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Consensus of multi-agent systems in the cooperation–competition network with inherent nonlinear dynamics: A time-delayed control approach

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ABSTRACT

In this paper, the consensus problem is investigated for a group of first-order agents in the cooperation–competition network, where agents can cooperate or even compete with each other, i.e., the elements in the coupling weight matrix of the graph can be either positive or negative. In order to solve this consensus problem, the whole network is firstly divided into two sub-networks, i.e., the cooperation sub-network and the competition sub-network, and then two kinds of time-delayed control schemes are designed in the competition sub-network. By combining the Lyapunov theory together with the synchronization manifold method, several effective sufficient conditions of consensus are provided without assuming that the interaction topology is strongly connected or contains a directed spanning tree, which means that the competition relationships could help the agents achieve consensus under the time-delayed control designed in the competition sub-network. Moreover, the results are also extended to the pure competition networks where all the elements in the weight matrices are either zeros or negative. Finally, some simulation examples are provided to validate the effectiveness of the theoretical analysis.

1. Introduction

Over the past few years, increasing attention has been paid to the study of multi-agent systems (MAS) across many fields of science and engineering. Generally, a multi-agent system is always composed of many interconnected agents, in which agents represent individual elements with their own dynamics and links represent certain relationships between their dynamics. Applications are ubiquitous in the real world, such as the World Wide Web [1] where the web pages as agents are connected by hyperlinks, the Social Network [2] in which the agents are persons and the links represent the relationships between them, and the Gene Regulatory Network [3] in which the genes as agents are connected by biological signals, etc.

Consensus is one of the most important issues in the multi-agent systems, which is always used to explain flocking of social animals and has been widely applied in many engineering areas such as air traffic control, wireless sensor networks and mobile robotic swarms [4–9]. The main objective of a consensus problem is to design an appropriate algorithm or interaction rule such that a group of agents converges to a consistent quantity of interest. Here, the algorithm or the interaction rule is usually called agreement protocol, and the consistent quantity depending on the initial states of all agents is called consensus state that may represent certain physical quantity such as attitude, position, temperature, voltage, etc.

In the past decade, researchers began to study consensus problems of multi-agent systems by considering different subsistent limitations, such as finite-time consensus [10–12], higher-order consensus [13–15], leader-following consensus [16–18], heterogeneous consensus [19–21], Adaptive consensus [22–24], etc. For example, Ma et al. [25] considered the consensus problem of second-order multi-agent systems with sampled data, where the sufficient consensus condition was derived and the upper bound of sampling interval was estimated by adopting a novel time-dependent Lyapunov function. Ren [26] further analyzed such second-order consensus problem in four cases, where several conditions were derived based on the interaction topology. Yu et al. [27] studied some necessary and sufficient conditions for second-order consensus in directed networks containing a directed spanning tree, where they proved that both the real and imaginary parts of the

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eigenvalues of the communication graph's Laplacian matrix play key roles in reaching such consensus. Xiao and Wang [28] studied asynchronous consensus problem for continuous-time multi-agent systems with discontinuous information transmission by adopting nonnegative matrix analysis and graph theory, and the asynchronous consensus problem was further investigated in [29]. Li et al. [30] considered the consensus problem of multi-agent systems with a time-invariant communication topology consisting of nodes with general linear dynamics, and introduced a novel framework that can describe the consensus of multi-agent systems and the synchronization of complex dynamic networks in a unified way. Liang et al. [31] investigated a new synchronization problem for an array of 2-D coupled dynamical networks where all the agents were governed by the Fornasini-Marchesini system, and derived several sufficient conditions by adopting the energy-like quadratic function. More recently, Wen et al. [32] studied the second-order consensus problem with communication constraints where each agent is assumed to share information only with its neighbors on some disconnected time intervals, which is then solved by a novel protocol with synchronous intermittent information feedback. Liu and Zhao [33] investigated the generalized output synchronization problem for dynamical networks using the output synchronization without assuming the negative definiteness property of the coupling matrix of the network. Ma and Lu [34] studied the cluster synchronization problem of a class of general complex dynamical networks, and the network topology was assumed to be directed and weakly connected. By the pinning control scheme, some simple control criteria were proposed.

Sometimes, the coupling delays [35,36] between agents need to be considered in real circumstance with practical reasons such as the communication congestion of the channels, the finite switching and spreading speed of the hardware and circuit implementation, the moving of the agents, etc. Lu et al. [37] proposed an adaptive scheme for the stabilization and synchronization of chaotic Lur'e systems with time-varying delay. Based on the work introduced in [18], Lin et al. [38] investigated consensus problems in networks of continuous-time agents with diverse time-delays and jointly-connected topologies, where several sufficient conditions were derived by adopting the Lyapunov–Krasovskii approach and it can be found that all the agents can reach consensus even though the communication structures among agents dynamically change over time so that the corresponding graphs may not be connected. Yu et al. [39] investigated the global synchronization problem of a generalized linearly hybrid coupled network with time-varying delay, and several effective sufficient conditions of global synchronization were obtained based on the Lyapunov function and the linear matrix inequality (LMI). Recently, Guan et al. [40] considered the consensus problem with system delay and multiple coupling delays via impulsive distributed control, and introduced the concept of control topology that describe the whole controller structure. Wang et al. [41] presented the coupled discrete-time stochastic complex network with randomly occurred nonlinearities and time delays. In their paper, several delay-dependent sufficient conditions were obtained which ensure the asymptotic synchronization in the mean square sense by employing a combination of LMI, the free-weighting matrix method and stochastic analysis theory. Qin et al. [42] studied the consensus problems for second-order agents under directed arbitrarily switching topologies with communication delay, and they proved that consensus can be reached if the delay is small enough.

However, almost all of the previous studies were only concerned with the cooperation network, i.e., the weight matrix of the network is assumed to be nonnegative, which may give rise to the problem that the network cannot fully represent the real physical object. Therefore, it needs to be further generalized in some cases. This paper focuses on studying the consensus problem in cooperation–competition networks where the entries of the corresponding weight matrices may be negative, which can further lead to the negative off-diagonal entries of the Laplacian matrices. Generally, such a cooperation–competition network can be divided into two sub-networks, i.e., the cooperation and competition sub-networks with the links having positive and negative weights, respectively. Since the competitions between agents may prevent their consensus, the situation introduced here is much more complicated than most former cases where only cooperation is considered. In this paper, a time-delayed control scheme is designed in the competition sub-network, which could overcome the negative factor and help the agents achieve consensus. Meanwhile, such control scheme is typically simple to be implemented. Based on the viewpoint of the synchronization manifold in [39,43], several sufficient conditions of consensus are then deduced by using the Lyapunov method and the linear matrix inequality (LMI).

The rest of the paper is organized as follows. In Section 2, some basic definitions in graph theory and related the mathematical preliminary results are presented. Then, the two kinds of time-delayed control schemes are described. In Section 3, main analytical results are established according to different time-delayed control schemes. In Section 4, numerical simulations are implemented to demonstrate the analytic results. Finally, the paper is concluded in Section 5.

2. Problem formulation

In this section, some basic definitions in graph theory, preliminary mathematical results, the system model, and two kinds of time-delayed control schemes are firstly introduced for subsequent use.

The mathematical notations which will be employed in this paper are presented as follows. Let R^n denote the n-dimensional real vector space, and the Euclidean norms of a vector $x \in R^n$ and a matrix $A \in R^{n\times n}$ are denoted by $\|x\| \triangleq \sqrt{x^T x}$ and $\|A\| \triangleq \sqrt{\lambda_{\max}(A^T A)}$ with $\lambda_{\max}(\cdot)$ and $\lambda_{\min}(\cdot)$ being the maximum and minimum eigenvalues of the matrix A, respectively. The matrix $A>0$ or $A<0$ denotes that A is symmetric and positive or negative definite matrix. Besides, the identity matrix of order n is denoted by I_n and the Kronecker product of matrices $A \in R^{n\times n}$ and $B \in R^{n\times n}$ is defined as

$$A \otimes B = \begin{bmatrix} a_{11}B & \cdots & a_{1n}B \\ \vdots & \ddots & \vdots \\ a_{n1}B & \cdots & a_{nn}B \end{bmatrix},$$

which satisfies the following properties:

$$\|I_P \otimes A\| = \|A \otimes I_N\| = \|A\|,$$
$$(A+B) \otimes C = A \otimes C + B \otimes C,$$
$$(A \otimes B)(C \otimes D) = (AC) \otimes (BD).$$

2.1. Topology description

In general, information exchanges between agents in a multi-agent system can be modeled by a network or graph [4,44]. Let $\mathfrak{R} = (V, \zeta, A)$ be a weighted directed network of N agents with a set of nodes $V = \{\pi_1, \pi_2, \cdots \pi_N\}$, a set of links $\zeta \subseteq V \times V$, and a weight matrix $A = [a_{ij}]$, which represents the communication topology. A link of $\mathfrak{R}$ is denoted by $e_{ij} = (\pi_i, \pi_j)$ which is associated with a nonzero weight, i.e. $(\pi_i, \pi_j) \in \zeta \Leftrightarrow a_{ij} \neq 0$, then the neighbor set of node π_i is denoted by $N_i = \{\pi_j | (\pi_j, \pi_i) \in \zeta\}$. Note that here $a_{ij} < 0$ also makes sense, which means that agent i competes with agent j,