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Image reconstruction under multiplicative speckle noise using total variation

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ABSTRACT

In this paper, we present a method for reconstructing images or volumes from a partial set of observations, under the Rayleigh distributed multiplicative noise model, which is the appropriate algebraic model in ultrasound (US) imaging. The proposed method performs a variable splitting to introduce an auxiliary variable to serve as the argument of the total variation (TV) regularizer term. Applying the *Augmented Lagrangian* framework and using an iterative alternating minimization method lead to simpler problems involving TV minimization with a least squares term. The resulting Gauss Seidel scheme is an instance of the Alternating Direction Method of Multipliers (ADMM) method for which convergence is guaranteed. Experimental results show that the proposed method achieves a lower reconstruction error than existing methods.

1. Introduction

Images acquired using any modality need to be denoised before further processing or analysis. The problem of estimating the image from the noisy and possibly incomplete observations is an ill-posed one [1], necessitating regularization or some assumption on the nature of the image. The statistical distribution of the noise and the algebraic observation model that leads to the image formation depends on the physics of the modality. Further, depending on hardware and sensing limitations as well as transmission errors, the image may need to be reconstructed from a set of partial observations. This is also the case in the sampling/acquisition methodology known as compressive sensing [2,3] wherein the image needs to be reconstructed from an undersampled set of incoherent observations.

Several methods exist to solve the denoising and reconstruction problems for the classical additive and Gaussian noise model, using non-smooth regularization such as *total variation* (TV) [4–6] which encourages the solution to be piece-wise smooth. Recent methods have focussed on the *augmented Lagrangian/alternating direction method of multipliers* (AL/ADMM) method [7,8] to solve the convex optimization formulations for these TV regularized problems [9–11] because of its computational speed. In this paper, we propose a TV and ADMM based method for image denoising and reconstruction from partial observations for the case when the noise is multiplicative and Rayleigh distributed, which is the model in Ultrasound (US) imaging for the radio frequency (RF) envelope image [12].

Ultrasound has emerged as a popular medical imaging modality in a number of medical imaging applications because of its low cost, wide reach, flexibility, lack of radiation, and intraoperability [13–15]. Because 2D US images are acquired as slices representing a thin plane from the volume, it is difficult to reproduce for follow-up, *i.e.*, image at the exact location again. Therefore three dimensional (3D) US imaging is being increasingly used for characterizing diseases such as carotid atherosclerosis, requiring a 3D volume to be reconstructed from a series of 2D slices. The slices can be acquired mechanically in a predetermined manner, or freehand wherein the user can manually position and orient the probe. It has also been reported that segmentation and classification based on 3D US has advantages compared to those based on 2D [16,17].

1.1. Related works

For the additive and Gaussian noise case, solvers for 2D and 3D reconstruction from partial data using TV regularization include the *Sparse Reconstruction by Separable Approximation* [18], *(Constrained) Split Augmented Lagrangian Shrinkage Algorithm* [11,19], split Bregman method [10], Fast TV deconvolution (FTVd) [9], and the Nesterov method based solvers, mxTV [20], and NESTA [21]. Sparse MRI [22] for MRI reconstruction also uses a TV regularizer term.

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A despeckling method for US images called *Rayleigh Log-Euclidean Total Variation* (RLTV) was proposed in [23,24], in which a logarithmic compression was used on the image, and then a TV regularizer term was applied on the transformed image. The resulting convex optimization problem was solved iteratively using Newton's method. The same denoising formulation was solved in *multiplicative image denoising by augmented Lagrangian* (MIDAL) method [25] in the context of Synthetic Aperture Radar (SAR) images, but solved using an AL/ADMM method. MIDAL also does not use a logarithmic compression. Another TV based denoising method for gamma distributed multiplicative speckle noise is the variational formulation based on mth root transformation called *linearized proximal alternating minimization algorithm* (LPAMA/mV) [26,27]. This method was proposed for the application of multi-look SAR images. The Nakagami distribution was assumed as the statistical model in the denoising method presented in [28]. A denoising method was presented in [29], assuming that the multiplicative noise (in natural images) was one-sided exponentially distributed, and with an ℓ_1 data fidelity term.

A variational model for deblurring under multiplicative noise was proposed in [30], which uses a quadratic penalty term and is strictly convex under mild conditions. The formulation is solved using a primal-dual algorithm.

The above listed methods solve the despeckling problem when there is no loss of pixels. To solve the harder problem of image reconstruction, in [24], the authors first perform a voxel interpolation over the grid to obtain a noisy image without missing pixels, and then apply RLTV to despeckle it.

Interpolation algorithms such as the Pixel Nearest Neighbor (PNN) [31], Voxel Nearest Neighbor (VNN) [31], and Pixel-Based Interpolation with Distance Weighting (PBM-DW) [32] do not use any regularization or *a priori* information about the volume to be reconstructed. A comprehensive review of interpolation methods for US reconstruction can be found in [14].

Other non-TV-based methods for 3D US reconstruction have been reported in the literature. These include the Cyclic Regularized Savitzky–Golay (CRSG) filter method [33] which estimates unobserved voxels through a local 3D least squares polynomial fitting. Results reported in this work showed that CRSG was able to obtain a lower normalized reconstruction error (0.032) than PNN-DW (0.047) in 3D synthetic experiments. Others such as [13] perform an interpolation and coordinate mapping over each unobserved voxel. Spline interpolation to connect regions across observed slices acquired freehand has also been proposed [34]. In this work, results were reported for different conditions of the carotid artery (normal or with plaque stenosis), without comparison with existing methods. A despeckling filter based on anisotropic diffusion without a linear approximation (DPAD) was proposed in [35] for denoising and separating the speckle component.

1.2. Contributions

In this paper, we extend the TV regularized despeckling formulation from [23,25] to the more general problem of estimating the image from a partial set of noisy pixels. This is a more difficult and ill-posed [1] problem than denoising, because some pixel/voxel values are unknown. This is a relevant problem from the point of view of reconstructing a 3D volume from a partial set of acquired 2D slices. We solve the resulting convex problem using an AL/ADMM approach which leads to an alternating minimization in which at every iteration a sequence of simpler problems has to be solved. The proposed method for reconstruction is a more general formulation of the method for solving the denoising problem alone. We test the proposed method with synthetic data simulating both linear mechanical and random freehand scanning, as well as real US images. Preliminary results were presented in [36], which showed that the proposed method is more accurate than interpolation methods, and is faster than all methods except the Pixel Nearest Neighbor (PNN) interpolation which is the crudest interpolation technique. In this paper, we compare our method against PNN interpolation followed by despeckling methods which take into account the statistical model. Synthetic experiments show that the proposed method achieves a lower mean square error than existing methods.

In Section 2, we formulate the optimization problems to be solved for estimating the despeckled image, with and without missing data. We present the proposed approach for solving the denoising and reconstruction problems in Section 3. In Section 4, we present experimental results on 2D and 3D reconstruction, with synthetic examples and real US images. Section 5 concludes the paper.

2. Problem formulation

The image is represented as a vector, say, in lexicographic ordering, as $\mathbf{x} \in \mathbb{R}^n$, where n is the number of pixels or voxels. When there is no loss of pixels, the dimensionality of the observed image $\mathbf{y}$ is the same as that of $\mathbf{x}$. Each element of $\mathbf{y}$ is the product of the corresponding element from $\mathbf{x}$ and the corresponding element from the noise field $\boldsymbol{\eta}$. The observation model is therefore the element-wise multiplication:

$$\mathbf{y} = \mathbf{x} \cdot \boldsymbol{\eta}. \tag{1}$$

In the case of partial observations, the number of elements of $\mathbf{y}$ is less than the size of $\mathbf{x}$. This is the case in the problem of inpainting, wherein pixels damaged or lost because of transmission errors have to be estimated [37–39]. The acquisition methodology of compressive sensing also involves observing an incomplete set of incoherent observations to speed up and simplify the sensing process and hardware [22]. When the number of observed pixels is $m < n$, we model the observation process as a multiplication of $\mathbf{x} \in \mathbb{R}^n$ by a linear operator $\mathbf{A} \in \mathbb{R}^{m \times n}$:

$$\mathbf{y} = (\mathbf{A}\mathbf{x}) \cdot \boldsymbol{\eta}. \tag{2}$$

In this case, $\mathbf{y}, \eta \in \mathbb{R}^m$. The matrix $\mathbf{A}$ maps a pixel or a voxel in the grid to a pixel in the set of observed slices, and discards the pixels or voxels in $\mathbf{x}$ which do not correspond to a pixel in $\mathbf{y}$. Hence, the matrix $\mathbf{A}$ is essentially the $n \times n$ identity matrix with $n-m$ rows (corresponding to non-observed voxels) removed. The position and orientation of each slice must be known to construct the matrix $\mathbf{A}$. For a denoising problem, *i.e.*, when all elements are observed ($m=n$), it is equal to the identity matrix $\mathbf{A} = \mathbf{I}$.

Assuming that the speckle field $\boldsymbol{\eta}$ is Rayleigh distributed, when there are no missing observations the likelihood is

$$p(\mathbf{y}|\mathbf{x}) = \prod_{i=1}^{n} \frac{y_i}{x_i} \exp\left(-\frac{y_i^2}{2x_i}\right), \tag{3}$$

where x_i is the ith element of the vector $\mathbf{x}$. After logarithmic compression, this leads to the log-likelihood function:

$$E(\mathbf{y},\mathbf{x}) = -\log(p(\mathbf{y}|\mathbf{x})) = \sum_{i=1}^{n} \left(\frac{y_i^2}{2x_i} + \log x_i\right). \tag{4}$$

In [40], a logarithmic compression $\mathbf{f} = \log(\mathbf{x})$ is applied and a TV regularizer term is applied on the transformed variable, leading to the convex optimization problem:

$$\min_{\mathbf{f}} \sum_i \left(\frac{y_i^2}{2} e^{-f_i} + f_i\right) + \frac{\lambda}{2} TV(\mathbf{f}), \tag{5}$$