Neurocomputing 144 (2014) 208–214

Contents lists available at ScienceDirect

Neurocomputing

journal homepage: www.elsevier.com/locate/neucom

New results on passivity analysis of memristor-based neural networks with time-varying delays

Leimin Wang [a,b], Yi Shen [a,b,*]

[a] School of Automation, Huazhong University of Science and Technology, Wuhan 430074, China
[b] Key Laboratory of Image Processing and Intelligent Control of Education Ministry of China, Wuhan 430074, China

ARTICLE INFO

Article history:
Received 17 January 2014
Received in revised form
4 May 2014
Accepted 6 May 2014
Communicated by Z. Wang
Available online 28 May 2014

Keywords:
Passivity
Memristor-based neural networks
Filippov solution
Time-varying delays

ABSTRACT

In this paper, the passivity problem of memristor-based neural networks (MNNs) with time-varying delays is investigated. New delay-dependent criteria are established for the passivity of MNNs. The time-varying delays of our paper are not necessary to be differentiable, so our results are less conservative, which enrich and improve the earlier publications. An example is given to demonstrate the effectiveness of the obtained results.

1. Introduction

Memtistor, as a contraction of memory and resistor, was originally theorized by Chua in 1971 [1]. It was predicted as the fourth circuit element (the other three are resistor, capacitor and inductor). In 2008, scientists at Hewlett-Packard Laboratories claimed to have found Chua's missing memristor based on an analysis of thin film of titanium dioxide [2]. In the past few years, memristor has received increasing research attention for its memory characteristic and nanometer dimension. From previous works [3,4], we know that memristor can be used to mimic the synaptic connections in a human brain. Hence, the model of memristor-based neural networks (MNNs) can be built to emulate the human brain where synapses are implemented with memristors. Recently, Wen and Zeng [8], Wu and Zeng [9,10], Zhang and Shen [11–14] have studied the MNNs with delays. A lot of significant results concerning the MNNs have been obtained [8–16].

It is well known that the passivity theory [34] plays an important role in the analysis of the stability of dynamical systems [32,33,39–41], and it has received a lot of attention since 1970s [17–26,34–38]. In fact, the passive properties of a system can keep the system internal stability. Recently, passivity properties have been related to MNNs [8,10]. In [8], passivity conditions of MNNs are obtained under the assumption that the time-varying delays are continuously differentiable, and the derivative of time-varying delay is bounded. In [10], passivity analysis is conducted with constant time delays. However, time delays can occur in an irregular fashion, and sometimes are not differentiable. Motivated by the above discussions, in this paper, we investigate the passivity of MNNs with time-varying delays which the delays are unnecessarily differentiable. New delay-dependent passivity conditions are established. The obtained results are more general and less conservative compared with the results in [8,10].

The organization of this paper is as follows. Some preliminaries are introduced in Section 2. In Section 3, new delay-dependent criteria are established for the passivity of MNNs in terms of LMIs. Then, an example is given to demonstrate the effectiveness of the obtained results in Section 4. Finally, conclusions and discussions are given in Section 5.

Notations: Throughout this paper, R^n denotes the n-dimensional Euclidean space. A^T and A^{-1} denote the transpose and the inverse of the matrix A, respectively. $A>0 (A\geq 0)$ means that the matrix A is symmetric and positive definite (semi-positive definite). $*$ represents the elements below the main diagonal of a symmetric matrix. I is the identity matrix with compatible dimension. diag $\{\cdots\}$ denotes a block-diagonal matrix.

* Corresponding author at: School of Automation, Huazhong University of Science and Technology, Wuhan 430074, China.
Tel.: +86 27 87543630; fax: +86 27 87543130.
E-mail addresses: leiminw89@126.com (L. Wang), yishen64@163.com (Y. Shen).

http://dx.doi.org/10.1016/j.neucom.2014.05.032
0925-2312/

2. System description and preliminaries

In this paper, we consider the memristor-based neural networks (MNNs) as follows:

$$\begin{cases} \dot{x}_i(t) = -d_i(x_i(t))x_i(t) + \sum_{j=1}^{n} a_{ij}(x_i(t))f_j(x_j(t)) \\ \quad + \sum_{j=1}^{n} b_{ij}(x_i(t))f_j(x_j(t-\tau_j(t))) + u_i(t), \\ y_i(t) = f_i(x_i(t)), t \geq 0, i,j = 1,2,\ldots,n, \end{cases} \tag{1}$$

where

$$d_i(x_i(t)) = \begin{cases} d_i^*, & -\dot{f}_i(x_i(t)) - \dot{x}_i(t) \leq 0, \\ d_i^{**}, & -\dot{f}_i(x_i(t)) - \dot{x}_i(t) > 0, \end{cases}$$

$$a_{ij}(x_i(t)) = \begin{cases} a_{ij}^*, & \theta_{ij}\dot{f}_j(x_j(t)) - \dot{x}_i(t) \leq 0, \\ a_{ij}^{**}, & \theta_{ij}\dot{f}_j(x_j(t)) - \dot{x}_i(t) > 0, \end{cases}$$

$$b_{ij}(x_i(t)) = \begin{cases} b_{ij}^*, & \theta_{ij}\dot{f}_j(x_j(t-\tau_j(t))) - \dot{x}_i(t-\tau_i(t)) \leq 0, \\ b_{ij}^{**}, & \theta_{ij}\dot{f}_j(x_j(t-\tau_j(t))) - \dot{x}_i(t-\tau_i(t)) > 0, \end{cases}$$

where

$$\theta_{ij} = \begin{cases} 1, & i \neq j, \\ -1, & i = j, \end{cases}$$

$x_i(t)$ is the state variable of the i-th neuron, $a_{ij}(x_i(t))$ and $b_{ij}(x_i(t))$ denote the feedback connection weight and the delayed feedback connection weight, respectively. $f_j : R \to R$ is bounded continuous function, $u_i(t)$ is an external input function, $\tau_j(t)$ corresponds to the transmission delay and satisfy $0 \leq \tau_j(t) \leq \tau, i,j = 1,2,\ldots,n.$ $d_i^* > 0$, $d_i^{**} > 0, a_{ij}^*, a_{ij}^{**}, b_{ij}^*, b_{ij}^{**}, i,j = 1,2,\ldots,n$ are all constant numbers. $y_i(t) = f_i(x_i(t)), t \geq 0, i = 1,2,\ldots,n$ is the output of system (1). The initial condition of system (1) is: $x(s) = \phi(s) = (\phi_1(s), \phi_2(s), \ldots, \phi_n(s))^T \in \mathcal{C}([-\tau, 0], R^n)$.

Obviously, system (1) is a discontinuous system, then its solution is different from the classic solution and cannot be defined in the conventional sense. In order to obtain the solution of system (1), some definitions and lemmas are given.

Definition 1. For a system with discontinuous right-hand sides:

$$\frac{dx}{dt} = F(x), \quad x(0) = x_0, \quad x \in R^n, \quad t \geq 0 \tag{2}$$

where $F(x) : R^n \longrightarrow R^n$ is discontinuous. A set-valued map is defined as

$$\Phi(x) = \bigcap_{\delta > 0} \bigcap_{\mu(N) = 0} \overline{co}[F(B(x,\delta) \backslash N)],$$

where $\overline{co}[E]$ is the closure of the convex hull of set E, $E \subset R^n$, $B(x,\delta) = \{y : \|y - x\| < \delta, x, y \in R^n, \delta \in R^+\}$, and $N \subset R^n$, $\mu(N)$ is Lebesgue measure of set N.

A solution in Filippov's sense [5] of system (2) with initial condition $x(0) = x_0 \in R^n$ is an absolutely continuous function $x(t), t \in [0,T], T > 0$, which satisfy $x(0) = x_0$ and differential inclusion:

$$\frac{dx}{dt} \in \Phi(x), \quad \text{for a.a. } t \in [0,T].$$

If $F(x)$ is bounded, then the set-valued function $\Phi(x)$ is upper semicontinuous with non-empty, convex and compact values [5]. Then the solution $x(t)$ of system (2) with initial condition exists and it can be extended to the interval $[0, +\infty)$ in the sense of Filippov.

By applying the theories of set-valued maps and differential inclusions [5–7], then system (1) can be rewritten as the following differential inclusion:

$$\dot{x}_i(t) \in -[\underline{d}_i, \overline{d}_i]x_i(t) + \sum_{j=1}^{n} [\underline{a}_{ij}, \overline{a}_{ij}]f_j(x_j(t))$$
$$+ \sum_{j=1}^{n} [\underline{b}_{ij}, \overline{b}_{ij}]f_j(x_j(t-\tau_j(t))) + u_i(t),$$
$$\text{for a.a. } t \geq 0, \ i,j = 1,2,\ldots,n, \tag{3}$$

where the output is $y_i(t) = f_i(x_i(t))$, $[\underline{\xi}_i, \overline{\xi}_i]$ is the convex hull of $[\underline{\xi}_i, \overline{\xi}_i], \underline{\xi}_i, \overline{\xi}_i \in R$. $\underline{d}_i = \min\{d_i^*, d_i^{**}\}$, $\overline{d}_i = \max\{d_i^*, d_i^{**}\}$, $\underline{a}_{ij} = \min\{a_{ij}^*, a_{ij}^{**}\}$, $\overline{a}_{ij} = \max\{a_{ij}^*, a_{ij}^{**}\}$, $\underline{b}_{ij} = \min\{b_{ij}^*, b_{ij}^{**}\}$, $\overline{b}_{ij} = \max\{b_{ij}^*, b_{ij}^{**}\}$. The other parameters are the same as in system (1).

Definition 2. A function $x(t) = (x_1(t), x_2(t), \ldots, x_n(t))^T$ is a solution of (1), with the initial condition $x(s) = \phi(s) = (\phi_1(s), \phi_2(s), \ldots, \phi_n(s))^T \in \mathcal{C}([-\tau, 0], R^n)$, if $x(t)$ is an absolutely continuous function and satisfies the differential inclusion (3).

Throughout this paper, we consider the following assumption for the activation functions:

(H1) For $j \in 1,2,\ldots,n, f_j$ is bounded and there exists constant $k_j > 0$ such that

$$0 \leq \frac{f_j(s_1) - f_j(s_2)}{s_1 - s_2} \leq k_j, f_j(0) = 0,$$

for all $s_1, s_2 \in R, s_1 \neq s_2$.

Lemma 1. *Suppose that assumption (H1) is satisfied, then solution $x(t)$ with initial condition $\phi(s) = (\phi_1(s), \phi_2(s), \ldots, \phi_n(s))^T \in \mathcal{C}([-\tau, 0], R^n)$ of (1) exists and it can be extended to the interval* $[0, +\infty)$.

Before giving our main results, a definition and a lemma [19] which are useful in the proof are given as follows.

Definition 3. System (1) is called passive if there exists a scalar $\gamma > 0$ such that

$$2\int_0^{t_p} y^T(s)u(s)\,ds \geq -\gamma \int_0^{t_p} u^T(s)u(s)\,ds$$

for all $t_p \geq 0$ and for all solution of (1) with $x(0) = 0$.

Lemma 2. *Given constant matrices $\Sigma_1, \Sigma_2, \Sigma_3$, where $\Sigma_1^T = \Sigma_1, \Sigma_2^T = \Sigma_2$, then*

$$\begin{pmatrix} \Sigma_1 & \Sigma_3 \\ \Sigma_3^T & -\Sigma_2 \end{pmatrix} < 0$$

is equivalent to the following conditions:

$$\Sigma_2 > 0 \quad \text{and} \quad \Sigma_1 + \Sigma_3 \Sigma_2^{-1} \Sigma_3^T < 0.$$

3. Main results

For presentation convenience, in the following, we denote $K = \text{diag}(k_1, k_2, \ldots, k_n)$, $D = \text{diag}(D_i)_n \times n, D_i = \min\{|\underline{d}_i|, |\overline{d}_i|\}$, $A = (A_{ij})_{n\times n}, A_{ij} = \max\{|\underline{a}_{ij}|, |\overline{a}_{ij}|\}, B = (B_{ij})_{n\times n}, B_{ij} = \max\{|\underline{b}_{ij}|, |\overline{b}_{ij}|\}$.

Theorem 1. *Suppose assumption (H1) holds. If there exist a scalar $\gamma > 0$, three symmetric matrices $P > 0, Q_1 > 0, Q_2 > 0$, three diagonal matrices $E > 0, F > 0, G > 0$, and matrices $R_{ij} (i,j = 1,2,3,4,5, i \leq j)$, such that the following two LMIs hold*

$$R = \begin{bmatrix} R_{11} & R_{12} & R_{13} & R_{14} & R_{15} \\ * & R_{22} & R_{23} & R_{24} & R_{25} \\ * & * & R_{33} & R_{34} & R_{35} \\ * & * & * & R_{44} & R_{45} \\ * & * & * & * & R_{55} \end{bmatrix} > 0, \tag{4}$$