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Asynchronous consensus of continuous-time Lagrangian systems with switching topology and non-uniform time delay

Morteza Mirzaei [a], Hajar Atrianfar [a,*], Noushin Mehdipour [a], Farzaneh Abdollahi [a,b]

[a] *The Center of Excellence on Control and Robotics Department of Electrical Engineering, Amirkabir University of Technology (Tehran Polytechnic), Tehran, Iran*

[b] *Department of Electrical and Computer Engineering, Concordia University, Montreal, Quebec, Canada*

HIGHLIGHTS

- A consensus protocol is proposed for a class of nonlinear continuous-time multi-agent systems with discontinuous information transmission between agents.
- The communication topology among the agents is switching.
- Agents receive their neighbors' information with bounded nonuniform time-varying delays.
- Communications are not continuous and updating neighbors information is asynchronous.
- This protocol tolerates arbitrary bounded time delays if the communication topology is modeled by a jointly weakly connected graph.

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ABSTRACT

This paper presents a novel asynchronous algorithm for consensus problem of a class of nonlinear continuous-time multi-agent systems with discontinuous information transmission between agents. The communication topology among the agents is switching and agents receive information in discrete time instants with a bounded nonuniform time delay. Using feedback linearization, Lagrangian dynamics of agents are transformed to double integrator dynamics. A state transformation is applied to obtain a non-delayed extended state space and then the proposed controller provides consensus. Simulation results show the effectiveness of the proposed algorithm.

1. Introduction

To control a group of autonomous vehicles, the main challenge is typically to have the whole group work in a cooperative fashion throughout a distributed protocol. Consensus is one of the fundamental topics of research in distributed control that refers to the group behavior in which, all agents asymptotically reach to an agreement through a local distributed protocol.

One pioneering work on consensus is generally referred to [1], where an asynchronous agreement problem was studied for distributed decision-making problems. Thereafter, some consensus algorithms were studied under various information-flow constraints as in [2–4]. As a direct extension of the study of the consensus problem for systems with simple dynamics such as single-integrator kinematics or double-integrator dynamics, a feedback control is introduced in [5] to achieve consensus for agents with general linear dynamics. The consensus problem of the multi-agent systems with nonlinear dynamics or with nonlinear consensus algorithms has also been studied in [6,7].

Limited communication speed for information transmission, computation and execution time for generating the control inputs are the main reasons of time delay in many practical systems. The main problem involved in consensus with time delay is to study the effects of time delay on the convergence and performance of consensus. Consensus of multi agent systems with single integrator dynamics and transmission delay is studied in [8]. In a similar manner, consensus with time delay was studied for systems with different dynamics, where the standard single-integrator dynamics are replaced by other more complex ones, such as double-integrator dynamics [9,10], general linear dynamics [11], rigid bodies [12] and general nonlinear dynamics [13].

* Corresponding author.
E-mail addresses: m-mirzaei@aut.ac.ir (M. Mirzaei), atrianfar@aut.ac.ir (H. Atrianfar), noushin.mehdipour@aut.ac.ir (N. Mehdipour), f_abdollahi@aut.ac.ir (F. Abdollahi).

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Communication links among agents are not usually fixed and they can be disconnected. To deal with this problem, switching communication topologies are proposed and some approaches have been studied in [8,14,15] to reach consensus with switching topologies. In [16], the consensus algorithm is extended for double integrator dynamics when the information exchange topologies switch randomly with time, and necessary and sufficient conditions are derived to guarantee state convergence.

Considering both linear and nonlinear dynamics, the main tools for stability analysis of the closed-loop system with switching topology or time delay are matrix theory [17], Lyapunov functions [18], frequency-domain approach [19] and the contraction principle [20].

Considering switching topology and communication delay simultaneously in one problem is a challenging problem which is studied in [21], where dynamics of agents are assumed to be single integrator and the approach is developed for group of agents with second order discrete-time dynamics in [22].

Besides time delay and switching topologies, one of the most important issues in information communication is quantity of communications. In discrete time dynamics [22], it is assumed that in each sampling time the agents have access to their neighbors information and in the continuous time dynamics [21], there is a continuous time information transmission among agents. However, in most applications, having a continuous communication is not possible; therefore, agents should update their neighbors information in discrete times, which can be synchronous or asynchronous. Asynchronous consensus in continuous-time multi agent systems with single integrator dynamics considering switching topology and time-varying delays is studied in [17].

The main contribution of this paper is to develop a consensus algorithm for multi agent systems with continuous time second order nonlinear dynamics considering both switching topologies and nonuniform time delay in the manner that the communications are not continuous and updating neighbors information is asynchronous, which is not studied at the best of authors knowledge.

It worth noting that even in large scale systems that can be compared with multi-agent systems, state estimation of these systems is studied considering internal time-delay; However information transmission among subsystems is continuous and interconnection of the subsystems is fixed [23].

Many rigid-body attitude dynamics and robotic manipulators can be modeled in the form of Lagrangian equations. Therefore, consensus of Lagrangian systems has numerous applications in control such as consensus of multiple spacecraft with switching interaction in the space. Another application is when there is a team of networked mobile vehicles equipped with robotic arms that hold sensors (e.g. iRobot Packbot Explorer), where the robotic arms on each mobile vehicle can be modeled by Euler–Lagrange equations. The proposed algorithm can be used to synchronize these robotic arms and sensors equipped on different mobile vehicles so that the team scan an area cooperatively. In this paper, synchronization of a team of mobile robots with robotic arms is studied [24].

In [25], a proportional and derivative (PD)-type synchronization controller with feedback of the coupled position error is proposed for position synchronization of multi-axis motions which guarantees asymptotic convergence of errors to zero. This result and also the one in [26] rely on a bidirectional or unidirectional ring communication topology. Authors in [27] present a 6 degrees of freedom (6-DOF) synchronization scheme for a deep space formation of spacecraft which rely on an all-to-all communication topology. Consensus of Euler–Lagrange systems is widely studied in [24]. Moreover consensus of Euler–Lagrange systems considering time delay and jointly weakly connected topologies is studied in [28]. However, in these papers the information transmission among agents is considered to be continuous. The main contribution of this paper is designing asynchronous controllers to make a group of Euler–Lagrange agents reach consensus in the presence of time-varying delays while the communication topology is switching and the information exchange among the agents is not continuous-time. Moreover updating the neighbors information is asynchronous. The controller is applied to a class of nonlinear systems with Lagrangian dynamics. Using feedback linearization, the nonlinear dynamics of agents are transformed to their equivalent linear double integrator dynamics. Typical tools such as stochastic matrix theory cannot be applied since the state matrix does not have the required features. Therefore, a state transformation is applied to obtain a non-delayed extended state space. By solving the resulting continuous-time model, sufficient conditions on control parameters and switching interaction topology are derived to make the whole network reach consensus under bounded arbitrary time-varying delays. It is noted that the system has a continuous-time dynamics and only the control input is piecewise constant. Therefore, the collective state vector of the system is computed by solving the continuous-time dynamics without any discretization.

The rest of the paper is organized as follows: In Section 2, some preliminaries on graph theory and some important lemmas are stated. The problem statement is presented in Section 3 afterwards the consensus algorithm is proposed in Section 4. In Section 5, simulation results show the effectiveness of the proposed algorithm and finally, conclusion is presented in Section 6.

2. Basic graph theory and preliminaries

2.1. Graph theory

To model the interactions among n agents, directed graphs like $\mathcal{G}(\mathcal{V},\mathcal{E})$ with the set of nodes $\mathcal{V} = \{1, 2, \dots, n\}$ and edges $\mathcal{E} \subseteq \mathcal{V} \times \mathcal{V}$ are used. The index set of neighbors of agent i is denoted by $\mathcal{N}_i = \{j \in \mathcal{V} : (i,j) \in \mathcal{E}\}$ [29]. An adjacency matrix $A = [a_{ij}]$ with nonnegative adjacency elements a_{ij} is introduced, where the adjacency elements are associated with the edges of the graph. In other words, if there exists an edge from node j to i e.g. agent i receives information from j, then $a_{ij} = w$, where w can be considered as the edge weight, otherwise $a_{ij} = 0$. An indegree matrix, is a diagonal matrix that is introduced as $D = \mathrm{diag}\{d_1, d_2, \dots, d_n\}$ where $d_i = \sum_{j=1}^{n} a_{ij}$, and Laplacian matrix can be expressed as $L = D - A$.

The directed graph $\mathcal{G}(\mathcal{V},\mathcal{E})$ has a directed spanning tree if and only if $\mathcal{G}(\mathcal{V},\mathcal{E})$ has at least one node with a directed path to all other nodes [30].

Definition 1. A matrix P is nonnegative, denoted as $P \geq 0$, if all of its entries are nonnegative.

Definition 2. A square nonnegative matrix is row stochastic if all of its row sums are 1. Every row-stochastic matrix has 1 as an eigenvalue with an associated eigenvector $\mathbf{1}_n$.

Definition 3. The row-stochastic matrix P is indecomposable and aperiodic (SIA) if $\lim_{k\to\infty} P^k = \mathbf{1}_n v^T$, where v is a column vector [30].

Definition 4. Let $A = [a_{ij}]_{r\times r}$ be a stochastic matrix, then

$$\delta(A) = \max_{j} \max_{i_1,i_2} |a_{i_1 j} - a_{i_2 j}|$$

measures how different the rows of A are.