol in the leader-following networks by the adaptive method. By using sampling technology, zero-order hold circuit, algebraic graph theory and matrix theory, the average-consensus problem of multi-agent systems with time-varying sampling interval was studied in [13]. In [14], based on the distributed observer method, the tracking problem was solved when there was an active leader in the second-order multi-agent systems. In [15], a novel control strategy for multi-agent systems with event-based broadcasting was presented, and it could guarantee either asymptotic convergence to average-consensus

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or convergence to a ball centred at the average-consensus. In [16], Cheng et al. studied the average consensus of second-order integral multi-agent systems under switching topologies and communication noises based on sampled-data. The result of [17] and [18] is about the leader–follower control problem based on output regulation theory, internal model principle and the distributed observer. In addition, many researchers have studied the high-order consensus problem of multi-agent systems [19,20].

In [1–19], all the agents in the multi-agent systems have the same dynamics. However, in the practical system, the dynamics of agents may be different. Nowadays, more and more researchers throw themselves into the study of heterogeneous multi-agent systems. In [21], a linear consensus protocol and a saturated consensus protocol were presented for the heterogeneous multi-agent systems which consisted of first-order integrator agents and second-order integrator agents. In [22], Zheng and Wang considered the consensus problem of heterogeneous multi-agent systems without velocity measurement information. The finite-time consensus problem of heterogeneous multi-agent systems was discussed by Zhu et al. in [23]. In [24], the consensus problem of heterogeneous multi-agent systems consisted of first-order integrator agents, second-order integrator agents and Euler–Lagrange agents was solved by the graph theory, Lyapunov direct method and Barbalat theorem. The containment problem of first-order and second-order integral multi-agent systems was considered in a noisy communication environment by Wang and Cheng in [25]. In [26], Liu et al. focused on the heterogeneous multi-agent systems consisted of second-order integrator agents and nonlinear dynamics agents. Based on the Lyapunov theory and novel decentralized adaptive strategy, the consensus problem was solved.

In this paper, motivated by [21,22,24], we consider that the control input of partial first-order integrator agents must be bounded due to the limitation of actuators. Some new control protocols with partial first-order integrator agents with input saturation are given to solve the consensus problem of heterogeneous multi-agent systems. The main contributions can be listed as follows:

(1) Partial first-order agents with input saturation are considered for a class of heterogeneous multi-agent systems which composed of first-order agents and second-order agents.
(2) We investigate the consensus problem of the heterogeneous multi-agent system under leaderless network and leader-following network, respectively. The proposed consensus control protocols can solve the consensus problem, aggregating problem and tracking problem, respectively.
(3) Distributed observers and auxiliary systems are taken to solve the consensus problem when there is an active leader in heterogeneous multi-agent system and its velocity information cannot be obtained by the followers.

The remainder of the paper is organized as follows. In Section 2, some preliminaries are briefly outlined. In Section 3, the consensus problem of heterogeneous multi-agent systems is discussed in the leaderless network and leader-following network, respectively. In Section 4, some simulation examples are presented to illustrate the theoretical results in Section 3. Finally, we get the conclusion in Section 5.

2. Preliminaries and problem formulation

2.1. Preliminaries

In this subsection, some important knowledge such as algebraic graph theory and LaSalle's invariance principle are reviewed. Algebraic graph theory is a branch of Mathematics, and the main research object is graph [27]. In the multi-agent systems, let $G=(V,E,A)$ be a weighted undirected graph, which consists of a set of vertices $V=\{v_1,v_2,\cdots,v_n\}$, a set of edges $E=\{e_{ij}=(v_i,v_j)\}\in V\times V$ and an adjacency matrix $A=[a_{ij}]\in\mathbb{R}^{n\times n}$. An edge of the graph G is denoted by an undirected pair of vertices $e_{ij}=(v_i,v_j)$, which indicates that agent i can receive information from agent j. We assume that the element of adjacency matrix $a_{ij}>0$ if $e_{ij}\in E$, otherwise $a_{ij}=0$ while $e_{ij}\notin E$, and $a_{ii}=0$ for all. From the foregoing, G is undirected, that is, $a_{ij}=a_{ji}$ and A is symmetric. We define an undirected graph is connected if there is a path between any two distinct vertices of the graph. In the multi-agent systems, the agent is defined as a leader which only sends information to other agents and cannot receive information from other agents, that is, $a_{n1}=a_{n2}=\cdots=a_{nn}=0$ and more than one element is not zero in the set $\{a_{1n},a_{2n},\cdots,a_{(n-1)n}\}$. Moreover, throughout this paper, let $\mathbb{R}$ be the set of real number and $\mathbb{R}^n$ be the n-dimensional real vector space. For a given vector X, $\|X\|$ denotes the Euclidean norm of a vector X.

Based on the above presentation, the LaSalle's invariance principle is introduced which will be used in the next part.

Definition 1. (Invariant set) ([28]): A set S is an invariant set for a dynamic system $\dot{x}=f(x)$ if every trajectory $x(t)$ which starts from a point in S remains in S for all time.

Lemma 1. ***(LaSalle's invariance principle)*** *([28]): Consider an autonomous system of the form $\dot{x}=f(x)$, with f continuous and let $V(x)$: $\mathbb{R}^n\to\mathbb{R}$ be a scalar function with continuous first partial derivatives. Assume that*

(1) When $\|x\|\to\infty$, $V(x)\to\infty$
(2) For any $x\in\mathbb{R}^n$, $\dot{V}(x)\le 0$

Let S be the set of all points within $\mathbb{R}^n$ where $\dot{V}(x)=0$ and M be the largest invariant set in S. Then, every solution $x(t)$ originating in $\mathbb{R}^n$ tends to M as $t\to\infty$.

2.2. Heterogeneous multi-agent systems

In this subsection, the composition of heterogeneous multi-agent systems, agent dynamics and the concept of consensus are presented. In this paper, the heterogeneous multi-agent system consists of first-order agents, first-order agents with input saturation and second-order agents. The first-order agent dynamics is given as follows:

$$\dot{x}_i(t)=u_i(t) \tag{1}$$

where $x_i(t)\in\mathbb{R}$ and $u_i(t)\in\mathbb{R}$ are the position information and control input, respectively, of agent i.

The first-order agent with input saturation dynamics is given as follows:

$$\dot{x}_i(t)=u_i(t)=f_i(\bullet) \tag{2}$$

where $f_i(\bullet)$ denotes the continuous function and it holds $\|f_i(\bullet)\|\le c$, $c\in\mathbb{R}^+$ induced by actuator constraint. $x_i(t)\in\mathbb{R}$ and $u_i(t)=\mathbb{R}$ are the position information and control input with input saturation, respectively, of agent i.

The second-order agent dynamics is given as follows:

$$\begin{cases}\dot{x}_i(t)=v_i(t)\\ \dot{v}_i(t)=u_i(t)\end{cases} \tag{3}$$

where $x_i(t)\in\mathbb{R}$,$v_i(t)\in\mathbb{R}$ and $u_i(t)\in\mathbb{R}$ are the position information, velocity information and control input, respectively, of agent i.