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Mean-square exponential stability for stochastic discrete-time recurrent neural networks with mixed time delays ☆

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ABSTRACT

In this paper, the mean-square exponential stability problem for discrete-time recurrent neural networks with time-varying discrete and distributed delays is investigated. Considering the delay distributions, a novel class of Lyapunov functional is introduced. By exploiting all possible information in mixed time delays, a sufficient condition for the whole system to be mean-square exponentially stable is given. Numerical examples are proposed to illustrate the effectiveness of the method, and show that by using the approach in this paper, the obtained results are less conservative than the existing ones.

1. Introduction

Within the past few decades, recurrent neural networks have received considerable attraction due to their wide potential applications in some areas, such as signal processing, pattern recognition, associative memories and optimization [1–3]. It is well known that stability is one of the preconditions for some optimization problems. With the development of computer technology, discrete-time recurrent neural networks are much more suitable to our digital life than continuous-time recurrent neural networks [4,5]. Besides, time-delays are inevitable in most recurrent neural networks, and they are the potential sources of instability and oscillation [6,7]. Therefore, the stability analysis of discrete-time delayed recurrent neural networks is an important issue, the corresponding studies can be found in [11,12] and the references therein. In [11], the discrete recurrent neural network system was modeled as a kind of stochastic system with time-varying delays and stochastic disturbance, then, a stability criterion was given via Lyapunov functional. In [12], without the traditional assumptions on the boundedness, monotony and differentiability of the activation functions, the free-weight matrix technology was used to get the stability condition of discrete-time recurrent neural networks, furthermore, the globally exponential stability criterion of the whole system was given.

On the other hand, another type of time-delay has attracted considerable interest, namely, distributed delay, that is because neural network usually has a spatial nature and there presence of an amount of parallel pathways with a variety of axon sizes and lengths [9]. Recently, several interesting research results for neural networks with mixed time delays have obtained, in [17], the stability analysis for a kind of neutral systems with sector-bounded nonlinearity and mixed time delays was given. Besides, the research on stochastic neural networks with Markovian jump parameters and mixed time delays was introduced in [18]. For recurrent neural network with multiple discrete delays and distributed delays, sufficient conditions for checking the global asymptotical stability of such systems were given via LMI in [19]. Best to the author's knowledge, the mean-square exponential stability analysis for recurrent neural networks with mixed time-delays has received little attention in the literature.

Motivated by the delay-distribution-dependent technology [20], the mean-square exponential stability criterion of discrete-time recurrent neural networks with discrete and distributed delays is investigated in this paper. Firstly, considering the delay distribution probability, the recurrent neural network is modeled as a kind of stochastic system and the stochastic variable is assumed to satisfy Bernoulli process. Then, a less conservative result is obtained via a novel class of Lyapunov functional, and the mean-square exponential

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stability criterion is given. Lastly, numerical examples are provided to show the effectiveness of proposed result.

Notation: Throughout the paper, $\mathbb{R}^n$ and $\mathbb{R}^{n\times m}$ denote the n- dimensional Euclidean space and the set of all $n\times m$ real matrices, respectively. The superscript "T" denotes the matrix transposition. The notation $X\geq Y$(or $X>Y$) means that X and Y are symmetric matrices and $X-Y$ is positive semidefinite (or positive definite). $\|\cdot\|$ is the Euclidean norm in $\mathbb{R}^n$.

2. Model formulation and preliminaries

Consider the following discrete-time recurrent neural network with time-varying delays:

$$x(k+1)=Cx(k)+AF(x(k))+BG(x(k-\tau(k)))+D\sum_{m=1}^{+\infty}\mu_m\alpha(x(k-m))+J \tag{1}$$

where $x(k)=[x_1(k),x_2(k),\ldots,x_n(k)]\in\mathbb{R}^n$ is the neural state vector, $x_i(k)$ is the state of the ith neuron at time k; $C=diag\{c_1,c_2,\ldots,c_n\}$ is the state feedback coefficient matrix ($\|c_i\|<1$); $A\in\mathbb{R}^{n\times n}$ is the connection weight matrix, $B\in\mathbb{R}^{n\times n}$ is the discretely delayed connection weight matrix and $D\in\mathbb{R}^{n\times n}$ is the system coefficient matrix; $J=[J_1,J_2,\ldots,J_n]^T\in\mathbb{R}^n$ is the exogenous input; $F(x(k))$ and $G(x(k))$ are the neuron activation functions, $\alpha(x(k))$ is nonlinear function, and satisfy

$$\begin{aligned}F(x(k))&=[F_1(x_1(k)),F_2(x_2(k)),\ldots,F_n(x_n(k))]^T\in\mathbb{R}^n\\G(x(k))&=[G_1(x_1(k)),G_2(x_2(k)),\ldots,G_n(x_n(k))]^T\in\mathbb{R}^n\\\alpha(x(k))&=[\alpha_1(x_1(k)),\alpha_2(x_2(k)),\ldots,\alpha_n(x_n(k))]^T\in\mathbb{R}^n\end{aligned}$$

Besides, $\tau(k)$ denotes the discrete time-varying delay, and $\tau_m\leq\tau(k)\leq\tau_M$, where τ_m and τ_M are the maximum and minimum of the allowable time delay bound, respectively. μ_m is a nonnegative constant and satisfies the convergent conditions [9], as

$$\sum_{m=1}^{+\infty}\mu_m<+\infty,\qquad\sum_{m=1}^{+\infty}m\mu_m<+\infty \tag{2}$$

Throughout the paper, the following assumptions are needed:

Assumption 1. For any $x,y\in\mathbb{R}$ $(x\neq y)$, $i\in\{1,2,\ldots,n\}$, the activation functions $F(x(k))$, $G(x(k))$ and nonlinear function $\alpha(x(k))$ satisfy

$$f_i^-\leq\frac{F_i(x)-F_i(y)}{x-y}\leq f_i^+,\quad g_i^-\leq\frac{G_i(x)-G_i(y)}{x-y}\leq g_i^+,\quad\alpha_i^-\leq\frac{\alpha_i(x)-\alpha_i(y)}{x-y}\leq\alpha_i^+$$

where f_i^-, f_i^+, g_i^-, g_i^+, α_i^- and α_i^+ are constants.

Under Assumption 1, [8] had got the results that the system has equilibrium point, define the equilibrium point as x^* and denote $y(k)=x(k)-x^*$, system (1) with stochastic disturbances can be written as

$$\begin{aligned}y(k+1)=&Cy(k)+Af(y(k))+Bg(y(k-\tau(k)))+D\sum_{m=1}^{+\infty}\mu_m\beta(y(k-m))\\&+\sigma(k,y(k),y(k-\tau(k)))\omega(k)\end{aligned} \tag{3}$$

where $y(k)=[y_1(k),y_2(k),y_3(k),\ldots,y_n(k)]^T$, $f(y(k))=F(x(k))-F(x^*)$, $g(y(k))=G(x(k))-G(x^*)$, $\beta(y(k))=\alpha(x(k))-\alpha(x^*)$. From Assumption 1, it is not difficult to conclude that

$$f_i^-\leq\frac{f_i(x)-f_i(y)}{x-y}\leq f_i^+,\quad g_i^-\leq\frac{g_i(x)-g_i(y)}{x-y}\leq g_i^+,\quad\alpha_i^-\leq\frac{\beta_i(x)-\beta_i(y)}{x-y}\leq\alpha_i^+$$

$\omega(k)$ is a scalar Wiener process on a probability space (Ω,F,P) with

$$\mathbb{E}\{\omega(k)\}=0,\quad\mathbb{E}\{\omega^2(k)\}=1,\quad\mathbb{E}\{\omega(i)\omega(j)\}=0(i\neq j)$$

Assumption 2. The noise intensity function vector $\sigma(\cdot,\cdot,\cdot):\mathbb{N}\times\mathbb{N}^n\times\mathbb{N}^n\rightarrow\mathbb{N}$ satisfies the Lipschitz condition and there exist constant δ_1, δ_2 such that the following inequality holds:

$$\sigma^T(k,x,y)\sigma(k,x,y)\leq\delta_1x^Tx+\delta_2y^Ty$$

Assumption 3. In real-time system, there exists a scalar τ_1 satisfying $\tau_m<\tau_1<\tau_M$, and the time-delay interval $[\tau_m,\tau_M]$ can be divided into two subintervals $[\tau_m,\tau_1)$ and $[\tau_1,\tau_M]$.

Then, a discrete stochastic variable $\theta(k)$ is defined as $\theta(k):\mathbb{N}\rightarrow\{0,1\}$, if $\theta(k)=1$, time delay $\tau(k)$ satisfies $\tau_m\leq\tau(k)<\tau_1$, whereas $\theta(k)=0$, $\tau_1\leq\tau(k)\leq\tau_M$. Denote $\tau_1(k)=\theta(k)\tau(k)$, $\tau_2(k)=(1-\theta(k))\tau(k)$, system (3) can be rewritten as

$$\begin{aligned}y(k+1)=&Cy(k)+Af(y(k))+\theta(k)Bg(y(k-\tau_1(k)))+(1-\theta(k))Bg(y(k-\tau_2(k)))\\&+D\sum_{m=1}^{+\infty}\mu_m\beta(y(k-m))+\theta(k)\sigma(k,y(k),y(k-\tau_1(k)))\\&\times\omega(k)+(1-\theta(k))\sigma(k,y(k),y(k-\tau_2(k)))\omega(k)\\=&\hat{y}(k)+\theta(k)\sigma(k,y(k),y(k-\tau_1(k)))\omega(k)+(1-\theta(k))\\&\times\sigma(k,y(k),y(k-\tau_2(k)))\omega(k)\end{aligned} \tag{4}$$

Denote $Prob\{\theta(k)=1\}=\mathbb{E}\{\theta(k)\}=\tilde{\theta}$, $Prob\{\theta(k)=0\}=1-\tilde{\theta}$, $\mathbb{E}\{(\theta(k)\tilde{\theta})^2\}=\tilde{\theta}(1-\tilde{\theta})$, and

$$\mathbb{E}\{\theta_i(k)\theta_j(k)\}=\begin{cases}\tilde{\theta}, & i=j\\0, & i\neq j\end{cases}$$

Thus, the system (4) is a kind of stochastic system and we need the following definition to investigate its stability:

Definition 1. The discrete-time stochastic neural network is said to be exponentially stable in mean square if there exist two positive constant $\delta>0$ and $0<\varepsilon<1$ such that

$$\mathbb{E}\{\|y(k)\|^2\}\leq\delta\varepsilon^k\|\phi(s)\|^2$$

where $\phi(s)$ is the initial function of $y(k)$, $s\in\mathbb{N}[-\tau_M,0]$.

Lemma 1 (*Zhu and Yang [10], Hou et al. [11]*). *For any constant matrix $M\in\mathbb{R}^{n\times n}$, $M=M^T>0$, integers $\gamma_2\geq\gamma_1$, vector function $\omega:[\gamma_1,\gamma_1+1,\ldots,\gamma_2]\rightarrow\mathbb{R}^{n\times n}$ such that the sums in the following are well defined, thus*

$$-(\gamma_2-\gamma_1+1)\sum_{i=\gamma_1}^{\gamma_2}\omega^T(i)M\omega(i)\leq-\left(\sum_{i=\gamma_1}^{\gamma_2}\omega(i)\right)^TM\left(\sum_{i=\gamma_1}^{\gamma_2}\omega(i)\right)$$

3. Main results

In this section, a sufficient condition is given to ensure the system (4) to be exponential stability in mean square.

Theorem 1. *The system (4) is said to be mean-square exponentially stable if there exist matrices $P>0$, $Q_i>0$ $(i=1,2,3,4)$, $Z>0$, $Z_1>0$, $U>0$, $H>0$, $W>0$, $R>0$, $S_1>0$, $S_2>0$ and scalars $\lambda_1>0$, $\lambda_2>0$ such that the following LMIs hold:*

$$\begin{aligned}&\tilde{\theta}(P+\tau_1^2Z+(\tau_1-\tau_m)^2Z_1)<\lambda_1I\\&(1-\tilde{\theta})(P+\tau_1^2Z+(\tau_M-\tau_1)^2Z_1)<\lambda_2I\\&\begin{bmatrix}\Pi & \Xi_{1,1}^TP & \tau_1\Xi_{1,2}^TZ & (\tau_M-\tau_1)\Xi_{1,3}^TZ_1 & (\tau_1-\tau_m)\Xi_{1,4}^TZ_1\\ * & -P & 0 & 0 & 0\\ * & * & -Z & 0 & 0\\ * & * & * & -Z_1 & 0\\ * & * & * & * & -Z_1\end{bmatrix}<0\end{aligned} \tag{5}$$