Robotics and Autonomous Systems 83 (2016) 94–105

Contents lists available at ScienceDirect

Robotics and Autonomous Systems

journal homepage: www.elsevier.com/locate/robot

Controllability analysis of a pair of 3D Dubins vehicles in formation

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HIGHLIGHTS

- The controllability analysis of two 3D Dubins vehicles constrained to maintain constant distance is proposed.
- Necessary and sufficient conditions for the existence of a limited control to steer the system between any two configurations are provided.
- The control laws that verify distance constraints are also furnished.
- The results are relevant for aerial or underwater vehicles that are e.g. physically constrained to a payload to be deployed.

ARTICLE INFO

Article history:
Received 17 September 2015
Accepted 30 May 2016
Available online 16 June 2016

Keywords:
Controllability
Vehicle formation
3D planning
Nonholonomic vehicles

ABSTRACT

In this paper we consider the controllability problem for a system consisting of a pair of Dubins vehicles moving in a 3D space (i.e. pair of *3D-Dubins* vehicles) while maintaining constant distance. Necessary and sufficient conditions for the existence of a limited control effort to steer the system between any two configurations are provided. The proposed controllability analysis and the developed motion planning algorithm are a step toward the solution of planning problems for example in case the robots are physically constrained to a payload to be deployed. Moreover, results obtained in this paper are relevant in order to solve formation control problems for multiple robots as aerial or underwater vehicles, which move in 3D spaces. Simulation results highlight the sufficiency of the obtained conditions showing that even from critical configurations an admissible control can be determined.

1. Introduction

Motion planning algorithms have been actively studied in the literature and there are several methods based e.g. on visibility graphs, potential field techniques or randomized sampling (see [1] and references therein). Several challenges may arise including issues related to nonholonomic and dynamic constraints, modeling uncertainty, noisy models, partial sensory data, and real-time computation.

Motion planning becomes particularly difficult and interesting (see e.g. [2]) when physical robots have to perform tasks in a truly 3D environment avoiding static or dynamic obstacles such as in disaster sites, underwater and aerial environments. In such scenario, multi-robot systems can perform tasks more efficiently than a single robot or can accomplish tasks not executable by a single one. Moreover, multi-robot systems have advantages, e.g. providing flexibility to the task execution by exploiting distributed sensing and actuation. Also in nature, several types of animals, such as insects, birds, or fishes, aggregate together, moving *en masse* or migrating in some directions, also known as swarm behavior. The term shoaling or schooling is used to refer specifically to swarm behavior in fishes which derives many benefits including also the increased hydrodynamic efficiency (cf. [3]).

The cooperation and coordination of multi-robot systems (i.e. formation control) has been object of considerable research efforts (see [4] for a detailed review and references therein). Formation control studies the problem of controlling multiple robots with different kinematics and sensory equipment so that they can maintain some given configuration constraints (e.g. distances) while moving as a whole group [5–7]. Many approaches of formation control have been proposed, such as behavior-based methods [8], leader–follower strategies [9,10] and virtual structure approaches [11]. Various kinds of nonholonomic vehicles have been considered, such as ground vehicles (e.g. in [12]), aircraft (e.g. in [13]) and underwater vehicles (e.g. in [14]).

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http://dx.doi.org/10.1016/j.robot.2016.05.015
0921-8890/

In order to solve challenging problems as motion planning algorithms and formation control as well as to plan optimal trajectories, it is important to analyze and prove the controllability of the system. A system is completely controllable if, for every pair of points q_1 and q_2 in the configuration space, there exists a control that steers the system from q_1 to q_2, [15,16].

Unlike other approaches, the analysis of a tight constraint on the distance to be maintained is herein considered. There are several application scenarios in which the motion of the robots can be physically constrained due to a load of large dimensions to be deployed. For example refer to [17,18] or to [19] where a group of quadrotor rigidly attached to a payload is considered. Other examples of those type of applications can be found in the aerospace robotics such as the JPL's Robot Colonie project where two rovers must transport a large box [20] and in underwater cooperative manipulation systems [21].

Besides the applications, the problem has several interesting theoretical aspects among which the control input set depends on the system configurations and classical controllability results cannot be directly applied. Moreover the high dimensional system is controlled by constrained 3-dimensional controls, and the range of admissible controls depends on the configuration variables. The solution of such constrained problems is also crucial for the solution of optimal control problems in which a minimum safety distance must be guaranteed during motion. Indeed, the optimal solution consists also of arcs along which the robot travel at constant distance, see e.g. [22].

In [23] the controllability of different pairs of identical nonholonomic vehicles (e.g. differential drive and car-like vehicles) moving in a plane while maintaining a constant distance has been proved. Results obtained have then been used in order to prove the controllability and design a motion planning algorithm for formations of planar Dubins vehicles, [13,24].

In this paper our purpose is to extend results of [13] to a system consisting of a pair of *3D-Dubins* vehicles moving in a 3D space while maintaining constant distance. Extension to the 3D case is not straightforward due to a more complex vehicle model and thus maneuvers between configurations must be accordingly computed. This paper completes our previous conference paper [25] which furnishes only sufficient conditions for controllability. Here we provide a more restrictive condition that is proved to be both necessary and sufficient. Finally, a motion planning algorithm to drive the considered system between initial and final configuration that verify the necessary and sufficient condition.

The paper is organized as follows. In Section 2 the model of two three dimensional Dubins vehicles are presented with the inputs and distance constraints to be verified. In Section 3 the effects of the controls on the system are evaluated in order to simplify the controllability analysis performed next. In Section 4 necessary conditions for the system controllability are obtained in terms of system internal configurations. In Section 5 several basic movements and associated control laws are obtained. Such movements are then combined to steer the system between any two configurations as described in Section 6. The necessary conditions are thus proven to be also sufficient for controllability if verified by the initial and final configurations. Finally, simulations results to highlight the verification of the constraints and the system's behavior under the proposed controls is reported in Section 7.

2. Problem definition

Consider a nonholonomic vehicle moving in a three dimensional space and let $\langle W \rangle = (O_w, X_w, Y_w, Z_w)$ be a fixed reference frame. In $\langle W \rangle$, the vehicle configuration is $\zeta(t) = (x(t), y(t), z(t), \varphi(t), \psi(t))$ where $\mathbf{q} = (x(t), y(t), z(t))$ is the position in $\langle W \rangle$ of the reference central point in the vehicle, $\varphi(t)$ is the angle formed by the vehicle heading and the plane $X_w \times Y_w$ and $\psi(t)$ is the angle formed by the projection of the vehicle heading on the plane $X_w \times Y_w$ and X_w axis (see Fig. 1).

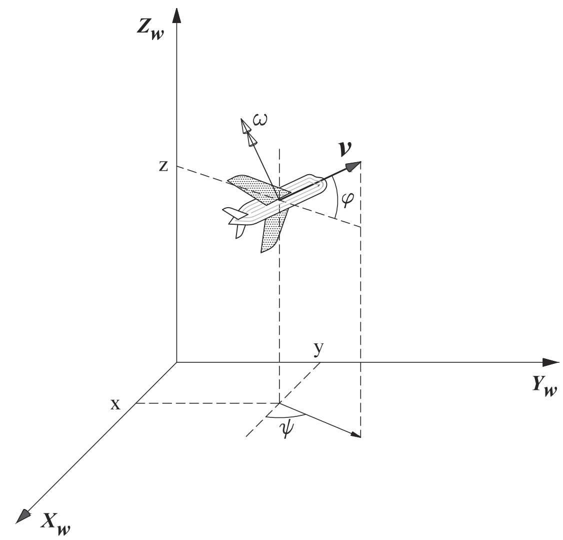

Fig. 1. A single *3D-Dubins* vehicle. The configuration ζ of the vehicle is described by three position variables x, y and z and two angular variable: φ is the angle formed by the vehicle heading and the plane $X_w \times Y_w$ and ψ is the angle formed by the projection of the vehicle heading on the plane $X_w \times Y_w$ and X_w axis. The control inputs are the forward velocity $v = 1$ and the angular velocity ω.

Given the forward velocity v of the vehicle, the velocity vector $\mathbf{v}$ in $\langle W \rangle$ is $\mathbf{v} = (v\cos\varphi\sin\psi,\ v\cos\varphi\cos\psi,\ v\sin\varphi)^T$. The kinematic model of the nonholonomic vehicle is (for the vehicle model for more details please refer to [26,27])

$$\begin{cases} \dot{\mathbf{q}} = \mathbf{v} \\ \dot{\mathbf{v}} = \mathbf{v} \times \boldsymbol{\omega} \end{cases} \tag{1}$$

where $\boldsymbol{\omega} = (\dot{\psi},\ \dot{\varphi}\sin\psi,\ -\dot{\varphi}\cos\psi)^T$.

In this paper we consider the *3D-Dubins* system that is described by system (1) subject to a constrained control effort $|\boldsymbol{\omega}| \leq \omega_M$. Moreover, without loss of generality, we consider $v = 1$. In such conditions, a *3D-Dubins* generates trajectories with bounded curvature, as the minimum radius is $r = \frac{v}{\omega_M}$.

Remark 1. For reader convenience we recall that the classical Dubins car is basically a unicycle vehicle moving on a plane with a constant positive forward velocity (usually normalized to 1) and a bounded angular velocity. The 3D-Dubins car here introduced is a generalization of the classical one to move in a 3D space. For detailed discussions on the model, the constraints, the controllability properties and optimal control results please refer to [28–30] and references therein.

Consider now the system consisting of a pair of *3D-Dubins* constrained to maintain constant the magnitude D of the distance vector $\mathbf{D}$ joining the centers of the two robots. The system is hence given by

$$\begin{cases} \dot{\mathbf{q}}_1 = \mathbf{v}_1 \\ \dot{\mathbf{q}}_2 = \mathbf{v}_2 \\ \dot{\mathbf{v}}_1 = \mathbf{v}_1 \times \boldsymbol{\omega}_1 \\ \dot{\mathbf{v}}_2 = \mathbf{v}_2 \times \boldsymbol{\omega}_2 \end{cases} \tag{2}$$

subject to the constraint of a constant magnitude of vector $\mathbf{D} = \mathbf{q}_2 - \mathbf{q}_1$ and limited control efforts

$$|\boldsymbol{\omega}_i| \leq \omega_M. \tag{3}$$