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Letters

New global exponential stability results for a memristive neural system with time-varying delays

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ABSTRACT

Recent research in memristor–CMOS neuromorphic learning systems has led to the practical realization of neuro-inspired learning architectures. At present, the deep understanding of nonlinear dynamical mechanisms governing memristive neural systems is still an open issue. In this paper, the global exponential stability problem is investigated for a class of memristive neural systems with time-varying delays. By employing comparison principle, some novel global exponential stability results are derived. These stability conditions also improve upon some existing results. In addition, the obtained results are convenient to estimate the exponential convergence rate. These theoretical studies are very useful in analyzing the composite behavior of complex memristor circuits.

1. Introduction

The neuronal synapse (or electrical synapse) is a crucial element in biological neural networks (or artificial neural networks). The resistance of a memristive system closely depends on its past historical states and exactly this functionality can be easily realized synaptic behavior in a brain-like system. With the development of applications, memristive nanodevices as synapses and conventional CMOS technology as neurons are widely adopted. In recent years, various kinds of hybrid memristor–CMOS neural learning systems have been proposed for building very large scale systems with a spike-timing-dependent-plasticity learning mechanism [1–14]. Such neuromorphic architecture can provide a great deal of inspiration for systems biologist, theoretical physicist and electronics engineer to design brain-like processing systems. This approach permits the use of basic electrical circuits as a model for biological systems, which promotes the development of hardware that mimics biological architectures in the nervous systems. In [1,2,5], some entirely new neuronal synaptic learning mechanisms are described through complex hierarchical nanoscale memristor structures. The physical architecture and design principles of brain-style neural associative memories are proposed in Pershin and Di Ventra [6]. Generally, memristive synaptic strength is the conductance of memristor. Thus, the composite behavior of multiple memristor circuits is extremely complex [3,4,8–14]. Consequently, the electrical characteristics are not yet fully understood. On the other hand, the electrical characteristics can more deeply reveal the associated biological mechanisms, such as learning and forgetting.

As pointed out in [3,8–14], analysis and design of memristive neurodynamic systems is particularly important when the promising characteristics of memristor–CMOS neuromorphic systems are used to revolutionize nanoelectronics. A problem is that memristive systems perform stateful logic operation, thereby inspiring varied resistance values of the memristors, i.e., from synapse ON to synapse OFF. This mechanism makes the nonlinear dynamic analysis and design inevitably a bottleneck [8–12]. From the point of view of cybernetics, a memristive neurodynamic system is basically a state-dependent nonlinear network cluster. Over the years, a lot of interesting concepts and properties on nonlinear systems have been reported, see [15–43]. Whereas, in the past decades, analysis and design of the state-dependent nonlinear network cluster is still an issue open to discussions, and the practical promotion for state-dependent nonlinear network cluster is not a reality yet. Consequently, nanoscale memristor neuromorphic learning systems suffer from high mismatch in general. And because of that, we more hope to reveal the dynamic evolution process.

Recently, Guo et al. [3] analyze the global exponential dissipativity of memristive recurrent neural system via Lyapunov method, *M*-matrix theory and LaSalle invariant principle. Within mathematical framework of the Filippov solution, the exponential

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stability about the memristive recurrent neural system is investigated in [8,13]. In [9] and [10], based on drive-response concept, differential inclusions theory and Lyapunov functional method, the exponential synchronization for coupled memristive neural systems is studied. In [11], some delay-dependent exponential stabilization criteria in terms of linear matrix inequalities for the memristive neural system are obtained via nonsmooth analysis and control theory. In order to investigate internal stability, some sufficient conditions ensuring the exponential passivity of memristive neural system with multiple time delays are derived in [12]. Periodic oscillation in neurodynamic systems is an interesting dynamic behavior. Zhang et al. [14] present some analytical results on the persistent periodic oscillation of a class of memristive recurrent neural systems.

However, in the existing literature, the obtained results are somewhat complicated, so proper advantage is not being taken of characteristics of memristive system. It is worth thinking deeply about that within mathematical framework of the Filippov solution, the comparison principle can be adopted according to the characteristics of memristive system, rather than the general Lyapunov method or the Lyapunov functional method. In addition, an important issue on neurodynamic systems is the convergence rate. The convergence rate can determine the speed of neural computations, which is not only theoretically interesting but also of practical importance to determine the stability. Most importantly, some existing results in the latest publications are based on some unsuitable assumption conditions [8–10,13,14], for all this, we need to re-develop strict logical inference and provide the right types of theoretical criteria.

This paper attempts to derive some less conservative conditions ensuring the global exponential stability for memristive neural system with time-varying delays. The main contributions of this paper can be summarized as follows: (1) In view of some inappropriate assumption conditions in the existing literature, we revise former logical inference and re-develop reasonable analytical framework. (2) The comparison principle represents a popular and effective methodology, which can solve the complexity of the model evoked by memristor nonlinearity and achieve good effect. (3) The exponential convergence rate in our criteria can be effectively estimated or calculated. As we all know, the memristive neurodynamic system is still quite incipient and to the best of our knowledge no existing result has been reported for exploring the related convergence rate. (4) The proposed method in this paper can be applied to the general nonlinear hybrid systems. Application of some novel techniques from the theory of nonlinear network cluster to memristive systems yields a deep insight into the nonlinear dynamics under investigation.

2. Preliminaries

Consider a class of memristive neurodynamic systems described by the following differential equations: for $i=1,2,\ldots,n$,

$$\dot{x}_i(t) = -x_i(t) + \sum_{j=1}^{n} a_{ij}(x_i(t)) f_j(x_j(t)) + \sum_{j=1}^{n} b_{ij}(x_i(t)) f_j(x_j(t-\tau_j(t))), \tag{1}$$

where $x_i(t)$ is the voltage of the capacitor $\mathbf{C}_i$, $0 \le \tau_i(t) \le \tau$ ($\tau > 0$ is a constant) is the time-varying delay, $f_i(\cdot)$ is feedback function satisfying $f_i(0)=0$, $a_{ij}(x_i(t))$ and $b_{ij}(x_i(t))$ represent memristor-based weights, and

$$a_{ij}(x_i(t)) = \frac{\mathbf{W}_{ij}}{\mathbf{C}_i} \times \text{sgin}_{ij}, \quad b_{ij}(x_i(t)) = \frac{\mathbf{M}_{ij}}{\mathbf{C}_i} \times \text{sgin}_{ij},$$

$$\text{sgin}_{ij} = \begin{cases} 1, & i \ne j, \\ -1, & i = j, \end{cases}$$

in which $\mathbf{W}_{ij}$ and $\mathbf{M}_{ij}$ denote the memductances of memristors $\mathbf{R}_{ij}$ and $\mathbf{F}_{ij}$, respectively. And $\mathbf{R}_{ij}$ represents the memristor between the feedback function $f_i(x_i(t))$ and $x_i(t)$, $\mathbf{F}_{ij}$ represents the memristor between the feedback function $f_i(x_i(t-\tau_i(t)))$ and $x_i(t)$.

According to the feature of pinched hysteresis loop and the characteristic of dynamical memristor resistance [8,12], then

$$a_{ij}(x_i(t)) = \begin{cases} \hat{a}_{ij}, & \text{sgin}_{ij} \dfrac{\mathrm{d}f_j(x_j(t))}{\mathrm{d}t} - \dfrac{\mathrm{d}x_i(t)}{\mathrm{d}t} \le 0, \\ \check{a}_{ij}, & \text{sgin}_{ij} \dfrac{\mathrm{d}f_j(x_j(t))}{\mathrm{d}t} - \dfrac{\mathrm{d}x_i(t)}{\mathrm{d}t} > 0, \end{cases} \tag{2}$$

$$b_{ij}(x_i(t)) = \begin{cases} \hat{b}_{ij}, & \text{sgin}_{ij} \dfrac{\mathrm{d}f_j(x_j(t-\tau_j(t)))}{\mathrm{d}t} - \dfrac{\mathrm{d}x_i(t)}{\mathrm{d}t} \le 0, \\ \check{b}_{ij}, & \text{sgin}_{ij} \dfrac{\mathrm{d}f_j(x_j(t-\tau_j(t)))}{\mathrm{d}t} - \dfrac{\mathrm{d}x_i(t)}{\mathrm{d}t} > 0, \end{cases} \tag{3}$$

for $i,j=1,2,\ldots,n$, where $\hat{a}_{ij}$, $\check{a}_{ij}$, $\hat{b}_{ij}$, and $\check{b}_{ij}$ are constants.

Throughout this paper, solutions of all the systems considered in the following are intended in Filippov's sense. $\text{co}\{\tilde{\Pi}, \hat{\Pi}\}$ denotes closure of the convex hull generated by real numbers $\tilde{\Pi}$ and $\hat{\Pi}$. E_n is an $n \times n$ identity matrix. Let $\overline{a}_{ij} = \max\{\hat{a}_{ij}, \check{a}_{ij}\}$, $\underline{a}_{ij} = \min\{\hat{a}_{ij}, \check{a}_{ij}\}$, $\overline{b}_{ij} = \max\{\hat{b}_{ij}, \check{b}_{ij}\}$, $\underline{b}_{ij} = \min\{\hat{b}_{ij}, \check{b}_{ij}\}$, $\tilde{a}_{ij} = \max\{|\hat{a}_{ij}|, |\check{a}_{ij}|\}$, $\tilde{b}_{ij} = \max\{|\hat{b}_{ij}|, |\check{b}_{ij}|\}$, for $i,j=1,2,\ldots,n$.

The initial condition of system (1) is assumed to be

$$x(t) = (x_1(t), x_2(t), \ldots, x_n(t))^T = \phi(t) = (\phi_1(t), \phi_2(t), \ldots, \phi_n(t))^T, \quad t_0 - \tau \le t \le t_0, \tag{4}$$

where $\phi_i(t) \in \mathcal{C}([t_0-\tau, t_0], \Re)$, $i=1,2,\ldots,n$.

In addition, from (2) and (3), it follows that $f_i(\cdot)(i=1,2,\ldots,n)$ are differentiable, and thus we have

$$k_i = \sup_{\hat{\chi} \ne \check{\chi}} \left| \frac{f_i(\hat{\chi}) - f_i(\check{\chi})}{\hat{\chi} - \check{\chi}} \right|, \quad i=1,2,\ldots,n, \quad \forall \hat{\chi}, \check{\chi} \in \Re. \tag{5}$$

where constants $k_i > 0$, $i=1,2,\ldots,n$.

By the theories of differential inclusions and set-valued maps, from (1), it follows that for $i=1,2,\ldots,n$,

$$\dot{x}_i(t) \in -x_i(t) + \sum_{j=1}^{n} \text{co}\{\hat{a}_{ij}, \check{a}_{ij}\} f_j(x_j(t)) + \sum_{j=1}^{n} \text{co}\{\hat{b}_{ij}, \check{b}_{ij}\} f_j(x_j(t-\tau_j(t))). \tag{6}$$

Clearly, for $i,j=1,2,\ldots,n$,

$$\text{co}\{\hat{a}_{ij}, \check{a}_{ij}\} = [\underline{a}_{ij}, \overline{a}_{ij}], \quad \text{co}\{\hat{b}_{ij}, \check{b}_{ij}\} = [\underline{b}_{ij}, \overline{b}_{ij}].$$

A solution $x(t) = (x_1(t), x_2(t), \ldots, x_n(t))^T$ (in the sense of Filippov) of system (1) with initial condition $x(s) = \phi(s)$, $s \in [t_0-\tau, t_0]$, is absolutely continuous on any compact interval of $[t_0, +\infty)$, and

$$\dot{x}_i(t) \in -x_i(t) + \sum_{j=1}^{n} \text{co}\{\hat{a}_{ij}, \check{a}_{ij}\} f_j(x_j(t)) + \sum_{j=1}^{n} \text{co}\{\hat{b}_{ij}, \check{b}_{ij}\} f_j(x_j(t-\tau_j(t))).$$

Definition 1. A constant vector $x = (x_1^*, x_2^*, \ldots, x_n^*)^T$ is called an equilibrium point of system (1), if for $i=1,2,\ldots,n$,

$$0 \in -x_i^* + \sum_{j=1}^{n} \text{co}\{\hat{a}_{ij}, \check{a}_{ij}\} f_j(x_j^*) + \sum_{j=1}^{n} \text{co}\{\hat{b}_{ij}, \check{b}_{ij}\} f_j(x_j^*).$$