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Robust stability analysis for discrete-time uncertain neural networks with leakage time-varying delay

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ABSTRACT

This paper is concerned with the stability problem for a class of discrete-time neural networks with time-varying delays in network coupling, parameter uncertainties and time-delay in the leakage term. By constructing triple Lyapunov–Krasovskii functional terms, based on Lyapunov method, new sufficient conditions are established to ensure the asymptotic stability of discrete-time delayed neural networks system. Convex reciprocal technique is incorporated to deal with double summation terms and the stability criteria are presented in terms of linear matrix inequalities (LMIs). Finally numerical examples are exploited to substantiate the theoretical results. It has also shown that the derived conditions are less conservative than the existing results in the literature.

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1. Introduction

Neural networks have become an important area of research in many areas including pattern recognition, associative memory, combinatorial optimization, fixed-point computation and signal processing [7,10]. Dynamical behaviors such as stability, instability, periodic oscillatory and chaos of the neural networks are known to be crucial in applications. Stability of neural networks is a prerequisite for many engineering problems, it received much research attention in recent years and many elegant results have been reported, for details see [14,24,27,36,38]. It is worth noticing that, when implementing the continuous-time recurrent neural networks for computer simulation, for experimental or computational purposes, it is essential to formulate a discrete-time system that is an analogue of the continuous-time recurrent neural networks. Merely, the discretization cannot always preserve the dynamics of the continuous-time counterpart even for a small sampling period [29]. Therefore, there is a crucial need to study the dynamics of the discrete-time neural networks.

Since time-delay inevitably occurs in the communication and response of neurons owing to the unavoidable finite switching speed of amplifiers in the electronic implementation of analog neural networks, it is more significant to study neural networks with time-delay, see [43,44,46], and references therein. It is well known that time-delay often causes undesirable dynamic behaviors such as performance degradation, and instability of the systems.

The stability analysis problem for neural networks with time-delay has attracted much attention and many sufficient conditions have been proposed to guarantee the asymptotic and exponential stability of neural networks with various types of time-delay such as constant, time-varying, random and distributed delays, see for example [3–5,12,13,15,16,21,34]. In [20], new improved delay-dependent stability criteria guaranteeing the global exponential stability have been obtained via a new augmented Lyapunov–Krasovskii functional (LKF). Stability analysis problem for a new class of discrete-time neural networks with randomly discrete and distributed time-varying delays has investigated in [37]. The state estimation problem for a class of discrete-time stochastic neural networks with random delays has been studied in [2]. In [25], synchronization problem has been considered for an array of linearly coupled neural networks with simultaneous presence of both the discrete and unbounded distributed time-delays. Recently, synchronization criteria for discrete-time coupled networks have discussed in [32] and a delay-dependent stability condition has presented in [40] by using the triple Lyapunov functional technique. Impulsive perturbations can also cause undesirable dynamical behaviors leading to poor performance. Moreover, impulsive neural networks model belongs to a new category of dynamical systems, which are neither purely continuous-time nor purely discrete-time ones, in recent years considerable attention has been paid to investigating the stability analysis of impulsive neural networks, see [43,45]. The problem of global exponential stability and exponential convergence

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rate for a class of impulsive discrete-time neural networks with time-varying delays has been studied in [42]. Also, a large number of interesting results have been reported on the stability of Markovian jump neural networks with time delays and impulsive perturbations in [33,43,44,45,47].

Neural networks with leakage delay are a class of one type of important neural networks. Time-delay in the leakage term has a great impact on the dynamics of neural networks since time-delay in the stabilizing negative feedback term has a tendency to destabilize a system, see [8,9,18,19]. Gopalsamy [8] initially investigated the dynamics of bidirectional associative memory (BAM) network model with leakage delay. Based on this work, authors in [19] considered the global stability for a class of nonlinear systems with leakage delay via LKF and LMI techniques. Authors in [18] extensively studied recurrent NNs with time-delay in the leakage term and their results dealt about the existence and uniqueness of the equilibrium point which is independent of time-delays and initial conditions. So, the time-delay in the leakage term does not affect the existence and uniqueness of the equilibrium point. The impulsive effects on existence–uniqueness and stability problems of neural networks with leakage delay have been studied in [17,18] through some analysis techniques on impulsive functional differential equations. More recently in [28], the passivity analysis has been addressed for neural networks of neutral type with Markovian jumping parameters and time-delay in the leakage term. Several results have been extensively considered the leakage delay in continuous-time neural networks, see [6,21,22,33]. Recently, authors in [31] have investigated delay-dependent robust synchronization analysis for coupled stochastic discrete-time neural networks with interval time-varying delays in networks coupling and a time-delay in leakage term with parameter uncertainties. Exponential stability for a class of discrete-time recurrent neural networks model with leakage delay and linear fractional uncertainties has discussed in [11].

Since leakage delay has more impact on dynamics of neural networks than other kinds of delay, it is of great importance to consider the leakage delay effects on dynamics of neural networks. Moreover, in the existing results, the leakage delay in the leakage term is usually a constant. In practice, the leakage delay is not a constant, so we ought to consider the neural networks with time-varying leakage delay. It is worth noting that in most of the above said references only continuous-time neural networks with leakage delay have been studied. However, it appears that very little attention is devoted to the investigation of stability for discrete-time neural networks with time-varying leakage delay. This motivates our study.

Based on the discussions, the aim of this paper is to study the asymptotic stability for a class of discrete-time dynamical networks with time-varying leakage delay and norm bounded parameter uncertainties. Choosing triple LKFs and utilizing some most updated techniques for achieving the refined delay-dependence, novel conditions are established in terms of LMIs. The feasibility of derived criteria can be easily checked by resorting to Matlab LMI Toolbox. Finally, numerical examples are included to show the effect of the leakage delay in the behavior of dynamical system.

This paper is organized as follows. Problem formulation and preliminaries are given in Section 2. Section 3 gives the main results of this paper. Numerical examples are demonstrated in Section 4 to illustrate the effectiveness of the proposed method. Finally, conclusions are drawn in Section 5.

Notations: Throughout this paper, $\mathcal{R}^n$ and $\mathcal{R}^{n\times n}$ denote the n-dimensional Euclidean space and the set of all $n\times n$ real matrices, respectively. The superscript T and (-1) denote the matrix transposition and matrix inverse, respectively. Matrices, if they are not explicitly stated, are assumed to have compatible dimensions. $\|\cdot\|$ is the Euclidean norm in $\mathcal{R}^n$. I is an identity matrix with appropriate dimension. The notation $*$ always denotes the symmetric block in one symmetric matrix.

2. Problem description and preliminaries

Consider the following discrete-time delayed neural networks system with time-varying leakage delay as

$$y(k+1)=Ay(k-\sigma(k))+B\hat{f}(y(k))+C\hat{g}(y(k-\tau(k)))+J, \tag{1}$$

where $y(\cdot)=[y_1(\cdot),\dots,y_n(\cdot)]^T\in\mathbb{R}^n$ is the state vector; $\hat{f}(\cdot)=[\hat{f}_1(\cdot),\dots,\hat{f}_n(\cdot)]^T\in\mathbb{R}^n$ and $\hat{g}(\cdot)=[\hat{g}_1(\cdot),\dots,\hat{g}_n(\cdot)]^T\in\mathbb{R}^n$ denote the activation functions; $J=[J_1,\dots,J_n]^T\in\mathbb{R}^n$ means a constant external input vector; $\sigma(k)$ represents the leakage delay satisfying $0<\sigma_m\le\sigma(k)\le\sigma_M$, where $\sigma_m,\ \sigma_M$ are known positive integers representing the lower and upper bounds of $\sigma(k)$; $\tau(k)$ describes the transmission delay satisfying $0<\tau_m\le\tau(k)\le\tau_M$, where $\tau_m,\ \tau_M$ are known positive integers representing the lower and upper bounds of $\tau(k)$.

Assumption 1 (*Liu et al. [27], Wang et al. [39]*). For any $s_1,\ s_2\in\mathbb{R}$, $s_1\ne s_2$, the continuous and bounded activation functions $\hat{f}_i(\cdot)$ and $\hat{g}_i(\cdot)$ satisfy

$$F_i^-\le\frac{\hat{f}_i(s_1)-\hat{f}_i(s_2)}{s_1-s_2}\le F_i^+,$$

$$G_i^-\le\frac{\hat{g}_i(s_1)-\hat{g}_i(s_2)}{s_1-s_2}\le G_i^+,\quad i=1,2,\dots,n,$$

where $F_i^-,\ F_i^+,\ G_i^-$, and G_i^+ are known constants.

Remark 1. In many electronic circuits, the input–output functions of amplifiers may be neither monotonically increasing nor continuously differentiable, hence non-monotonic functions can be more appropriate to describe the neuron activation functions in designing and implementing an artificial neural network. Assumption 1 was first proposed in [27,39] and has been subsequently studied in many neural network papers [1,36,40]. The constants $F_i^-,\ F_i^+,\ G_i^-$, and G_i^+ in Assumption 1 are allowed to be positive, negative, or zero. So this condition is more general than the usual sigmoid functions and Lipschitz conditions. Such a description is precise in quantifying the lower and upper bounds of the activation functions.

In order to simplify our proof, we shift the equilibrium point of (1) to the origin. Assume $y^*=[y_1^*,\ y_2^*,\dots,y_n^*]^T$ is an equilibrium point of (1) and let $x_i(k)=y_i(k)-y_i^*$, $f_i(x_i(k))=\hat{f}_i(y_i(k))-\hat{f}_i(y_i^*)$, $g_i(x_i(k-\tau(k)))=\hat{g}_i(y_i(k-\tau(k)))-\hat{g}_i(y_i^*)$. Then, the neural networks system (1) can be transformed as

$$x(k+1)=Ax(k-\sigma(k))+Bf(x(k))+Cg(x(k-\tau(k))), \tag{2}$$

where $x(k)=[x_1(k),\ x_2(k),\dots,x_n(k)]^T$, $x(k-\sigma(k))=[x_1(k-\sigma(k)),\ x_2(k-\sigma(k)),\dots,x_n(k-\sigma(k))]^T$, $f(x(k))=[f(x_1(k)),\ f(x_2(k)),\dots,f(x_n(k))]^T$, $g(x(k-\sigma(k)))=\ [g(x_1(k-\sigma(k))),\ g(x_2(k-\sigma(k))),\dots,g(x_n(k-\sigma(k)))]^T$. By Assumption 1, it can be verified readily that the functions $f_i(\cdot)$, $g_i(\cdot)$, $i=1,\ 2,\dots,n$, satisfy $F_i^-\le(f_i(s_1)-f_i(s_2))/(s_1-s_2)\le F_i^+$, $G_i^-\le(g_i(s_1)-g_i(s_2))/(s_1-s_2)\le G_i^+$ for any $s_1\ne s_2$ and $f_i(0)=g_i(0)=0$.

The initial condition associated with the model is

$$x(s)=\phi(s),\quad s=-\rho,\ -\rho+1,\ -\rho+2,\dots,1. \tag{3}$$

where $\rho=\max\{\sigma_M,\ \tau_M\}$.

The following lemmas will be useful in establishing the stability results.

Lemma 1 (*Liu and Zhang [26], Park et al. [30]*). *Let $f_1,\ f_2,\dots,f_N:\mathbb{R}^m\mapsto\mathbb{R}$ have positive values in an open subset D of $\mathbb{R}^m$. Then, the reciprocally convex combination of f_i over D satisfies*

$$\min_{\{\alpha_i|\alpha_i>0,\sum_i\alpha_i=1\}}\sum_i\frac{1}{\alpha_i}f_i(k)=\sum_i f_i(k)+\max_{g_{i,j}(k)}\sum_{i\ne j}g_{i,j}(k)$$

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