Neurocomputing 151 (2015) 817–826

Contents lists available at ScienceDirect

Neurocomputing

journal homepage: www.elsevier.com/locate/neucom

Mixed noise removal by weighted low rank model

Jielin Jiang, Jian Yang*, Yan Cui, Lei Luo

School of Computer Science and Engineering, Nanjing University of Science and Technology, Nanjing 210094, China

ARTICLE INFO

Article history:
Received 13 January 2014
Received in revised form
20 August 2014
Accepted 7 October 2014
Communicated by X. Gao
Available online 28 October 2014

Keywords:
Mixed noise removal
Weighted low rank model
Low rank approximation
Low rank representation

ABSTRACT

Mixed noise removal has been a challenging task due to the complex noise distribution. One representative type of mixed noise is the additive white Gaussian noise (AWGN) coupled with impulse noise (IN). Most mixed noise removal methods first detect and restore impulse pixels using median-type filters, and then perform AWGN removal. Such mixed noise removal methods, however, are less effective in preserving image structures, and tend to over-smooth image details. In this paper, we present a novel mixed noise removal method by proposing a weighted low rank model (WLRM). By grouping image nonlocal similar patches as a matrix, we reconstruct the clean image by finding the weighted low rank approximation or representation of the matrix. IN can be well suppressed by the adaptive weight setting, while the image global structure and local edges can be well preserved via the low rank model fitting. The weight setting and low rank model fitting are jointly optimized in WLRM. Our experiments validate that WLRM leads to very promising mixed noise removal results in terms of both quantitative measure and visual perception.

1. Introduction

Noise removal is a classical and fundamental problem in image processing and low level vision, which aims to reconstruct a plausible estimate of the original image from its noisy observation. In the processes of image acquisition and transmission, noise corruption is often inevitable due to low illumination, high speed transmission rate, and so on. The prior knowledge of noise statistics is crucial for the design of noise removal algorithm. Specifically, the additive white Gaussian noise (AWGN), impulse noise (IN) and the mixture of them are the most commonly encountered noises in the literature [13–19], and they can represent a majority of noises corrupted in natural images.

AWGN is the most widely studied noise model and it characterized by adding to each image pixel a value independently sampled from a zero-mean Gaussian distribution [13]. Traditional linear filters such as mean filtering can smooth noise efficiently but will blur the edges in the meantime. In order to solve this problem, nonlinear filtering methods have been developed. The bilateral filter (BF) [1] has good capability in edge preservation. It estimates each pixel as the weighted average of the local neighbors and the weights are determined by both the intensity and spatial location. The nonlocal means (NLM) filtering method [2] can be viewed as a significant extension of BF based on the fact that similar patches may not be necessarily spatial neighbors. BM3D [3] is a well-known method and has been a benchmark in removing AWGN, by grouping the nonlocal similar patches into a 3D cube based on the norm distance function between different patches and then a shrinkage in 3D transform domain is used. Zhang et al. [4] grouped the similar blocks into a matrix and applied principal component analysis (PCA) for AWGN denoising. Recently, the sparse representation and dictionary learning based methods have been attracting significant attention in image restoration. KSVD [5,35] initiates the study of learning an over-complete dictionary from natural images for denoising. By using sparse representation and nonlocal self-similarity regularization jointly, centralized sparse representation (CSR) has lead to state-of-the-art AWGN removal performance [6]. Very recently, Dong et al. [7] connected low-rank methods with simultaneous sparse coding and proposed spatially adaptive iterative singular-value thresholding algorithm (SAIST), which has also shown powerful capability to remove AWGN.

IN is characterized by replacing a portion of an image's pixel values with random noise values, leaving the rest unchanged. Salt-and-pepper impulse noise (SPIN) and random-valued impulse noise (RVIN) are the two types of IN. An image corrupted by SPIN shows dark pixels in bright regions and bright pixels in dark regions. Nonlinear filters such as median filters have been dominantly used for removing IN due to its good denoising property and high computational efficiency. However, the defect of median filters is that the image detailed structures can be destroyed, which makes the denoised image looks unnatural. Based on this reason, various improvements of median filters have been proposed to better preserve the details [8–12].

* Corresponding author. Tel.: +86 25 8431 7297x307.
E-mail address: csjyang@njust.edu.cn (J. Yang).

http://dx.doi.org/10.1016/j.neucom.2014.10.017
0925-2312/

In many cases, images may not be corrupted by only a single statistical model of noise, but mixed types of noise such as the mixture of AWGN and IN. The mixture of AWGN and IN makes the denoising problem much more difficult than either one of them because of the very different characteristics of the two types of noises. Filters that can remove these mixed noise have also been proposed [13–19]. The trilateral filter (TF) [13] incorporates the rank-order absolute difference (ROAD) statistics into the BF [1] framework for IN detection. A modified two-phase method to smooth images corrupted by IN and AWGN mixed noise was proposed by Cai et al. [14], and the computational performance of this method is further improved in [15]. HDIR filter [17] removes the mixed noise by the kernel regression framework based on a Bayesian classification of the input pixels. Ji et al. [18] presented a patch-based algorithm to remove mixed noise from video data by using low rank matrix without strong assumptions on the statistical properties of noise, and fixed point iteration is used for solving nuclear norm related minimization problem. A new IN detection mechanism based on robust outlyingness ratio (ROR) is proposed in [19]. All the image pixels are divided into four clusters based on ROR and different decision rules are adopted to detect IN in each cluster, the detection process contains two stages and different thresholds are used.

Almost all existing mixed noise (AWGN with IN) removal methods follow a two-phase framework: IN pixel detection and removal, followed by AWGN removal, where IN pixel detection usually done by median filters, which can lose some details of image structure. Recently, the low rank approximation (LRA) [20] and low rank representation (LRR) [21,22] methods have shown powerful capability in signal approximation and subspace segmentation. Inspired by their success, in this paper we propose a novel and effective model for mixed noise removal, namely weighted low rank model (WLRM). We group image nonlocal similar patches as a matrix, and reconstruct the clean image by finding the weighted low rank approximation or representation of the matrix. The weights can be adaptively set to indicate and suppress IN, and the low rank model fitting can ensure the preservation of image global structure and local edges. The proposed WLRM performs weight setting and low rank model fitting jointly but not separately, and our experiments clearly show its superiority to other mixed noise removal methods.

The rest of the paper is organized as follows. The noise model is given in Section 2. In Section 3, we describe the proposed WLRM in detail. Section 4 gives experimental results. Conclusion is made in Section 5.

2. Noise model

Denote by $\boldsymbol{x}$ an image and $x_{i,j}$ its pixel at location (i,j). Let $\boldsymbol{y}$ be the noisy observation of $\boldsymbol{x}$. For AWGN model, $\boldsymbol{y}=\boldsymbol{x}+\boldsymbol{n}$, where $\boldsymbol{n}$ follows i.i.d. zero-mean Gaussian distribution. For impulse noise (IN), two most common types are salt-and-pepper impulse noise (SPIN) and random-valued impulse noise (RVIN). Denote the dynamic range of $\boldsymbol{y}$ as $[d_{min}, d_{max}]$, the SPIN model can be described as follows: $y_{i,j}=d_{min}$ with probability $s/2$, $y_{i,j}=d_{max}$ with probability $s/2$, and $y_{i,j}=x_{i,j}$ with probability $1-s$. The RVIN can be defined as: $y_{i,j}=d_{i,j}$ with probability r, and $y_{i,j}=x_{i,j}$ with probability $1-r$, where $0\le r\le 1$ and $d_{i,j}$ is identically and uniformly distributed random value within $[d_{min}, d_{max}]$.

In this paper, we consider the mixed noise of AWGN and IN. More specifically, we consider two kinds of mixed noise model: "AWGN+SPIN" and "AWGN+RVIN+SPIN". For the case of "AWGN+SPIN", the signal observation model can be described as

$$y_{i,j}=\begin{cases} d_{min} & \text{with probability } s/2 \\ d_{max} & \text{with probability } s/2 \\ x_{i,j}+n_{i,j} & \text{with probability } 1-s. \end{cases} \tag{1}$$

For the "AWGN+RVIN+SPIN" noise, the observation model is

$$y_{i,j}=\begin{cases} d_{min} & \text{with probability } s/2 \\ d_{max} & \text{with probability } s/2 \\ d_{i,j} & \text{with probability } r(1-s) \\ x_{i,j}+n_{i,j} & \text{with probability } (1-r)(1-s). \end{cases} \tag{2}$$

3. Methodology

3.1. Low rank approximation (LRA) and low rank representation (LRR)

In recent years, sparse coding has been successfully used in face recognition [23,24] and image restoration (IR) [25–27]. As the 2-dimensional extension of 1-dimensional sparse coding techniques, low rank technology has also been developed, and two representative low rank methods are low rank approximation (LRA) [20] and low rank representation (LRR) [21,22].

LRA aims to recover the desired matrix $\boldsymbol{X}$ from the given observation matrix $\boldsymbol{Y}$. By assuming the noise is dense and matrix $\boldsymbol{X}$ is of low rank, LRA can be formulated as $\min_{\boldsymbol{X}}\|\boldsymbol{Y}-\boldsymbol{X}\|_F^2$, s.t. $\mathrm{rank}(\boldsymbol{X})\le r$. In many case, r is unknown, and the LRA model can be converted into

$$\min_{\boldsymbol{X}} \mathrm{rank}\ (\boldsymbol{X})+\frac{\beta}{2}\|\boldsymbol{Y}-\boldsymbol{X}\|_F^2, \tag{3}$$

where $\beta>0$ is a constant. This problem is in general NP hard. As a common practice in rank minimization problems, the rank function of $\boldsymbol{X}$ can be replaced by its nuclear $\|\boldsymbol{X}\|_*$, resulting in the following convex optimization problem:

$$\min_{\boldsymbol{X}} \|\boldsymbol{X}\|_*+\frac{\beta}{2}\|\boldsymbol{Y}-\boldsymbol{X}\|_F^2. \tag{4}$$

This problem can be solved by a singular value thresholding algorithm [28].

Recently, Liu et al. [21,22] proposed a low rank representation (LRR) for subspace clustering. For the given data matrix $\boldsymbol{Y}$, LRR seeks for a low rank representation $\boldsymbol{Z}$ of $\boldsymbol{Y}$ over itself:

$$\min_{\boldsymbol{Z}} \|\boldsymbol{Z}\|_*+\frac{\beta}{2}\|\boldsymbol{Y}-\boldsymbol{Y}\boldsymbol{Z}\|_{l_p}. \tag{5}$$

When $\|.\|_F^2$ is used, Eq. (5) can be converted into

$$\min_{\boldsymbol{Z}} \|\boldsymbol{Z}\|_*+\frac{\beta}{2}\|\boldsymbol{Y}-\boldsymbol{Y}\boldsymbol{Z}\|_F^2. \tag{6}$$

The optimal solution $\tilde{\boldsymbol{Z}}$ can be obtained based on the matrix singular value decomposition of $\boldsymbol{Y}$ [29].

From Eqs. (4) and (6), we can see that the main difference between LRA and LRR lies in that LRA assumes that the underlying data structure is a single low-rank subspace and LRR assumes that the data is drawn from a union of multiple subspaces. Usually, LRA is used for data approximation and LRR is used for subspace segmentation.

3.2. The proposed denoising model

Recently, LRA has been used for noise removal [18], the basic idea of this method is converting the problem of noise removal from the stack of matched patches to a low rank matrix completion problem. When mixed noise is considered, the authors used