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Finite-time $\mathcal{H}_\infty$ synchronization control for semi-Markov jump delayed neural networks with randomly occurring uncertainties ☆

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ABSTRACT

This paper studies the problem of finite-time $\mathcal{H}_\infty$ synchronization control for semi-Markov jump delayed neural networks with randomly occurring uncertainties. The randomly occurring parameter uncertainties follow certain mutually uncorrelated Bernoulli distributed white noise sequences. By employing a Markov switching Lyapunov functional and a weak infinitesimal operator, a criterion is obtained to ensure that the resulting error system is stochastically finite-time stable and master system synchronizes with the slave system over a finite-time interval accordingly. Based on this, a clear expression for the desired controller is given by using a simple matrix decoupling. The effectiveness of the proposed method is demonstrated by employing a simulation example.

1. Introduction

It is well known that time delays are commonly encountered in neural networks, and its existence is frequently a source of oscillation and instability [1]. Therefore, delayed neural networks (DNNs) have been a hot research topic. DNNs play a key role in different areas such as artificial intelligence, signal and image processing, industrial automation, and so on. These applications heavily depend on the dynamical behaviors. Thus, the dynamical behaviors of DNNs have attracted a great deal of attentions and many significant advances have been reported in the literature; see, e.g. [2–6], and the reference therein.

As a basic phenomenon in networks, the synchronization problem of DNNs has induced increasing concern and many effective methods to establish the synchronization criteria of DNNs have been presented in the literature. To name just a few, some adaptive synchronization criteria were established for DNNs in [7–12]. Some global exponential synchronization criteria for DNNs were provided in [13,14]. However, most of these criteria are on the account of Lyapunov stability defined over an infinite time interval. Due to exist bad transient characteristics, it may be unavailable in some practical situations. Therefore, in some practical engineering applications, one can pay close attention on the synchronization behavior over a finite-time (or called short-time interval) [15–17]. In addition, the problem of $\mathcal{H}_\infty$ synchronization control has also been addressed. The authors of [18] presented a multiple delayed state-feedback control design for exponential $\mathcal{H}_\infty$ synchronization problem of a class of DNNs with multiple time-varying discrete delays. The authors of [19] established some $\mathcal{H}_\infty$ synchronization criteria of a class of time-delayed chaotic systems with external disturbance. However, there is little research effort devoted to the finite-time $\mathcal{H}_\infty$ synchronization control problem for DNNs. This is one of our motivations to deal with this work.

On the other hand, jumping connection or switching topology often appears in a network owing to link failures or new creation [20–27]. Markov jumping parameters in studying synchronization of neural networks are wildly considered [28–33]. However, due to the fact that the jump time of the Markov chain is exponentially distributed, unrelated to the sojourn-time, the transition rate should be often assumed to be constant, which brings some limitations in applications of Markov jump neural networks. To linkage such a gap, much attention has been paid to the study of systems with semi-Markov jump parameters. The advantage of semi-Markov jump model lies in the fact that it is not only dependent on transition probabilities but also dependent on sojourn time, and as a result, which has wider applications than the Markov jump systems. Based on this consideration, the authors of [34,35] studied semi-Markov jump linear systems and obtained the sufficient conditions for stochastic stability of semi-Markov jump linear systems. It should be pointed out that although the importance of semi-Markov jump model has been

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broadly recognized, no related results have been reported for the finite-time synchronization of neural networks. Moreover, the uncertainty inevitably exists in many practical systems [36–38]. For example, the randomly occurring parameter uncertainties (first proposed in [39]) have not been fully considered in addressing the finite-time synchronization problem.

Motivated by the aforementioned discussion, the main contributions of the paper are, therefore, two-fold: (1) we make the first attempt to deal with the problem of finite-time $\mathcal{H}_\infty$ synchronization control for delayed neural networks with randomly occurring uncertainties. The randomly occurring parameter uncertainties follow certain mutually uncorrelated Bernoulli distributed white noise sequences; (2) the semi-Markov jump model is for the first time introduced to the study of DNNs. The objective of this paper is to design a mode-independent controller such that the resulting synchronization error system is stochastically finite-time bounded with a prescribed $\mathcal{H}_\infty$ performance level γ. By using a stochastic analysis method, a sufficient condition for the solvability of the problem is obtained. A mode-independent controller can be constructed by using a simple matrix decoupling approach. Finally, a numerical example illustrates the proposed approach.

Notation: Throughout this paper, for symmetric matrices X and Y, the notation $X \geq Y$ (respectively, $X > Y$) means that the matrix $X-Y$ is positive semi-definite (respectively, positive definite); I is the identity matrix with appropriate dimension. The notation M^T represents the transpose of the matrix M; $\lambda_{\max}(M)$ (respectively, $\lambda_{\min}(M)$) means the largest (respectively, smallest) eigenvalue of the matrix M; $\mathcal{E}\{\cdot\}$ denotes the expectation operator with respect to some probability measure $\mathcal{P}$; $\mathcal{L}_2[0,\infty)$ is the space of square-integrable vector functions over $[0,\infty)$; $|\cdot|$ refers to the Euclidean vector norm. Matrices, if not explicitly stated, are assumed to have compatible dimensions. In symmetric block matrices or complex matrix expressions, we employ an asterisk $(*)$ to represent a term that is induced by symmetry.

2. Problem formulation

Given a probability space $(\Omega, \mathcal{F}, \mathcal{P})$, where Ω is the sample space, and $\mathcal{P}$ is the probability measure on $\mathcal{F}$. Let $\{\sigma(t), t \geq 0\}$ be a continuous-time discrete-state semi-Markov process and take values in finite set $\mathcal{S} = \{1, 2, \ldots, \mathcal{N}\}$ with transition probability matrix $\Pi \triangleq \{\pi_{ij}(h)\}$ given by

$$\Pr\{\sigma(t+h)=j \mid \sigma(t)=i\} = \begin{cases} \pi_{ij}(h)h + o(h), & i \neq j, \\ 1+\pi_{ii}(h)h + o(h), & i = j, \end{cases}$$

where $o(h)$ is the little-o notation defined as $\lim_{h\to 0}(o(h)/h)=0$, and $\pi_{ij}(h) \geq 0$, for $i \neq j$, is the transition rate from mode i at time t to mode j at time $t+h$ and

$$\pi_{ii}(h) = -\sum_{j \in S, j \neq i} \pi_{ij}(h) \tag{1}$$

In this paper, we consider the following semi-Markov jump DNNs, which defined in the probability space $(\Omega, \mathcal{F}, \mathcal{P})$, and its state equation is described by the dynamical equation:

$$\begin{aligned}\dot{x}(t) = & -A(\sigma(t))x(t) + (B(\sigma(t)) + \beta_1(t)\Delta B(\sigma(t),t))f(x(t)) \\ & + (C(\sigma(t)) + \beta_2(t)\Delta C(\sigma(t),t))f(x(t-\tau(t))) + I(t)\end{aligned} \tag{2}$$

$$\tilde{Z}(t) = D(\sigma(t))x(t), \tag{3}$$

where $x(t) = (x_1(t), x_2(t), \ldots, x_n(t))^T \in \mathbb{R}^n$, $x_k(t), k \in [1,n]$ denotes the state vector of the kth neuron at time t; $f(x(t)) = (f_1(x_1(t)), f_2(x_2(t)), \ldots, f_n(x_n(t)))^T \in \mathbb{R}^n$ are the neuron activation functions; $I(t) = (I_1(t), I_2(t), \ldots, I_n(t))^T \in \mathbb{R}^n$ is an external input vector; $A(\sigma(t)) > 0$ is a diagonal matrix for each $\sigma(t) \in S$; $B(\sigma(t))$ and $C(\sigma(t))$ are the connection weight matrices. To simplify the notation, we denote $A_i = A(\sigma(t)), \Delta B_i(t) = \Delta B(\sigma(t),t)$, for each $\sigma(t) = i \in S$, and the other symbols are similar denoted. The real-valued matrices $\Delta B_i(t)$ and $\Delta C_i(t)$ represent the time-varying parameter uncertainties and are assumed to be of the following form:

$$\left[\Delta B_i(t) \ \ \Delta C_i(t)\right] = M_i F_i(t)\left[N_{1,i} \ \ N_{2,i}\right], \tag{4}$$

where for each i, M_i, N_{1i} and N_{2i} are known real constant matrices, and $F_i(t)$ is an unknown time-varying matrix function satisfying

$$F_i(t)^T F_i(t) \leq I, \quad \forall t.$$

The function $\tau(t)$ denotes the time-varying delay of the considered systems, and is assumed to satisfy

$$0 < \tau(t) \leq h, \quad \dot{\tau}(t) \leq \mu.$$

In this paper, the parameter uncertainties described in (4) are randomly occurring with two stochastic variables $\beta_l(t), l=1,2$. The stochastic variables $\beta_l(t), l=1,2$, are mutually independent Bernoulli-distributed white sequences, which obey the following probability distribution laws:

$$\Pr\{\beta_1(t)=1\} = \mathcal{E}\{\beta_1(t)\} = \beta_1, \quad \Pr\{\beta_1(t)=0\} = 1-\beta_1,$$
$$\Pr\{\beta_2(t)=1\} = \mathcal{E}\{\beta_2(t)\} = \beta_2, \quad \Pr\{\beta_2(t)=0\} = 1-\beta_2,$$

where $\beta_1 \in [0,1]$ and $\beta_2 \in [0,1]$ are known constants. Throughout this paper, we suppose that $\beta_1(t)$, $\beta_2(t)$ and $\sigma(t)$ are mutually independent.

Assumption 1 (*Wang et al. [40]*). Every activation function $f_l(\bullet)$ in (2) is continuous and bounded and satisfies

$$\psi_l^- \leq \frac{f_l(y)-f_l(x)}{y-x} \leq \psi_l^+, \quad l=1,2,\ldots,n, \tag{5}$$

where $f_l(0)=0$, y, $x \in \mathbb{R}$, $y \neq x$, and ψ_l^- and ψ_l^+ are known real scalars, and they may be positive, negative, or zero, which means that the resulting activation functions may be nonmonotonic and more general.

In the paper, semi-Markov jump DNNs $(\sum)$ are considered as the master system, and a corresponding slave system $\left(\hat{\sum}\right)$ can be described by the following dynamical equation:

$$\begin{aligned}\dot{y}(t) = & -A_i y(t) + (B_i + \beta_1(t)\Delta B_i(t))f(y(t)) \\ & + (C_i + \beta_2(t)\Delta C_i(t))f(y(t-\tau(t))) + I(t) + u(t) + E_i\omega(t),\end{aligned} \tag{6}$$

$$\hat{Z}(t) = D_i y(t), \tag{7}$$

where $y(t)$ and $f(y(t))$ have the similar definition as $x(t)$ and $f(x(t))$ in (2), and $u(t) \in \mathbb{R}^s$ is the appropriate control input that will be designed in order to obtain a certain control objective. E_i is a known real constant matrix for every $i \in S$; $\omega(t) \in \mathbb{R}^q$ is the disturbance input that belongs to $\mathcal{L}_2[0,\infty)$ and satisfies

$$\int_0^\infty \omega^T(t)\omega(t)\, d(t) \leq \mathcal{W}. \tag{8}$$

In this paper, without loss of generality, we assume that $x(t) = y(t) = 0$ when $t \in [-h, 0)$. Define $e(t) = y(t) - x(t)$, then the error system can be written as follows:

$$\begin{aligned}\dot{e}(t) = & -A_i e(t) + (B_i + \beta_1(t)\Delta B_i(t))g(e(t)) \\ & + (C_i + \beta_2(t)\Delta C_i(t))g(e(t-\tau(t))) + E_i\omega(t) + u(t),\end{aligned} \tag{9}$$

$$Z(t) = D_i e(t), \tag{10}$$

where $g(e(t)) = f(y(t)) - f(x(t))$. Then, in light of (5), it is easy to see that function $g(e(t))$ and $g(e(t-\tau(t)))$ satisfy the following condition for any $z \in \mathbb{R}$, $z \neq 0$:

$$\psi_l^- \leq \frac{g_l(z)}{z} \leq \psi_l^+, \quad l=1,2,\ldots,n. \tag{11}$$