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# Hybrid-state driven autonomous control for planar bipedal locomotion



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#### HIGHLIGHTS

- Formulation of realistic foot-ground impact model.
- Explicit analytic solution for control law without a priori walking gait optimization.
- Transparent control design strategy based on dynamically coordinated motion control primitives.
- Novel velocity control algorithm by selective activation of ground contact point.
- Walking Performance is demonstrated for a wide range of velocities and ground slopes.

#### ARTICLE INFO

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#### ABSTRACT

The focus of this paper is on the development of a human inspired autonomous control scheme for a planar bipedal robot in a hybrid dynamical framework to realize human-like walking projected onto sagittal plane. In addition, a unified modelling scheme is presented for the biped dynamics incorporating the effects of various locomotion constraints due to varying feet-ground contact states, unilateral ground contact force, contact friction cone, passive dynamics associated with floating base etc. along with a practical impact velocity map on heel strike event. The autonomous control synthesis is formulated as a two-level hierarchical control algorithm with a hybrid-state based supervisory control in outer level and an integrated set of constrained motion control primitives, called task level control, in inner level. The supervisory level control is designed based on a human inspired heuristic approach whereas the task level control is formulated as a quadratic optimization problem with linear constraints. The explicit analytic solution obtained in terms of joint acceleration and ground contact force is used in turn to generate the joint torque command based on inverse dynamics model of the biped. The proposed controller framework is named as Hybrid-state Driven Autonomous Control (HyDAC). Unlike many other bipedal control schemes, HyDAC does not require a preplanned trajectory or orbit in terms of joint variables for locomotion control. Moreover, it is built upon a set of basic motion control primitives similar to those in human walk which provides a transparent and easily adaptable structure for the controller. These features make HyDAC framework suitable for bipedal walk on terrain with step and slope discontinuities without a priori gait optimization. The stability and agility of the proposed control scheme are demonstrated through dynamic model simulation of a 12-link planar biped having similar size and mass properties of an adult sized human being restricted to sagittal plane. Simulation results show that the planar biped is able to walk for a speed range of 0.1-2 m/s on level terrain and for a ground slope range of +/-20 deg for 1 m/s speed.

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#### 1. Introduction

The bipedal locomotion control is identified as a challenging problem by the control community due to its multiphase, hybrid nature and the unilateral characteristics of ground contact forces. The underactuation during heel or toe centred rolling motion and the intermittent ground impacts introduce additional complexity. The essence of bipedal walking control is to sustain near-periodic gaits with certain desired postural pattern while the biped is steered forward with the specified velocity over a fairly known terrain. There are basically three approaches reported in the literature for the design of bipedal locomotion control, viz. heuristic control methods, passive dynamic walking approach with minimal control, and analytical approach based on







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Abbreviations

Descrete event state <i>F<sub>c</sub></i> Ground contact force vector expressed in { <i>O<sub>0</sub></i> } <i>F<sub>g</sub></i> Ground contact force vector expressed in { <i>O<sub>0</sub></i> } <i>F<sub>g</sub></i> Ground contact force vector expressed in { <i>O<sub>0</sub></i> } <i>G</i> Gravitational force vector <i>g</i> Acceleration due to gravity <i>G</i> Combined Coriolis, Centrifugal and Gravity torque vector <i>g<sub>p</sub></i> Rows of <i>g</i> corresponding to the passive joints <i>h<sub>com</sub></i> Height of biped's hip joint in nominal straight knee stance posture Hes Hybrid event state <i>J<sub>c</sub></i> Feet-ground contact Jacobian w.r.t. translational motion <i>J<sub>cp</sub></i> Columns of <i>J<sub>c</sub></i> matrix corresponding to the passive joints <i>J<sub>g</sub></i> Feet-ground contact Jacobian expressed w.r.t. { <i>O<sub>g</sub></i> } <i>J<sub>hip</sub></i> Jacobian of hip joint expressed in { <i>O<sub>0</sub></i> } frame <i>K<sub>v</sub></i> Task loop velocity error gain <i>K<sub>v</sub></i> Number of feet tips in ground contact state <i>n<sub>j</sub></i> Number of feet tips in ground contact state <i>n<sub>j</sub></i> Number of feet tips in ground contact state <i>n<sub>j</sub></i> Number of opints for planar biped { <i>O<sub>0</sub></i> } Right handed inertial frame attached to any foot-ground contact point with <i>O<sub>g</sub>X<sub>g</sub></i> along the ground <i>q</i> Generalized position vector <i>R<sub>g</sub></i> 2D rotation matrix from { <i>O<sub>0</sub></i> } frame to { <i>O<sub>g</sub></i> } frame <i>X<sub>p</sub></i> Postural state of biped with dimension 2 <i>n<sub>p</sub></i> <i>x<sup>p</sup></i> <i>G</i> Goal point on heel impact in terms of <i>x<sub>p</sub></i> <i>X</i> State vector of biped given by ( <i>q</i> ; <i>q̇</i> ) <i>y<sub>go</sub></i> Intercept of extended ground line with <i>O<sub>0</sub>Y<sub>0</sub></i> axis <i>A</i> Impact reset map operating on <b>x</b> <i>r</i> , <i>r<sub>d</sub></i> Generalized force vector acting at biped joints and the corresponding command <i>π<sub>m</sub></i> Physical feet-ground contact state vector <i>π<sub>c</sub></i> Active feet-ground contact state vector defined for control <i>σ<sub>g</sub></i> Ground slope <i>σ<sub>gd</sub></i> Effective ground slope including terrain step discon- tinuity <i>μ<sub>c</sub></i> Dry friction parameter of foot-ground contact <i>model</i> <i>θ<sub>tor</sub></i> Inertial orientation of torso link <i>θ<sub>dma</sub></i> Orientation of	$D_p$	Rows of D matrix corresponding to the passive joints
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$ \begin{aligned} & \mathbb{H}_{es} & \text{Hybrid event state} \\ J_e & \text{Feet-ground contact Jacobian w.r.t. translational motion} \\ J_{cp} & \text{Columns of } J_c \text{ matrix corresponding to the passive joints} \\ J_g & \text{Feet-ground contact Jacobian expressed w.r.t. } \{O_g\} \\ J_{hip} & \text{Jacobian of hip joint expressed in } \{O_0\} \text{ frame} \\ K_p & \text{Task loop velocity error gain} \\ M_{bp} & \text{Total mass of biped links} \\ n_c & \text{Number of feet tips in ground contact state} \\ n_l & \text{Number of feet tips in ground contact state} \\ n_l & Number of fight handed inertial frame of reference also represented as O_0X_0Y_0Z_0.\{O_0\} Right handed coordinate frame attached to any foot-ground contact point with \overline{O_gX_g} along the groundq$ Generalized position vector $R_g$ 2D rotation matrix from $\{O_0\}$ frame to $\{O_g\}$ frame $V_{fc}$ Forward velocity command for biped along ground plane $(x_g, y_g)$ Coordinates of a point on ground expressed w.r.t. $\{O_0\}$ $\mathbf{x}_p$ Postural state of biped with dimension $2n_p$ $\mathbf{x}_p^0$ Goal point on heel impact in terms of $\mathbf{x}_p$ $\mathbf{x}$ State vector of biped given by $(q; \dot{q})$ $y_{go}$ Intercept of extended ground line with $\overline{O_0Y_0}$ axis $\Delta$ Impact reset map operating on $\mathbf{x}$ $\Gamma$ , $\Gamma_d$ Generalized force vector acting at biped joints and the corresponding command $\pi_m$ Physical feet-ground contact state vector defined for $\sigma_{gd}$ Ground slope $\sigma_{gd}$ Effective ground slope including terrain step discontinuity $\mu_c$ Dry friction parameter of foot-ground contact model $\theta_{tor}$ Inertial orientation of torso link $\theta_{dna}$ Orientation of dynamic neutral axis in inertial frame $\rho_{gc}$ Clearance between transit foot bottom and ground $\mathbf{v}_{re}$ $\mathbf{v}_{re}$ $\mathbf{v}_{re}$ between transit foot bottom and ground $\mathbf{v}_{re}$ $\mathbf{v}_{re}$ $\mathbf{v}_{re$	тпр	stance posture
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J <sub>cp</sub> Columns of J <sub>c</sub> matrix corresponding to the passive jointsJ <sub>g</sub> Feet-ground contact Jacobian expressed w.r.t. { $O_g$ } J <sub>hip</sub> Jacobian of hip joint expressed in { $O_0$ } frame $K_p$ K <sub>p</sub> Task loop position error gain $K_v$ M <sub>bp</sub> Total mass of biped links n <sub>c</sub> Number of feet tips in ground contact state $n_l$ Number of joints for planar biped { $O_0$ }Right handed inertial frame of reference also represented as $O_0X_0Y_0Z_0$ .{ $O_g$ }Right handed coordinate frame attached to any foot-ground contact point with $\overline{O_gX_g}$ along the groundqGeneralized position vector $R_g$ $Z_p$ Portation matrix from { $O_0$ } frame to { $O_g$ } frame $V_{fc}$ Forward velocity command for biped along ground plane( $x_g, y_g$ )Coordinates of a point on ground expressed w.r.t. { $O_0$ } $\mathbf{x}_p^p$ Goal point on heel impact in terms of $\mathbf{x}_p$ $\mathbf{x}_g$ $\mathcal{Y}_r$ $\mathcal{Y}_r$ $\mathcal{Y}_r$ $\mathcal{Y}_r$ $\mathcal{Y}_r$ $\mathcal{Y}_q$ $\mathcal{Y}_0$ $\mathcal{Y}_q$ <th>JC</th> <th>motion</th>	JC	motion
<i>μ</i> pFormation of <i>jjj</i>	I	Columns of $L_{\rm c}$ matrix corresponding to the passive
$J_{g}$ Feet-ground contact Jacobian expressed w.r.t. {O <sub>g</sub> } J <sub>hip</sub> Jacobian of hip joint expressed in {O <sub>0</sub> } frame K <sub>p</sub> Task loop position error gain M <sub>bp</sub> Total mass of biped links n <sub>c</sub> Number of feet tips in ground contact state n <sub>J</sub> Number of joints for planar biped {O <sub>0</sub> } Right handed inertial frame of reference also represented as O <sub>0</sub> X <sub>0</sub> Y <sub>0</sub> Z <sub>0</sub> . {O <sub>g</sub> } Right handed coordinate frame attached to any foot-ground contact point with $\overrightarrow{O_g}X_g$ along the ground q Generalized position vector R <sub>g</sub> 2D rotation matrix from {O <sub>0</sub> } frame to {O <sub>g</sub> } frame V <sub>fc</sub> Forward velocity command for biped along ground plane (x <sub>g</sub> , y <sub>g</sub> ) Coordinates of a point on ground expressed w.r.t. {O <sub>0</sub> } <b>x</b> <sub>p</sub> Postural state of biped with dimension 2n <sub>p</sub> <b>x</b> <sub>p</sub> <b>x</b> State vector of biped given by (q; $\dot{q}$ ) y <sub>go</sub> Intercept of extended ground line with $\overline{O_0Y_0}$ axis Δ Impact reset map operating on <b>x</b> Γ, Γ <sub>d</sub> Generalized force vector acting at biped joints and the corresponding command π <sub>m</sub> Physical feet-ground contact state vector $\sigma_g$ Ground slope $\sigma_{gd}$ Effective ground slope including terrain step discon- tinuity $\mu_c$ Dry friction parameter of foot-ground contact $\theta_{dna}$ Orientation of dynamic neutral axis in inertial frame $\rho_{gc}$ Clearance between transit foot bottom and ground	Jcp	ioints
	I.	Feet-ground contact lacobian expressed w.r.t. $\{O_n\}$
	Jg Ihin	Jacobian of hip joint expressed in $\{O_0\}$ frame
ImplementationTask loop velocity error gain $M_{bp}$ Total mass of biped links $n_c$ Number of feet tips in ground contact state $n_l$ Number of joints for planar biped $\{O_0\}$ Right handed inertial frame of reference also represented as $O_0X_0Y_0Z_0$ . $\{O_g\}$ Right handed coordinate frame attached to any foot-ground contact point with $\overrightarrow{O_gX_g}$ along the ground $q$ Generalized position vector $R_g$ 2D rotation matrix from $\{O_0\}$ frame to $\{O_g\}$ frame $V_{fc}$ $V_{fc}$ Forward velocity command for biped along ground plane $(x_g, y_g)$ Coordinates of a point on ground expressed w.r.t. $\{O_0\}$ $\mathbf{x}_p$ Postural state of biped with dimension $2n_p$ $\mathbf{x}_p^o$ Goal point on heel impact in terms of $\mathbf{x}_p$ $\mathbf{x}$ State vector of biped given by $(q; \dot{q})$ $y_{go}$ Intercept of extended ground line with $\overline{O_0Y_0}$ axis $\Delta$ Impact reset map operating on $\mathbf{x}$ $\Gamma$ , $\Gamma_d$ Generalized force vector acting at biped joints and the corresponding command $\pi_m$ Physical feet-ground contact state vector $\sigma_g$ Ground slope $\sigma_{gd}$ Effective ground slope including terrain step discon- tinuity $\mu_c$ Dry friction parameter of foot-ground contact $\theta_{dna}$ Orientation of dynamic neutral axis in inertial frame $\rho_{gc}$	$K_n$	Task loop position error gain
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$\begin{aligned} q & \text{Generalized position vector} \\ R_g & 2D \text{ rotation matrix from } \{O_0\} \text{ frame to } \{O_g\} \text{ frame} \\ V_{fc} & \text{Forward velocity command for biped along ground} \\ plane \\ (x_g, y_g) & \text{Coordinates of a point on ground expressed w.r.t,} \\ \{O_0\} \\ \mathbf{x}_p & \text{Postural state of biped with dimension } 2n_p \\ \mathbf{x}_p^0 & \text{Goal point on heel impact in terms of } \mathbf{x}_p \\ \mathbf{x} & \text{State vector of biped given by } (q; \dot{q}) \\ y_{go} & \text{Intercept of extended ground line with } \overrightarrow{O_0Y_0} \text{ axis} \\ \Delta & \text{Impact reset map operating on } \mathbf{x} \\ \Gamma, \Gamma_d & \text{Generalized force vector acting at biped joints and} \\ \pi_m & \text{Physical feet-ground contact state vector } \\ \pi_c & \text{Active feet-ground contact state vector defined for control} \\ \sigma_{g} & \text{Ground slope} \\ \sigma_{gd} & \text{Effective ground slope including terrain step discontinuity} \\ \mu_c & \text{Dry friction parameter of foot-ground contact model} \\ \\ \theta_{tor} & \text{Inertial orientation of torso link} \\ \\ \theta_{dna} & \text{Orientation of dynamic neutral axis in inertial frame} \\ \end{array}$		foot-ground contact point with $\overrightarrow{OX}$ along the
$q \qquad \text{Generalized position vector} \\ R_g \qquad 2D \ rotation matrix from {O_0} \ frame \ to {O_g} \ frame \\ V_{fc} \qquad \text{Forward velocity command for biped along ground} \\ plane \\ (x_g, y_g) \qquad \text{Coordinates of a point on ground expressed w.r.t,} \\ {O_0} \\ \mathbf{x}_p \qquad \text{Postural state of biped with dimension } 2n_p \\ \mathbf{x}_p \qquad \text{Goal point on heel impact in terms of } \mathbf{x}_p \\ \mathbf{x} \qquad \text{State vector of biped given by } (q; \dot{q}) \\ y_{go} \qquad \text{Intercept of extended ground line with } \overrightarrow{O_0Y_0} \text{ axis} \\ \Delta \qquad \text{Impact reset map operating on } \mathbf{x} \\ \Gamma, \Gamma_d \qquad \text{Generalized force vector acting at biped joints and} \\ the corresponding command \\ \pi_m \qquad \text{Physical feet-ground contact state vector defined for control} \\ \sigma_g \qquad \text{Ground slope} \\ \sigma_{gd} \qquad \text{Effective ground slope including terrain step discontinuity} \\ \mu_c \qquad \text{Dry friction parameter of foot-ground contact model} \\ \theta_{tor} \qquad \text{Inertial orientation of torso link} \\ \theta_{dna} \qquad \text{Orientation of dynamic neutral axis in inertial frame} \\ \rho_{gc} \qquad \text{Clearance between transit foot bottom and ground} \\ \end{cases}$		ground contact point with $O_g X_g$ along the
$q$ Contrained position vector $R_g$ 2D rotation matrix from $\{O_0\}$ frame to $\{O_g\}$ frame $V_{fc}$ Forward velocity command for biped along ground plane $(x_g, y_g)$ Coordinates of a point on ground expressed w.r.t, $\{O_0\}$ $\mathbf{x}_p$ Postural state of biped with dimension $2n_p$ $\mathbf{x}_p^0$ Goal point on heel impact in terms of $\mathbf{x}_p$ $\mathbf{x}$ State vector of biped given by $(q; \dot{q})$ $y_{go}$ Intercept of extended ground line with $\overline{O_0Y_0}$ axis $\Delta$ Impact reset map operating on $\mathbf{x}$ $\Gamma$ , $\Gamma_d$ Generalized force vector acting at biped joints and the corresponding command $\pi_m$ Physical feet-ground contact state vector $\pi_c$ Active feet-ground contact state vector defined for control $\sigma_g$ Ground slope $\sigma_{gd}$ Effective ground slope including terrain step discon- tinuity $\mu_c$ Dry friction parameter of foot-ground contact model $\theta_{dna}$ Orientation of dynamic neutral axis in inertial frame $\rho_{gc}$	a	Ceneralized position vector
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$\begin{aligned} & (x_g, y_g) & \text{Coordinates of a point on ground expressed w.r.t,} \\ & \{O_0\} \\ & \mathbf{x}_p & \text{Postural state of biped with dimension } 2n_p \\ & \mathbf{x}_p^o & \text{Goal point on heel impact in terms of } \mathbf{x}_p \\ & \mathbf{x} & \text{State vector of biped given by } (q; \dot{q}) \\ & \mathbf{y}_{go} & \text{Intercept of extended ground line with } \overrightarrow{O_0Y_0} \text{ axis} \\ & \Delta & \text{Impact reset map operating on } \mathbf{x} \\ & \Gamma, \Gamma_d & \text{Generalized force vector acting at biped joints and} \\ & \text{the corresponding command} \\ & \pi_m & \text{Physical feet-ground contact state vector defined for control} \\ & \sigma_g & \text{Ground slope} \\ & \sigma_{gd} & \text{Effective ground slope including terrain step discontinuity} \\ & \mu_c & \text{Dry friction parameter of foot-ground contact model} \\ & \theta_{tor} & \text{Inertial orientation of torso link} \\ & \theta_{dna} & \text{Orientation of dynamic neutral axis in inertial frame} \\ & \rho_{gc} & \text{Clearance between transit foot bottom and ground} \\ \end{aligned}$	Ve	Forward velocity command for bined along ground
$ \begin{array}{ll} (x_g, y_g) & \text{Coordinates of a point on ground expressed w.r.t,} \\ \{O_0\} \\ \mathbf{x}_p & \text{Postural state of biped with dimension } 2n_p \\ \mathbf{x}_p^o & \text{Goal point on heel impact in terms of } \mathbf{x}_p \\ \mathbf{x} & \text{State vector of biped given by } (q; \dot{q}) \\ \hline \mathbf{y}_{go} & \text{Intercept of extended ground line with } \overrightarrow{O_0Y_0} \text{ axis} \\ \Delta & \text{Impact reset map operating on } \mathbf{x} \\ \Gamma, \Gamma_d & \text{Generalized force vector acting at biped joints and} \\ \pi_m & \text{Physical feet-ground contact state vector } \\ \pi_c & \text{Active feet-ground contact state vector defined for control} \\ \sigma_g & \text{Ground slope} \\ \sigma_{gd} & \text{Effective ground slope including terrain step discontinuity} \\ \mu_c & \text{Dry friction parameter of foot-ground contact model} \\ \theta_{tor} & \text{Inertial orientation of torso link} \\ \theta_{dna} & \text{Orientation of dynamic neutral axis in inertial frame} \\ \rho_{gc} & \text{Clearance between transit foot bottom and ground} \\ \end{array}$	• jc	nlane
$\{O_0\}$ $\mathbf{x}_p$ Postural state of biped with dimension $2n_p$ $\mathbf{x}_p^0$ Goal point on heel impact in terms of $\mathbf{x}_p$ $\mathbf{x}$ State vector of biped given by $(q; \dot{q})$ $y_{go}$ Intercept of extended ground line with $\overline{O_0Y_0}$ axis $\Delta$ Impact reset map operating on $\mathbf{x}$ $\Gamma, \Gamma_d$ Generalized force vector acting at biped joints and the corresponding command $\pi_m$ Physical feet-ground contact state vector $\pi_c$ Active feet-ground contact state vector defined for control $\sigma_g$ Ground slope $\sigma_{gd}$ Effective ground slope including terrain step discon- tinuity $\mu_c$ Dry friction parameter of foot-ground contact model $\theta_{dna}$ Orientation of dynamic neutral axis in inertial frame $\rho_{gc}$ Clearance between transit foot bottom and ground	$(\mathbf{x}_{\alpha}, \mathbf{v}_{\alpha})$	Coordinates of a point on ground expressed w.r.t.
$\mathbf{x}_p$ Postural state of biped with dimension $2n_p$ $\mathbf{x}_p^o$ Goal point on heel impact in terms of $\mathbf{x}_p$ $\mathbf{x}$ State vector of biped given by $(q; \dot{q})$ $y_{go}$ Intercept of extended ground line with $\overline{O_0Y_0}$ axis $\Delta$ Impact reset map operating on $\mathbf{x}$ $\Gamma, \Gamma_d$ Generalized force vector acting at biped joints and the corresponding command $\pi_m$ Physical feet-ground contact state vector $\pi_c$ Active feet-ground contact state vector defined for control $\sigma_g$ Ground slope $\sigma_{gd}$ Effective ground slope including terrain step discon- tinuity $\mu_c$ Dry friction parameter of foot-ground contact model $\theta_{dna}$ Orientation of dynamic neutral axis in inertial frame $\rho_{gc}$	(g, Jg)	$\{O_0\}$
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Coriolis and Centrifugal force Coefficient matrix

Joint space inertia matrix

formal mathematical models. A brief description of each of these approaches are given below.

The complexity of bipedal locomotion has prompted many researchers to pursue heuristic approach for the realization of practically stable bipedal walking gaits. The most popular among them is known as Zero Moment Point (ZMP) concept, proposed in the late 1960's by Vukobratović et al. [1,2]. It states that as long as the ZMP of a biped stays within the foot support polygon, the biped cannot fall by tipping over the edges of its feet. ZMP concept was later extended by A. Goswami by introducing Foot Rotation Indicator (FRI) point useful for quantifying the instability associated with foot rotation about stance toe [3]. Different variants of ZMP based controls were used in many practical humanoids such as ASIMO, WABIAN-2, LOLA etc. [4-6]. However, it has been proved that ZMP criterion is neither necessary nor sufficient for bipedal gait stability [7] and it leads to inefficient bent-knee type locomotion. Another heuristically motivated stability concept is based on the regulation of centroidal angular momentum of biped [8]. This is also not a necessary requirement for bipedal stability and it often leads to unnaturally looking gaits [9]. J.E. Pratt et al. proposed velocity regulation scheme for bipedal gait by steering the swing foot towards the, 1-step capture region [9,10] based on a linear inverted pendulum model of biped. Virtual model control is another intuitive approach proposed by J.E. Pratt et al. [11]. Heuristic methods make use either simplified models of biped [12,10] or simulation based optimization using the full biped model [6] for generating the empirical control expressions. From accuracy point of view, the latter approach is superior to the former.

The second approach known as passive dynamic walking was developed by Tad McGeer in the late 1980's [13]. Unlike traditional robots, which expend energy for controlled actuation by using motors, McGeer showed that a human-like frame can walk itself down a slope without requiring powered joints. McGeer's initial configuration for passive dynamic walker relies only on the natural swinging of the limbs under gravity to move forward down a slope. Later, Steven H. Collins and his associates have demonstrated a 3D passive dynamic walker with two legs and knees by extending the concept of McGeer [14]. Extensions to passive dynamic walker have been made later on by adding minimal actuation to the joints [15–17]. Even though this approach leads to energetically efficient gaits, it lacks robustness in the presence of large disturbances [10].

In the analytical control approach, biped is modelled as a floating base robotic manipulator with holonomic feet-ground contact constraints along with intermittent swing leg ground impact represented by suitable impact velocity maps. The periodic stability of the walking gait is analysed based on the method of Poincaré sections. There are various control methods developed in the above framework like virtual constraints and hybrid zero dynamics (HZD) [7,18-20], energy shaping [21], numerical optimization [22], and controlled Routhian reduction [23]. The above analytical approaches are in general mathematically rigorous, and computationally intensive. For example, in HZD based virtual constraint method, the control solution along with the periodic orbit is obtained as the solution of a nonlinear optimization problem. The formulation of the orbit-control optimization problem with all the associated holonomic and nonholonomic constraints makes the solution guite hard to obtain. Moreover, multiple local minima exist due to the nonconvexity of formulation. Another drawback of such control laws which make use of a priori computed gait is that, the control is not robust against external force disturbances [10] and unexpected changes in ground slopes or step discontinuities within a gait and as a result, the ground contact constraints may get violated during control execution. A comprehensive review of the models, feedback control and open problems of 3D bipedal robotic walking is given in the survey paper by J.W. Grizzle et al. [24].

Another area of active research is focused on achieving human like walking gaits [25,26]. Any unnatural stability criterion will impose unnecessary constraints on walking gaits affecting its Download English Version:

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