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Consensus of discrete-time linear multi-agent systems with Markov switching topologies and time-delay ☆

Yongquan Pei, Jitao Sun *

Department of Mathematics, Tongji University, Shanghai 200092, China

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ABSTRACT

This paper investigates the average consensus problems of the discrete-time linear multi-agent systems (LMAS) with Markov switching topologies. The average consensus protocol is a time-delay feedback switching controller. Compared with existing controllers, it is switching with time-delay. The constant time-delay exists in the signal feedback, and the time-varying time-delay exists in the state feedback. Firstly, we introduce a concept of the average consensus in this stochastic system. Then, we develop a new signal mode to simplify this challenging problem. By Lyapunov technique, two LMIs determinate theorems of average consensus are provided. Then we can find a controller to solve such problems effectively by these theorems. And the last theorem also reveals that we can determinate consensus by the maximum and minimum nonzero eigenvalues of the Laplacian matrices. Finally, a numerical example is given to illustrate the efficiency of our results.

1. Introduction

In recent years, the consensus problems of multi-agent systems have attracted intensive attention in the literature, because it has been demonstrated that the consensus problems have a variety of applications in various areas including formation control [1], synchronization [2], flocking [3] and sensor networks [4]. However, most of the consensus problems have been mainly concerned with agents which are modeled by a no-self-driven dynamics [5–7]. In other words, the dynamic of the agent is the controller. Without control, the state of the agent will be a constant vector.

Recently, research interest in multi-agent systems has been devoted to the linear multi-agent systems (LMAS), i.e., the agents are modeled by linear systems. Several researchers considered the leader-following consensus [8,9], the output regulation problem [10], the distributed containment control problem [11], the event-based consensus problem [12], LMAS with both missing measurements and parameter uncertainties [13] and selecting leader agents [14]. Most works of the consensus problems in the LMAS focused on the continuous-time dynamics [10–12,14]. Results about the discrete-time dynamics are less [9,8]. In this paper, we investigate the consensus problems of the discrete-time LMAS. Consensus problems of nonlinear multi-agent systems [15] will be further studied in the later work.

Switching phenomenon widely exists in the real world. Due to the nodes of network are moving, the communication link between two agents may disappear or reestablish. Consider to this, we assume that the communication topologies are Markov switching in this paper. Many papers also investigated the consensus problems of the multi-agent systems with switching topology [5,16]. Refs. [9,17] studied that on LMAS, but they assumed that the state matrix is stable. Different from above papers, the state matrix and the input matrix are unconstrained, and the feedback controller is Markov switching with time-delay in this paper. To the best of our knowledge, there have been few reports that solve this problem by Markov switching controller.

Time-delays are frequently encountered in practical systems such as engineering, communications and biological systems. Ref. [18] solved the consensus problems of LMAS by a time-delay feedback controller, and the time-delay is constant. But, in the feedback switching controller, time-delays may occur in the feedback of the switching signal also. For this reason, we design the controller by state and signal feedbacks with time-delay. We assume that the state feedback is time varying time-delay, and the signal feedback is constant time-delay. This is a new controller for LMAS, and the study is full of challenging.

The rest of this paper is organized as follows. Section 2 introduces some graph knowledge and property of Kronecker product. Section 2.2 presents the consensus problem of discrete-time LMAS with Markov switching topologies, and defines the

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* Corresponding author. Tel.: +86 21 65983241x1307; fax: +86 21 65981985.
E-mail addresses: yqpei0@gmail.com (Y. Pei), sunjt@sh163.net (J. Sun).

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average consensus of the stochastic systems. In Section 3, we give two sufficient conditions of consensus. By these theorems we can find a controller such that the LMAS is average consensus. Section 4 gives a numerical example to illustrate the efficiency of our results. Concluding remarks are finally stated in Section 5.

Notation: The following notation will be used throughout this paper. $\mathbf{1}$ ($\mathbf{0}$) is a compatible dimension vector with all elements to be one (zero). I_N is the $N \times N$-dimensional identity matrix, and I is the identity matrix of compatible dimensions. The notation $*$ always denotes the symmetric block in one symmetric matrix. The transpose of matrix A is denoted by A^T. The shorthand $diag\{\cdots\}$ denotes the block diagonal matrix. $\|\cdot\|$ refers to the Euclidean norm for vectors. $E(\cdot)$ stands for the mathematical expectation operator. $\otimes$ denotes the Kronecker product of matrices. Some properties of Kronecker product are useful in this paper: $(A \otimes B)^T = A^T \otimes B^T$, $(A \otimes B)(C \otimes D) = (AC) \otimes (BD)$, $A \otimes B + A \otimes C = A \otimes (B+C)$.

2. Preliminaries and problem formulations

In this section, we introduce some basic concepts in graph theory (more information is available in [19]), and the average consensus problems in stochastic systems.

2.1. Preliminary

Let $\mathcal{G} = (\mathcal{V}, \mathcal{E}, \mathcal{A})$ be a *graph* of order N, where $\mathcal{V} = \{\nu_1, \nu_2, \ldots, \nu_N\}$ is the set of nodes, $\mathcal{E} \in \mathcal{V} \times \mathcal{V}$ is the set of edges, and $\mathcal{A} = (a_{ij})_{N \times N}$ is the *weighted adjacency matrix*. The node indexes belong to a finite index set $\mathcal{I} = \{1, 2, \ldots, N\}$. $(i,j) \in \mathcal{E}$ denotes there is a edge connect ν_i and ν_j, and ν_i can receive information from ν_j. In the following, it is stipulated that $(i,j) \in \mathcal{E}$ if and only if $a_{ij} > 0$ and $a_{ii} = 0$ for $i \in \mathcal{I}$. $\mathcal{G}$ is called an *undirected graph*, if $a_{ij} = a_{ji}$ for all $i,j \in \mathcal{I}$. If there exists a sequence of edges $(i,i_1),(i_1,i_2),\ldots,(i_k,j) \in \mathcal{E}$ for any two agents $i,j \in \mathcal{I}$, $\mathcal{G}$ is called a *connected graph*.

The matrix $\mathcal{L} = (l_{ij})_{N \times N}$ is the *Laplacian matrix* of $\mathcal{G}$, where

$$l_{ij} = \begin{cases} -a_{ij} & i \neq j \\ \sum_{k=1}^{N} a_{ik} & i = j. \end{cases}$$

Lemma 1 (*Godsil et al.* [19]). *Let $\mathcal{G} = (\mathcal{V}, \mathcal{E}, \mathcal{A})$ be a weighted undirected graph with Laplacian $\mathcal{L}$, and $\lambda_1 \le \cdots \le \lambda_N$ be the eigenvalues of $\mathcal{L}$. If $\mathcal{G}$ is connected, $0 = \lambda_1 < \lambda_2 \le \cdots \le \lambda_N$.*

In a multi-agent network with N agents, the information flow between agents can be described by a graph $\mathcal{G} = (\mathcal{V}, \mathcal{E}, \mathcal{A})$. The node ν_i in graph $\mathcal{G}$ corresponds to agent i in the networks. $(i,j) \in \mathcal{E}$ expresses that the information of the agent j can be spread to agent i.

2.2. Consensus problems on Markov switching graphs

This paper considers the consensus problems of discrete-time LMAS with Markov switching topologies. We assume that the set of nodes are invariant, and the edges may disappear or reestablish with the switching of the topologies. Then we denote the switching topologies as $\mathcal{G}(r(k)) = (\mathcal{V}, \mathcal{E}(r(k)), \mathcal{A}(r(k)))$, where $\{r(k), k \in \mathbb{Z}_+\}$ is a discrete-time Markov chain, with finite state space $\Upsilon = \{0, 1, \ldots, d-1\}$. The state transition matrices of $\{r(k)\}$ are $P = (p_{ij})$, where $p_{ij} = \Pr\{r(k+1) = j | r(k) = i\} \ge 0$, for $i,j \in \Upsilon$, denote the transition probability from i to j. In this paper, all systems are defined on a complete probability space (Ω, F, P). For all $\upsilon \in \Upsilon$, the Laplacian matrix of $\mathcal{G}(\upsilon)$ is denoted by $\mathcal{L}(\upsilon)$.

Assumption 1. For all $\nu \in \Upsilon$, $\mathcal{G}(\upsilon)$ is undirected and connected. Then the eigenvalues of $\mathcal{L}(\upsilon)$ can be denoted by $0 = \lambda_1(\upsilon) < \lambda_2(\upsilon) \le \cdots \le \lambda_N(\upsilon)$.

The dynamics of all agents are discrete-time linear systems:

$$x_i(k+1) = Ax_i(k) + Bu_i(k), \quad i \in \mathcal{I}, \tag{1}$$

where $x_i(k) \in \mathbb{R}^n$ and $u_i(k) \in \mathbb{R}^m$ represent the state and the control input of agent i respectively. $A \in \mathbb{R}^{n \times n}$ and $B \in \mathbb{R}^{n \times m}$ are the state and the input matrix respectively. And the average consensus protocol is designed as

$$u_i(k) = K(r(k-\tau)) \sum_{j=1}^{N} a_{ij}(r(k))(x_j(k-\sigma(k)) - x_i(k-\sigma(k))), \quad i \in \mathcal{I}, \tag{2}$$

where $K(\upsilon) \in \mathbb{R}^{m \times n}$, for all $\upsilon \in \Upsilon$, is the gain matrix to be designed. $\tau \in \mathbb{N}$ is a constant delay occurring in the mode signal $r(k)$. $\sigma(k) \in \mathbb{N}$ is the time-varying delay of the state feedback, and satisfies $\sigma_{min} \le \sigma(k) \le \sigma_{max}$, where $\sigma_{min}, \sigma_{max} \in \mathbb{N}$ are constants. Denote $\sigma_h = \sigma_{max} - \sigma_{min}$.

Let $x(k) = [x_1^T(k), \ldots, x_N^T(k)]^T$. Then systems (1) under the protocol (2) can be written as

$$x(k+1) = I_N \otimes Ax(k) - \mathcal{L}(r(k)) \otimes BK(r(k-\tau))x(k-\sigma(k)). \tag{3}$$

We define the *center* of $x(k)$ as $\overline{x}(k) \triangleq (1/N)\sum_{i=1}^{N} x_i(k)$. Since $\mathbf{1}^T \mathcal{L}(\upsilon) = \mathbf{0}^T$, for all $\upsilon \in \Upsilon$, the following property of the center in the systems (3) is in force:

$$\overline{x}(k+1) = \frac{1}{N}(\mathbf{1}^T \otimes I_n)x(k+1) = A\overline{x}(k). \tag{4}$$

In this paper, the main problem of interest is to get the determinate conditions of the average consensus in the stochastic multi-agent systems (3). Inspired by [20], the definition of average consensus is given as follows.

Definition 1 (*Mean Square Average Consensus, MSAC*). Multi-agent systems (1) under the protocol (2) are said to reach a MSAC, if

$$\lim_{k \to \infty} E\{\|x_i(k) - \overline{x}(k)\|^2 | x_0, r_0\} = 0, \quad i \in \mathcal{I},$$

for any initial conditions $x_0 = \{x(-\sigma_{max}), x(-\sigma_{max}+1), \ldots, x(0)\}$ and $r_0 = \{r(-\tau), x(-\tau+1), \ldots, x(0)\}$.

3. The main results

Inspired by [21,22], we extend the state space of switching signal in the beginning.

Let $\Upsilon^{\tau+1} = \underbrace{\Upsilon \times \Upsilon \times \cdots \times \Upsilon}_{\tau+1}$, and define a *vector switching signal* $s(k) = [r(k), r(k-1), \ldots, r(k-\tau)]^T$ with finite state space $\Upsilon^{\tau+1}$. The transition probability from ν to η denotes as $\tilde{p}_{\nu\eta}$, for any $\nu, \eta \in \Upsilon^{\tau+1}$. For any $\nu = [\nu_0, \nu_{-1}, \ldots, \nu_{-\tau}]^T$, $\eta = [\eta_0, \eta_{-1}, \ldots, \eta_{-\tau}]^T \in \Upsilon^{\tau+1}$, we can get $\tilde{p}_{\nu\eta} = p_{\nu_0\eta_0}$, if $[\nu_0, \ldots, \nu_{-\tau+1}]^T = [\eta_{-1}, \ldots, \eta_{-\tau}]^T$, otherwise $\tilde{p}_{\nu\eta} = 0$.

Furthermore, let $\tilde{\mathcal{L}}(s(k)) \triangleq \mathcal{L}(r(k))$ and $\tilde{K}(s(k)) \triangleq K(r(k-\tau))$. Then system (3) can be written as

$$x(k+1) = I_N \otimes Ax(k) - \tilde{\mathcal{L}}(s(k)) \otimes B\tilde{K}(s(k))x(k-\sigma(k)). \tag{5}$$

Lemma 2. *The system (5) reaches a MSAC if and only if the system (6) satisfies $\lim_{k \to \infty} E\{\|y(k)\|^2 | x_0, r_0\} = 0$ for any initial conditions x_0, r_0, where $\overline{\mathcal{L}}(\nu)$ is a symmetric matrix with the eigenvalues $\lambda_2(\nu_0), \ldots, \lambda_N(\nu_0)$, for all $\nu = [\nu_0, \nu_{-1}, \ldots, \nu_{-\tau}]^T \in \Upsilon^{\tau+1}$.*

$$y(k+1) = I_{N-1} \otimes Ay(k) - \overline{\mathcal{L}}(s(k)) \otimes B\tilde{K}(s(k))y(k-\sigma(k)) \tag{6}$$

Proof. We define $\delta_i(k) \triangleq x_i(k) - \overline{x}(k)$, $i \in \mathcal{I}$, and $\delta(k) = [\delta_1^T(k), \ldots, \delta_N^T(k)]^T$. So the multi-agent systems (5) reach a MSAC, if and