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Delay-dependent stability criteria of uncertain Markovian jump neural networks with discrete interval and distributed time-varying delays ☆

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ABSTRACT

In this paper, a class of uncertain neural networks with discrete interval and distributed time-varying delays and Markovian jumping parameters (MJPs) are carried out. The Markovian jumping parameters are modeled as a continuous-time, finite-state Markov chain. By using the Lyapunov–Krasovskii functionals (LKFs) and linear matrix inequality technique, some new delay-dependent criteria is derived to guarantee the mean-square asymptotic stability of the equilibrium point. Numerical simulations are given to demonstrate the effectiveness of the proposed method. The results are also compared with the existing results to show the less conservativeness.

1. Introduction

It is well known that, many kinds of neural networks such as cellular neural networks, Hopfield neural networks, Cohen–Grossberg neural networks, recurrent neural networks (RNNs), complex dynamical networks (CDNs), bidirectional associative memory (BAM) neural networks, chaotic neural networks (CNNs) and static neural networks (SNNs) have been studied, since their extensive applications in different fields such as fault diagnosis, pattern recognition, signal processing and parallel computation [1–6]. Some of these applications require the equilibrium points of the designed network to be stable. Since axonal signal transmission time delays often occur in various neural networks, and may also cause undesirable dynamic network behaviors such as oscillation and instability. Thus it is important to study the stability of neural networks [7–9].

On the other hand Markovian jump neural networks (MJNNs) can be regarded as a special class of hybrid systems, which can model dynamic systems whose structures are subject to random abrupt parameter changes resulting from component or interconnection failures, sudden environment changes, changing subsystem interconnections, and so forth [10,11]. A neural network may have finite modes, which may jump from one to another at various time. It is shown that such jumping can be determined by a Markovian chain [12]. Much work on MJNNs has been reported in the literature [13–16]. A great number of results on the stability and estimation problems related to such neural networks (NNs) have appeared in the recent years [17]. Applications of this kind of neural networks can be found in modeling production systems, economic systems, and other practical systems.

The phenomena of time-delays are very often encountered in various physical systems, like communication systems, nuclear reactors, aircraft stabilization, ship stabilization, models of lasers, manual control and systems with lossless transmission lines, for example see [18–22]. Stability is always required for the real-world applications of neural networks, since their potential applications to solve some previously unsolvable problems and improve system performance in many fields such as pattern recognition, fault diagnosis, signal processing and parallel computation. Some of these applications require the equilibrium points of the designed network to be stable. Thus, stability analysis is one of the fundamental research issues in the study of neural networks. In the past decade, lots of research efforts have been devoted to the stability analysis of neural networks with time delays. This is because time delays are unavoidable in neural networks and, more importantly, the existence of time delays often makes a neural network unstable.

In practice, interval time delays exist in biological and artificial neural networks due to the finite switching speed of neurons and amplifiers. That is, the range of delay varies in an interval for which

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the lower bound is not restricted to zero. The interval time-varying delays are used to indicate the promoted speed of signals is finite and uncertain in systems [23,24]. And, note that while signal propagation is sometimes instantaneous and can be modeled with discrete delays, it may also be distributed during a certain time period so that distributed delays are incorporated into the model. The time-varying delays are assumed to be unbound. The distributed delays were studied in [25] due to the spatial nature of a neural network with an amount of parallel pathways of a variety of axon sizes and lengths in many cases. Therefore, both discrete time-varying and distributed time-varying delays should be taken into account when modeling a realistic neural network [26–28]. However, delay-dependent stability criteria of uncertain Markovian jump neural networks with interval and distributed time-varying delays has not done yet. Which motivates our work.

From the above discussions, in this paper we studied the delay-dependent stability criteria of uncertain Markovian jump neural networks with interval and distributed time-varying delays. A sufficient condition has been obtained in terms of Linear matrix inequalities. Lyapunov–Krasovskii functionals together with the zero function guarantee the asymptotic stability of the neural networks. The developed results in this paper are generally less conservative than some existing methods. To show the less conservativeness of our result, numerical simulations are given.

Notations: Throughout this paper, $\mathcal{R}^n$ and $\mathcal{R}^{n\times n}$ denote, respectively, the n-dimensional Euclidean space and the set of all $n\times n$ real matrices. For a matrix B and two symmetric matrices A and C, $\left[\begin{smallmatrix} A & B\\ * & C\end{smallmatrix}\right]$ denote the symmetric matrix, where the notation $*$ represents the entries implied by symmetry. A^T and A^{-1} denote the matrix transpose and inverse of A respectively. We say $X>0$ means that the matrix X is real symmetric positive definite with appropriate dimensions. I denotes the identity matrix with appropriate dimensions. Let $(\Omega,\mathcal{F},\mathcal{P})$ be a complete probability space which is related to an increasing family $(\mathcal{F}_t)_{t>0}$ of σ algebras $(\mathcal{F}_t)_{t>0}\subset\mathcal{F}$, where Ω is the sample space, $\mathcal{F}$ is σ algebra of subsets of the sample space and $\mathcal{P}$ is the probability measure on $\mathcal{F}$. Let $\|f\|_2=\sqrt{\int_0^\infty \|f\|^2\,dt}, f(t)\in L_2[0,\infty)$, $\|f\|$ refers to the Euclidean norm of the function $f(t)$ at the time t. $L_2[0,\infty)$ is the space of square integrable vectors on $[0,\infty)$.

2. Problem statement

Consider the following delayed Markovian jump neural networks with discrete interval and distributed time-varying delays:

$$\dot{y}(t)=-A(r(t))y(t)+B(r(t))g(y(t))+C(r(t))g(y(t-\tau(t)))$$
$$+D(r(t))\int_{t-d(t)}^{t} g(y(s))\,ds+\nu(t), \tag{1}$$

where $y(t)=[y_1(t),y_2(t),\ldots,y_n(t)]^T\in\mathcal{R}^n$ is the state vector of the network at time $t\ge 0$, n corresponds to the number of neurons, $A=\text{diag}\{a_1,\ldots,a_n\}$ is a diagonal matrix with $a_i>0$, $i=1,2,\ldots,n$. B, C and D represent the connection weight matrix, the discretely delayed connection weight matrix and the distributively delayed connection weight matrix, respectively. $g(y(t))=[g(y_1(t)),\ldots,g(y_n(t))]^T\in\mathcal{R}^n$ denotes the neuron activation function at time t. $\nu(t)$ denotes the external inputs at time t. $\tau(t)$ and $d(t)$ denote the time-varying and distributed delays, respectively, and are assumed to satisfy

$$\tau_1\le\tau(t)\le\tau_2,\quad \dot{\tau}(t)\le\mu_1,\quad 0\le d(t)\le d,\quad \dot{d}(t)\le\mu_2,$$

where $\tau_2>\tau_1>0$, d, μ_1 and μ_2 are constants.

Assume that $y^*=[y_1^*,y_2^*,\ldots,y_n^*]^T$ is an equilibrium point of Eq. (1), one can derive from (1) that the transformation $x(t)=y(t)-y^*$ transforms system (1) into the following uncertain Markovian jump neural networks with discrete interval and distributed time-varying delays:

$$\dot{x}(t)=-A(r(t))x(t)+B(r(t))f(x(t))+C(r(t))f(x(t-\tau(t)))$$
$$+D(r(t))\int_{t-d(t)}^{t} f(x(s))\,ds, \tag{2}$$

where $x(t)$ is the state vector of the transformation system and $f(x(t))=g(x(t)+y^*)-g(y^*)$.

The matrices $A(r(t))=A(r(t))+\Delta A(r(t))$, $B(r(t))=B(r(t))+\Delta B(r(t))$, $C(r(t))=C(r(t))+\Delta C(r(t))$ and $D(r(t))=D(r(t))+\Delta D(r(t))$ are real constant matrices with appropriate dimensions for all $r(t)\in\mathcal{N}$. In that $A(r(t))$, $B(r(t))$, $C(r(t))$ and $D(r(t))$ are real-valued known constant matrices. And $\Delta A(r(t))$, $\Delta B(r(t))$, $\Delta C(r(t))$ and $\Delta D(r(t))$ are real-valued unknown matrices representing time-varying parameter uncertainties, and are assumed to be of the form

$$[\Delta A(r(t))\ \ \Delta B(r(t))\ \ \Delta C(r(t))\ \ \Delta D(r(t))]$$
$$=M(r(t))F(r(t))[E_1(r(t))\ \ E_2(r(t))\ \ E_3(r(t))\ \ E_4(r(t))], \tag{3}$$

where $M(r(t))$, $E_1(r(t))$, $E_2(r(t))$, $E_3(r(t))$ and $E_4(r(t))$ are known real constant matrices for all $r(t)\in\mathcal{N}$ and $F(r(t))$ is the uncertain time-varying matrix satisfying

$$F^T(r(t))F(r(t))\le I,\quad \forall r(t)\in\mathcal{N}. \tag{4}$$

Let $\{r(t), t\ge 0\}$ be a right-continuous Markov chain on a probability space $(\Omega,\mathcal{F},\mathcal{P})$ taking values in a finite state space $\mathcal{N}=\{1,2,\ldots,m\}$ with generator $\Pi=\{\pi_{ij}\}$ given by

$$P\{r_{t+\Delta}=j\,|\,r(t)=i\}=\begin{cases}\pi_{ij}\Delta+o(\Delta), & i\ne j\\ 1+\pi_{ij}\Delta+o(\Delta), & i=j\end{cases}$$

where $\Delta>0$, $\lim_{t\to+\infty}o(\Delta)/\Delta=0$, $\pi_{ij}\ge 0$ is the transition rate from i to j if $j\ne i$ while $\pi_{ii}=-\sum_{j=1,j\ne i}^{m}\pi_{ij}$. The initial system mode probability vector is defined by

$$\pi(0)=[\pi_i(0)\cdots\pi_m(0)]^T.$$

Note that the system mode probability vector $\pi(t)$ can be found via

$$\dot{\pi}(t)=\Pi^T\pi(t).$$

The Markov process transition rate matrix Π is defined by

$$\Pi=\begin{bmatrix}\pi_{11} & \pi_{12} & \cdots & \pi_{1n}\\ \pi_{21} & \pi_{22} & \cdots & \pi_{2n}\\ \vdots & \vdots & \ddots & \vdots\\ \pi_{n1} & \pi_{n2} & \cdots & \pi_{nn}\end{bmatrix}$$

Here, we assume that the Markov process is irreducible.

In order to obtain our main results, the activation functions in (1) are assumed to satisfy the following assumption.

Assumption 1. Each activation function in system (1) $f_i(t)$ $(i=1,2,\ldots,n)$ is continuous and bounded, and satisfies the following conditions:

$$0\le\frac{f_i(k_1)-f_i(k_2)}{k_1-k_2}\le L_i,$$

where L_i $(i=1,2,\ldots,n)$ are some constants and k_1, $k_2\in\mathcal{R}$, $k_1\ne k_2$.

Lemma 2.1 (*Syed Ali and Saravanakumar [29]*). *For any scalars $\tau(t)\le 0$ and any constant matrix $Q\in\mathcal{R}^{n\times n}$, $Q=Q^T>0$, the following inequality holds:*

$$-\int_{t-\tau(t)}^{t}\dot{x}^T(s)Q\dot{x}(s)\,ds\le\tau(t)\xi^T(t)VQ^{-1}V^T\xi(t)+2\xi^T(t)V[x(t)-x(t-\tau(t))],$$