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New delay-dependent stability criteria for switched Hopfield neural networks of neutral type with additive time-varying delay components

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ABSTRACT

The objective of this paper is to analyze the stability of switched Hopfield neural networks of neutral type with additive time-varying discrete delay components and finitely distributed delay. By constructing a suitable Lyapunov–Krasovskii functional with triple and four integral terms and by using Finsler's lemma, a new delay-dependent stability criterion is derived in terms of linear matrix inequalities to ensure the asymptotic stability of the equilibrium point of the considered neural networks. The derived criteria use the information of the upper delay bounds, which may lead to conservativeness. Finally, numerical example is provided to demonstrate the effectiveness of the obtained theoretical results.

1. Introduction

A neural network is a network that performs computational tasks such as associative memory, pattern recognition, model identification, and signal and image processing. Due to this reason, neural networks (especially recurrent neural networks, Hopfield neural networks and cellular neural networks) have been studied extensively during the past decades and results have been published in the literature [1–6]. In applications point of view, the prerequisite property for neural networks is the stability and among different neural networks, stability of Hopfield neural networks has been extensively studied because of their immense applications in various fields [7–11]. Moreover, it is well known that, when designing a neural network to solve a problem such as linear program or pattern recognition, we need to guarantee that the neural network model is globally asymptotically stable.

Time-delay is an inevitable factor in implementing neural networks and often occurs in neural networks owing to the finite switching speed of the amplifiers and the inherent communication time of neurons. This feature for neural networks is one of the key causes leading to instability or performance degradation and is increasingly recognized as an important issue for broader usage. Literature has seen a tremendous results on the stability of neural networks with time-delays, see [12–16]. It is worth noticing that all the before mentioned works are with time-delays appearing in singular or simple form in the state variable. A new type of neural network model with two additive time-varying delay components has been introduced in [17] and the authors therein have pointed out that since signal transmissions may experience a few segment of networks and the conditions of network transmission may differ for each other, it may naturally induce successive delays with different properties. Considering this type of neural networks for stability analysis will find more applications in the real world. Moreover, it is usual to consider constant fixed time-delays in the models, which serve as a good approximation in simple circuits having a smaller number of cells. But it is quite common that neural networks usually have a spatial nature due to the presence of an amount of parallel pathways of a variety of axon sizes and lengths. Thus, it is more reasonable to model them with distributed delays. In addition, it can be seen that the existing neural network models in many cases cannot characterize the properties of a neural reaction process exactly due to the complicated dynamic properties of the neural cells. It is natural and important that systems will contain some information about the derivative of the past state for further describing and modeling the dynamics for such complex neural reactions. Neural networks of this new type are called as neutral neural networks or neural networks of neutral type and the stability analysis for this type of networks has

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been rarely investigated, see [18–23]. In [18], a class of delayed neural networks described by nonlinear delay differential equations of the neutral type has been considered and a sufficient condition for the existence, uniqueness and global exponential stability of an equilibrium point has been derived. In [22], by improving the condition of the above, sufficient criterion for the global asymptotic stability of delayed cellular neural networks of neutral type has been derived. Moreover, a new criterion for the global asymptotic stability of bidirectional associative memory neural networks of neutral type has been derived in [20], which has been further improved in [21]. In all the above-mentioned works, the neutral type delays were considered to be constant delays and not time-varying.

In recent years, with the enormous development of intelligent control, hybrid systems have also been investigated for their wide spread applications. As a special class of hybrid systems, switched systems are considered as nonlinear systems consisting of a finite number of subsystems described by differential or difference equations and a rule specifying which subsystem will be activated at each instant of time. Many physical phenomena and practical applications such as autonomous transmission systems, computer disc cover, room temperature control, power electronics, and chaos generators can be mathematically modeled in the framework of a switched system. Now in practice, switched neural networks, whose individual subsystems are a set of neural networks have found their applications in the field of high-speed signal processing and artificial intelligence. Based on this aspect, a great deal of research has been done on the stability analysis of switched neural networks and accordingly results have been published in the literature, see [24–28].

It should be noted that, in recent years, switched Hopfield neural networks with time-varying delays have been investigated extensively [29–31,44]. Although, there exist a number of papers dealing with the stability of switched Hopfield neural networks, to the best of authors' knowledge there has been no paper with the results for the stability of switched Hopfield neural networks of neutral type comprising successive time-varying delay components and distributed delay. This definitely shows that there exists room for extra improvements. Perhaps, one of the differences mainly lies on the fact that Finsler's lemma has been applied to reduce the number of decision variables and computational burden.

Enlightened by the above ideals, in this paper, we investigate the global asymptotic stability of the equilibrium point of switched Hopfield neural networks of neutral type with successive time-varying delays and distributed delay. A novel Lyapunov–Krasovskii functional (LKF) with triple and four integral terms is constructed and a tighter upper bound for the derivative of LKF is derived by utilizing Jensen's inequality technique and Finsler's lemma. The stability criteria ensuring the asymptotic stability of the equilibrium point are derived in the LMI framework. The obtained criteria strongly use the information on the time-varying delays and their upper bounds.

The outline of the paper is as follows: in Section 2, delayed Hopfield neural networks of neutral type with additive time-varying delays are introduced and the main results of the paper are derived in Section 3. Section 4 deals with numerical examples that are used to show the effectiveness of the proposed theoretical results and finally conclusions are drawn in Section 5.

Notations and preliminaries: In this paper, R^n denotes the n-dimensional Euclidean space. For a matrix X, $X>0$ (<0) represents a positive (negative) definite symmetric matrix and X^T, X^{-1} denotes the transpose and the inverse of a square matrix X, respectively. I denotes the identity matrix with compatible dimension. The symbol $\star$ in a symmetric matrix represents the elements that are induced by symmetry. The shorthand $diag\{\cdot\}$ stands for a diagonal or block diagonal matrix.

Before stepping further for the problem under study, few lemmas which facilitate the presentation of our main results are introduced.

Lemma 1 (*Kwon et al. [32], Tian and Zhong [33]*). *For any constant matrix $X \in R^{n\times n}$, $X=X^T>0$, two scalars a and b, $a<b$ such that the integrations concerned are well defined, then*

$$-\frac{(b-a)^2}{2}\int_a^b\int_\theta^b x^T(s)Xx(s)\,ds\,d\theta \le -\left(\int_a^b\int_\theta^b x(s)\,ds\,d\theta\right)^T X\left(\int_a^b\int_\theta^b x(s)\,ds\,d\theta\right),\ -(b-a)\int_a^b x^T(s)Xx(s)\,ds \le -\left(\int_a^b x(s)\,ds\right)^T X\left(\int_a^b x(s)\,ds\right).$$

Lemma 2 (*de Oliveira and Skelton [34]*). *Let $\zeta \in R^n$, $\Phi=\Phi^T \in R^{n\times n}$ and $\Gamma \in R^{m\times n}$ such that $rank(\Gamma)<n$. The following statements are equivalent:*

1. $\zeta^T\Phi\zeta<0,\ \forall \Gamma\zeta=0,\ \zeta\neq 0,$
2. $\Gamma^{\perp^T}\Phi\Gamma^{\perp}<0,$
3. $\exists d\in R: \Xi-d\Gamma^{\perp^T}\Gamma<0,$
4. $\exists \mathcal{F}\in R^{n\times m}: \Phi+\mathcal{F}\Gamma+\Gamma^T\mathcal{F}^T<0.$

Remark 1. Lemma 2 covers standard Finsler lemma and has been successfully applied in analysis and synthesis of both continuous-time and discrete-time systems with delays. An advantage on applying this lemma is that it helps in reducing the number of decision variables which results in less computational burden.

2. Stability analysis problem

In this section we introduce the delayed Hopfield neural networks for which the stability analysis is going to be carried out. Consider the following delayed Hopfield neural network model of neutral type with successive time-varying delay components and distributed delay as

$$\begin{aligned}\dot{x}(t)&=-Dx(t)+Af(x(t))+Bf(x(t-\tau_1(t)-\tau_2(t)))\\&\quad+C\int_{t-\sigma(t)}^{t} f(x(s))\,ds+E\dot{x}(t-\rho(t))+J,\\x(t)&=\varphi(t),\quad t\in[-\overline{\tau},0],\end{aligned}\tag{1}$$

where $x(t)=[x_1(t),x_2(t),\dots,x_n(t)]^T$ is the neural state vectors, D is a positive diagonal matrix, A, B, C, E are the connection weight matrix, the discretely delayed connection weight matrix, the distributively delayed connection weight matrix and coefficient matrix of the time derivative of the delayed states, respectively. $f(x(t))=[f_1(x_1(t)),f_2(x_2(t)),\dots,f_n(x_n(t))]^T$ denotes the activation function. $\tau_1(t),\tau_2(t)$ are continuous time-varying functions that represent the two delay components in the state, $\sigma(t)$ and $\rho(t)$ are distributive and neutral delays, respectively which satisfy $0\le\tau_1(t)\le\tau_1$, $0\le\tau_2(t)\le\tau_2$, $0\le\sigma(t)\le\sigma$, $0\le\rho(t)\le\rho$, where $\tau_1,\tau_2,\sigma,\rho$ are some positive scalars. Moreover, we assume that $\dot{\tau}_1(t)\le\mu_1,\dot{\tau}_2(t)\le\mu_2,\dot{\rho}(t)\le\rho'$.Also we denote $\tau(t)=\tau_1(t)+\tau_2(t)$, $\tau=\tau_1+\tau_2$ and $\mu=\mu_1+\mu_2$. $J=[J_1,J_2,\dots,J_n]^T$ is the constant external input vector. $\varphi_i(t)$ $(i\in N)$ is continuous on $[\overline{\tau},0]$, $\overline{\tau}=\max\{\tau_1,\tau_2,\sigma,\rho\}$, the norm is defined by

$$\|\varphi\|_{\overline{\tau}}=\max\left\{\sup_{-\overline{\tau}\le s\le 0}\|\varphi(s)\|,\ \sup_{-\overline{\tau}\le s\le 0}\|\dot{\varphi}(s)\|\right\}.\tag{2}$$

Assumption 1. The activation function $f_j(\cdot)$ is bounded and there exist constants δ_j^-,δ_j^+ such that $\delta_j^-<\delta_j^+$ and

$$\delta_j^-\le\frac{f_j(u)-f_j(v)}{u-v}\le\delta_j^+,\quad \forall u,v\in R, u\neq v,\ j\in N.\tag{3}$$

For the sake of simplicity, we denote

$$\Delta_1=diag\{\delta_1^-,\delta_2^-,\dots,\delta_n^-\},\quad \Delta_2=diag\{\delta_1^+,\delta_2^+,\dots,\delta_n^+\},$$