Neurocomputing 151 (2015) 729–736

Contents lists available at ScienceDirect

Neurocomputing

journal homepage: www.elsevier.com/locate/neucom

Synchronization of complex dynamical networks with discrete time delays on time scales

Quanxin Cheng [a], Jinde Cao [a,b,*]

[a] Department of Mathematics, Southeast University, Nanjing 210096, China
[b] Department of Mathematics, Faculty of Science, King Abdulaziz University, Jeddah 21589, Saudi Arabia

ARTICLE INFO

Article history:
Received 5 May 2014
Received in revised form
8 August 2014
Accepted 10 October 2014
Communicated by H. Jiang
Available online 18 October 2014

Keywords:
Exponential synchronization
Lyapunov functional
Complex network
Time scale
Delta derivative

ABSTRACT

This paper investigates the global exponential synchronization problem in arrays of complex networks with time delays based on the theory of calculus on time scales, Lyapunov functional and linear matrix inequality technique, and derives several sufficient criteria to ensure the global exponential synchronization for the considered networks. It is shown that the synchronization conditions of complex dynamical networks on time scales are different from those derived for conventional continuous or discrete complex networks. Moreover, the presented results of this paper indicate that the globally synchronization problems with both discrete time case and continuous time case can be addressed in a unified framework.

© 2014 Elsevier B.V. All rights reserved.

1. Introduction

Networks are ubiquitous in the real world, such as Internet, World Wide Web, social networks, ecosystems, and global economic network. The topologies of complex networks have been widely studied, since the topology property itself is very interesting, and also, it shows great influence on the collective dynamics of complex dynamic networks. As a typical collective behavior in complex networks, synchronization or consensus problem has been widely studied in the past few years [1–5]. In [1], Bauso et al. studied the consensus for the networks with unknown but bounded disturbances. The consensus problem in second- or higher-order multi-agent systems was investigated in [2–4]. Obviously, there exist great benefit of having synchronization or consensus in physics, biology or some engineering applications, such as secure communication, image processing, harmonic oscillation generation, formation control and flocking [6–10]. In [6], Olfati-Saber studied the flocking of multi-agent dynamic systems. In [7], Barahona and Pecora introduced the synchronization in small-world networks. Synchronization via pinning control on general complex networks was discussed by Yu et al. in [9], and so on. The synchronization in arrays of complex networks has attracted increasing research attention in various research fields in recent years. Both continuous-time synchronization [11–16] and discrete-time synchronization [5,17,18] protocols have been extensively studied in the previous literature. For continuous time system, in [11], Cao et al. considered synchronization problem for an array of delayed neural networks with hybrid coupling, in [14], Lu et al. studied the exponential synchronization of linearly coupled neural networks with impulsive disturbances, in [15], the chaos synchronization is discussed for complex dynamical networks. The global synchronization control of general delayed discrete-time networks with stochastic coupling and disturbances was investigated in [17]. In [5], the synchronization criteria are derived for linearly coupled networks of discrete time systems. The global synchronization for discrete-time stochastic complex networks with randomly occurred nonlinearities and mixed timedelays was studied in [18]. Hence, most of the previous investigations dealt with synchronization problems in continuous-time and discrete-time cases, respectively. Obviously, it is not consistent with some real networked systems. In real-world systems, the interaction among agents can happen at any time, maybe some continuous time intervals accompanying some discrete moments. So it is necessary and meaningful to consider both continuous-time and discrete-time cases at the same time in networked systems. In this paper, we will combine continuous-time and discrete-time cases together and design the consensus/synchronization protocols under a unified framework. To overcome the aforementioned shortcomings, we will study the globally exponential synchronization problems of complex

* Corresponding author at: Department of Mathematics, Southeast University, Nanjing 210096, China.
E-mail addresses: chengqx168@163.com (Q. Cheng), jdcao@seu.edu.cn (J. Cao).

http://dx.doi.org/10.1016/j.neucom.2014.10.033
0925-2312/© 2014 Elsevier B.V. All rights reserved.

networks by using the theory and method of time scales calculus in this paper. And the results of the synchronization criteria are desirable.

The theory of time scales has a tremendous potential for applications in some mathematical models of real processes and phenomena studied in physics, population dynamics, biotechnology, economics and so on. Empirical results show that the theory of time scales is not only a pure theoretical field of mathematics but also a useful tool to deal with many practical problems. The field of dynamic equations on time scales contains links and extends the classical theory of differential and difference equations. Recently, the theory of time scale calculus has been applied in neural networks. In [19], Chen and Du studied the global exponential stability of delayed BAM networks on time scale. In [20] the consensus criteria are derived for a class of multi-agent systems on time scales. Motivated by the above-mentioned results, as an attempt, we will study globally exponential synchronization problems of complex dynamical networks on time scales in this paper.

2. Notations and preliminaries

In order to obtain the main results, some elementary notions and lemmas in the theory of time scales are presented as follows.

In 1980s, Stefan Hilger initiated the theory of time scale calculus. Bohner and Peterson developed and consummated it [21–23]. This novel and fascinating type of mathematics is more general and versatile than the traditional theories of differential and difference equations as it can, under one framework, mathematically describe continuous and discrete hybrid processes and hence is the optimal way forward for accurate and malleable mathematical modeling.

Throughout this paper, $\mathbb{N}$ and $\mathbb{Z}$ denote the positive integer collection and integer collection, respectively. $\mathbb{R}^n$ and $\mathbb{R}^{n\times m}$ denote the n-dimensional Euclidean space and the set of all $n\times m$ real matrices, respectively. $\mathbb{T}$ is a time scale. Set $[a,b]_{\mathbb{T}}:=\{t\in\mathbb{T}, a\le t\le b\}$. $\mathbb{T}^+=\{t\in\mathbb{T}, t\ge 0\}$. $P>0$ means that matrix P is real, symmetric and positive definite. I and O denote the identity matrix and the zero matrix with compatible dimensions, respectively; and diag $\{\cdots\}$ stands for a block-diagonal matrix. The superscript 'T' stands for a matrix transposition. The Kronecker product of matrices $Q\in\mathbb{R}^{m\times n}$ and $R\in\mathbb{R}^{p\times q}$ is a matrix in $\mathbb{R}^{mp\times nq}$ and denoted as $Q\otimes R$. Let $\tau>0$ and $C([-\tau,0]_{\mathbb{T}};\mathbb{R}^n)$ denote the family of continuous functions φ from $[-\tau,0]_{\mathbb{T}}$ to $\mathbb{R}^n$ with the norm $\|\varphi\|=\sup_{-\tau\le\theta\le 0}\|\varphi(\theta)\|$, where $\|\cdot\|$ is the Euclidean norm in $\mathbb{R}^n$.

A time scale $\mathbb{T}$ is an arbitrary nonempty closed subset of the real set $\mathbb{R}$ with the topology and ordering inherited from $\mathbb{R}$. Assume that $0\in\mathbb{T}$, $\mathbb{T}$ is unbounded above, that is, sup $\mathbb{T}=\infty$. The forward and backward jump operators $\sigma,\rho:\mathbb{T}\to\mathbb{T}$ and the graininess $\mu:\mathbb{T}\to\mathbb{R}^+$ are defined, respectively, by $\sigma(t):=\inf\{s\in\mathbb{T}: s>t\}$; $\rho(t):=\sup\{s\in\mathbb{T}: s<t\}$; $\mu(t):=\sigma(t)-t$.

We put $\inf\varnothing:=\sup\mathbb{T}$ and $\sup\varnothing:=\inf\mathbb{T}$, where $\varnothing$ denotes the empty set.

A point t is said to be left-dense if $t>\inf\mathbb{T}$ and $\rho(t)=t$, right-dense if $t<\sup\mathbb{T}$ and $\sigma(t)=t$, left-scattered if $\rho(t)<t$ and right-scattered if $\sigma(t)>t$. If $\mathbb{T}$ has a left-scattered maximum m, then we define $\mathbb{T}^k$ to be $\mathbb{T}-\{m\}$. Otherwise $\mathbb{T}^k=\mathbb{T}$.

A function $f:\mathbb{T}\to\mathbb{R}$ is called right-dense continuous provided it is continuous at right-dense point of $\mathbb{T}$ and the left side limit exists (finite) at left-dense point of $\mathbb{T}$. The set of all right-dense continuous functions on $\mathbb{T}$ is defined by $C_{rd}=C_{rd}(\mathbb{T})=C_{rd}(\mathbb{T},\mathbb{R})$.

A function $f:\mathbb{T}\to\mathbb{R}$ is called regressive provided

$$1+\mu(t)f(t)\neq 0,\quad \forall\ t\in\mathbb{T}.$$

Definition 2.1 (*Bohner and Peterson [21]*). For a function $f:\mathbb{T}\to\mathbb{R}, t\in\mathbb{T}^k$, the delta derivative of $f(t)$, $f^{\Delta}(t)$, is the number (if it exists) with the property that, for a given $\varepsilon>0$, there exists a neighborhood U of t such that

$$|[f(\sigma(t))-f(s)]-f^{\Delta}(t)[\sigma(t)-s]|<\varepsilon|\sigma(t)-s|,$$

for all $s\in U$.

For all $t\in\mathbb{T}^k$, one can get

$$f(\sigma(t))=f(t)+\mu(t)f^{\Delta}(t).$$

If f and g are two differentiable functions, then the product rule for the derivative of product $f\cdot g$ is that

$$(f\cdot g)^{\Delta}=f^{\Delta}\cdot g+f^{\sigma}\cdot g^{\Delta}=f^{\Delta}\cdot g^{\sigma}+f\cdot g^{\Delta}.$$

Definition 2.2. A function $F:\mathbb{T}^k\to\mathbb{R}$ is called a delta-antiderivative of $f:\mathbb{T}\to\mathbb{R}$ provided $F^{\Delta}=f$ holds for all $t\in\mathbb{T}^k$. In this case, the integral of f is defined by

$$\int_a^t f(s)\,\Delta s=F(t)-F(a)\quad \text{for } t\in\mathbb{T}.$$

Then we have

$$\left(\int_a^t f(s)\,\Delta s\right)^{\Delta}=f(t)\quad \text{for } t\in\mathbb{T}^k.$$

Let $A=(a_{ij})_{1\le i\le m, 1\le j\le n}$ be an $m\times n$-matrix-valued function on $\mathbb{T}$. We say that A is differentiable on $\mathbb{T}$ provided each entry of A is differentiable on $\mathbb{T}$. In this case, we put

$$A^{\Delta}=(a_{ij}^{\Delta})_{1\le i\le m, 1\le j\le n}.$$

Similarly, we denote that $A^{\sigma}=(a_{ij}^{\sigma})$.

Lemma 2.1 (*Bohner and Peterson [21]*). *Suppose Φ and Ψ are differentiable $n\times n$-matrix-valued functions. Then*

(1) $(\Phi+\Psi)^{\Delta}=\Phi^{\Delta}+\Psi^{\Delta}$;
(2) $(\alpha\Phi)^{\Delta}=\alpha\Phi^{\Delta}$ *if α is a constant*;
(3) $(\Phi\Psi)^{\Delta}=\Phi^{\Delta}\Psi^{\sigma}+\Phi\Psi^{\Delta}$.

The addition '$\oplus$' is defined by $p\oplus q:=p+q+\mu pq$. The set of all regressive functions on a time scale $\mathbb{T}$ forms an Abelian group under the addition '$\oplus$'. The additive inverse in this group is denoted by $\ominus p:=-p/(1+\mu p)$. Then the subtraction $\ominus$ on the set of regressive functions is defined by $p\ominus q:=p\oplus(\ominus p)$. It can be shown easily that $p\ominus q=-(p-q)/(1+\mu q)$. The set of all regressive and right-dense continuous functions will be denoted by $\mathcal{R}=\mathcal{R}(\mathbb{T})=\mathcal{R}(\mathbb{T},\mathbb{R})$.

We denote that $\mathcal{R}^+=\mathcal{R}^+(\mathbb{T},\mathbb{R})=\{f\in\mathcal{R}: 1+\mu(t)f(t)>0$, for all $t\in\mathbb{T}\}$. Obviously, $\mathcal{R}^+$ is the set of all positively regressive elements of $\mathcal{R}$. One can easily verify that if $f\in\mathcal{R}^+$, then $\ominus f\in\mathcal{R}^+$.

Definition 2.3. If $p\in\mathcal{R}$, then the generalized exponential function $e_p(t,s)$ is defined by

$$e_p(t,s)=\exp\left\{\int_s^t \xi_{\mu(\tau)}(p(\tau))\,\Delta\tau\right\}\quad \text{for } s,t\in\mathbb{T}$$

with the cylinder transformation

$$\xi_h(z)=\begin{cases}\dfrac{Log(1+hz)}{h} & \text{if } h\neq 0,\\ z & \text{if } h=0.\end{cases}$$

If $p\in\mathcal{R}$, then the exponential function $e_p(t,t_0)$ is the only solution of the initial value problem

$$y^{\Delta}=py,\quad y(t_0)=1,$$

on a time scale $\mathbb{T}$.

Download English Version:

https://daneshyari.com/en/article/412055

Download Persian Version:

https://daneshyari.com/article/412055

Daneshyari.com