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Adaptive output synchronization of complex delayed dynamical networks with output coupling

Jin-Liang Wang [a,b], Huai-Ning Wu [b,*]

[a] School of Computer Science & Software Engineering, Tianjin Polytechnic University, Tianjin 300387, China
[b] Science and Technology on Aircraft Control Laboratory, School of Automation Science and Electrical Engineering, Beihang University, Beijing 100191, China

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ABSTRACT

In the present paper, two kinds of adaptive output synchronization problems for a complex delayed dynamical network with output coupling are investigated, that is, the cases with positive definite output matrix and with semi-positive definite output matrix. For the former, by using adaptive control method, a sufficient condition is obtained to guarantee the output synchronization of the complex dynamical network. In addition, a pinning adaptive output synchronization criterion is also derived for such network model. Then we extend these results to the case when the output matrix is semi-positive definite. Finally, two numerical examples are provided to illustrate the effectiveness of the proposed results.

1. Introduction

In the real world, complex networks can be seen everywhere, and have been considered as a fundamental tool to understand dynamical behavior and the response of real systems such as food webs, communication networks, social networks, power grids, cellular networks, World Wide Web, metabolic systems, disease transmission networks, and many others [1,2]. The topology and dynamical behavior of various complex networks have been extensively studied by researchers [3,4]. Especially, as one of the most significant and interesting dynamical properties of the complex networks, synchronization has received much of the focus in recent years. So far, a great many important results on synchronization have been obtained for various complex networks such as time invariant, time-varying, and impulsive network models; see [5–20] and relevant references therein.

It should be noticed that the state synchronization of complex networks with state coupling was considered in these articles (see also the above-mentioned references). Practically, there are two kinds of coupling forms in complex networks: state coupling and output coupling. As we know, many phenomena in nature can be modeled as complex networks with output coupling [21,22]. Nevertheless, there are very few works on complex dynamical networks with output coupling [21–23]. To our knowledge, Jiang et al. [21] first introduced a complex network model with output coupling. Some conditions for synchronization were established based on the Lyapunov stability theory. In [22], Chen proposed a complex network model with output coupling and random sensor delay. A sufficient synchronization condition was given to ensure that the proposed network model is exponentially mean-square stable. One should note that the state synchronization was investigated in [21,22]. It is well known that the node state in complex networks is difficult to be observed or measured, even the node state cannot be observed or measured at all. Moreover, in many circumstances, only part states are needed to make the synchronization to come true. For these phenomena, it is more interesting to study the output synchronization of complex networks [23,24]. For instance, Wang and Wu [24] discussed the output synchronization of a class of impulsive complex dynamical networks with time-varying delay. By constructing suitable Lyapunov functionals, some useful conditions were obtained to guarantee the local and global exponential output synchronization of the impulsive complex networks. Unfortunately, very few authors have considered the output synchronization for complex networks with output coupling [23]. So, it is essential to further study the output synchronization of complex dynamical networks with time-varying delay and output coupling.

Recently, the synchronization control problem of complex dynamical networks has become a very hot topic in both theoretical research and practical applications. Among the existing results, some researchers focused on adaptive control [5,6,25–32] on complex dynamical networks by applying state feedback controllers. For instance, Zhou et al. [5] investigated the locally and globally adaptive synchronization of an uncertain complex dynamical network. In [26], Ji et al. proposed an adaptive control method to achieve the lag synchronization between uncertain complex dynamical network

* Corresponding author. Tel.: +86 10 82317301; fax: +86 10 82317332.
E-mail address: whn@buaa.edu.cn (H.-N. Wu).

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having delayed coupling and a nonidentical reference node. Wang et al. [32] proved that the states of a weighted complex dynamical network with time-varying delay can globally asymptotically synchronize onto a desired orbit under the designed controllers, and the adaptive controllers have strong robustness against asymmetric coupling matrix, time-varying weights, delays, and noise. However, the node state often cannot be observed or measured, which makes controller design very difficult. Moreover, these adaptive control strategies [5,6,25,27,28,30–32] are based on a special solution of an isolate node of the networks, which may be difficult to obtain in some engineering applications. To overcome these difficulties, some adaptive output feedback controllers are proposed in this paper.

Motivated by the above discussions, in this paper, we propose a complex delayed dynamical network with output coupling. The objective of this paper is to study the adaptive output synchronization of the proposed network model. By constructing appropriate Lyapunov functionals and utilizing adaptive control technique, some sufficient conditions on output synchronization are derived for the proposed network model.

The rest of this paper is organized as follows. In Section 2, our mathematical model of complex delayed dynamical network is presented and some preliminaries are given. The main results of this paper are given in Sections 3 and 4. In Section 5, two numerical examples are provided to illustrate the effectiveness of the theoretical results. Finally, Section 6 concludes the investigation.

2. Network model and preliminaries

Let $\mathbb{R}^n$ be the n-dimensional Euclidean space and $\mathbb{R}^{n\times m}$ be the space of $n\times m$ real matrices. $P\in\mathbb{R}^{n\times n}\geqslant 0$ ($P\in\mathbb{R}^{n\times n}\leqslant 0$) means that matrix P is symmetric and semi-positive (semi-negative) definite. $P\in\mathbb{R}^{n\times n}>0$ ($P\in\mathbb{R}^{n\times n}<0$) means that matrix P is symmetric and positive (negative) definite. I_n denotes the $n\times n$ real identity matrix. B^T denotes the transpose of matrix B. $\otimes$ denotes the Kronecker product of two matrices. $\lambda_m(\cdot)$ and $\lambda_M(\cdot)$ denote the minimum and the maximum eigenvalue of the corresponding matrix, respectively. $C([-\tau,0],\mathbb{R}^n)$ is a Banach space of continuous functions mapping the interval $[-\tau,0]$ into $\mathbb{R}^n$ with the norm $\|\phi\|_\tau=\sup_{-\tau\le\theta\le 0}\|\phi(\theta)\|$, where $\|\cdot\|$ is the Euclidean norm.

In this paper, we consider a complex network consisting of N identical nodes with diffusive and output coupling, in which each node is an n-dimensional dynamical system. The mathematical model of the network can be described as follows:

$$\begin{cases}\dot{x}_i(t)=f(x_i(t))+a\sum_{j=1}^{N}G_{ij}y_j(t-\tau(t))+u_i(t),\\ y_i(t)=Cx_i(t)\end{cases}\tag{1}$$

where $i=1,2,\ldots,N$. The function $f(\cdot)$, describing the local dynamics of the nodes, is continuous and capable of producing various rich dynamical behaviors; $x_i(t)=(x_{i1}(t),x_{i2}(t),\ldots,x_{in}(t))^T\in\mathbb{R}^n$ is the state variable of node i; $y_i(t)=(y_{i1}(t),y_{i2}(t),\ldots,y_{in}(t))^T\in\mathbb{R}^n$ is the output vector of node i; $u_i(t)\in\mathbb{R}^n$ is the control input; $\tau(t)$ is the time-varying delay with $0\le\tau(t)\le\tau$; a is a positive real number, which represents the overall coupling strength; the output matrix $C=\mathrm{diag}(c_1,c_2,\ldots,c_n)$ is a semi-positive definite matrix; $G=(G_{ij})_{N\times N}$ represents the topological structure of network and coupling strength between nodes, where G_{ij} is defined as follows: if there exists a connection between nodes i and j, then $G_{ij}=G_{ji}>0$; otherwise, $G_{ij}=G_{ji}=0$ $(i\neq j)$, and the diagonal elements of matrix G are defined by

$$G_{ii}=-\sum_{\substack{j=1\\ j\neq i}}^{N}G_{ij},\quad i=1,2,\ldots,N.$$

In this paper, we always assume that complex network (1) is connected. Since network (1) is connected in the sense of having no isolated clusters, which means that the coupling matrix G is irreducible. The initial condition associated with the complex network (1) is given in form

$$x_i(s)=\Phi_i(s),\quad y_i(s)=C\Phi_i(s)$$

where $i=1,2,\ldots,N$, $\Phi_i(s)\in C([-\tau,0],\mathbb{R}^n)$.

Remark 1. In [21], Jiang, Tang and Chen first introduced a complex network model with output coupling without time delay. However, time delays always exist in complex networks due to the finite speeds of transmission and/or the traffic congestion, and most of the delays are notable. So it is crucial for us to take the delay into the consideration when we study complex networks. Moreover, absolute constant delay may be scarce and delays are frequently varied with time. Therefore, a complex network model with output coupling and time-varying delay is considered in this paper. In addition, another issue of importance deserving attention is the fact that many existing complex network models with state coupling can be represented by (1) through an appropriate choice of the parameters, e.g., see also [33–37].

Remark 2. In recent years, a lot of efforts have been made to study the adaptive state synchronization of complex networks with state coupling [25–32]. Unfortunately, the node state in complex networks is difficult to be observed or measured, even the node state cannot be observed or measured at all. Moreover, in many circumstances, only part states are needed to make the synchronization to come true. Therefore, it is more interesting to study the output synchronization of complex networks. To the best of our knowledge, this is the first paper to consider the adaptive output synchronization of complex delayed dynamical networks with output coupling, which is a very important and interesting problem.

In what follows, we introduce some useful definitions and lemmas.

Definition 2.1. The complex network (1) is said to achieve output synchronization if

$$\lim_{t\to+\infty}\|y_i(t)-\frac{1}{N}\sum_{j=1}^{N}y_j(t)\|=0\quad\text{for all } i=1,2,\ldots,N.$$

Lemma 2.1 (*See Yu et al. [38]*). *Suppose that $G=(G_{ij})_{N\times N}$ is a real symmetric and irreducible matrix, where*

$$G_{ij}\ge 0\ (i\neq j),\quad G_{ii}=-\sum_{\substack{j=1\\ j\neq i}}^{N}G_{ij}.$$

Then,

(1) 0 *is an eigenvalue of matrix G with multiplicity* 1 *and all the other eigenvalues of G are strictly negative.*
(2) *The largest nonzero eigenvalue $\lambda_2(G)$ of the matrix G satisfies*
$$\lambda_2(G)=\max_{x^T\mathbf{1}_N=0,x\neq 0}\frac{x^TGx}{x^Tx}.$$
(3) *For any $\eta=(\eta_1,\eta_2,\ldots,\eta_N)^T\in\mathbb{R}^N$,*
$$\eta^TG\eta=-\frac{1}{2}\sum_{i=1}^{N}\sum_{j=1}^{N}G_{ij}(\eta_i-\eta_j)^2.$$

3. Adaptive output synchronization of complex delayed dynamical network with positive definite output matrix

This section discusses the adaptive output synchronization of complex delayed dynamical network (1) with positive definite output matrix. Several output synchronization criteria are obtained by