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Stochastic adaptive optimal control of under-actuated robots using neural networks ☆

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A B S T R A C T

Stochastic adaptive optimal control of robotic manipulators with a passive joint which has neither an actuator nor a brake is investigated. Firstly, the under-actuated system is decomposed into two subsystems with the first $n-1$ joints subsystem fully actuated while the second one unactuated. Secondly, a reference model for the first subsystem is derived by using the Linear Quadratic Regulator (LQR) optimization approach which guarantees the motion tracking and achieves the minimized moving accelerations. Instead of leaving the unactuated joint dynamics uncontrolled, the reference trajectory for the last joint is designed to indirectly affect the movements such that the desired trajectory can be achieved. Radial Basis Function neural networks (RBFNNs) have been employed to design the adaptive reference control and to construct a reference trajectory generator for the last joint. The stability and the optimal tracking performance in finite time have been rigorously established by theoretic analysis. Simulation studies show the effectiveness of the proposed control approach.

1. Introduction

Under-actuated robots have received considerable research attention in the last two decades (see, e.g., [1–5,7–20]). In contrast to conventional robot for which each joint has one actuator and its degree of freedom equals the number of actuators, an under-actuated robot has passive joints equipped with no actuators. Though the passive joints are not actuated but they can be controlled by using the dynamic coupling with the active joints, i.e., these passive joints can be indirectly driven by other active joints [3]. The zero torque at the passive joints results in a second-order nonholonomic constraint. In robotics, nonholonomic constraints formulated as nonintegrable differential equations containing time-derivatives of generalized coordinates (velocity, acceleration etc.) are mainly studied (see, e.g., [8–17]).

One of the most interesting consequences of nonholonomy in robotic systems is that it allows one to control the configuration of the whole mechanism with a reduced number of inputs [19]. These mechanisms arise in a number of situations, ranging from non-prehensile manipulation [21] to robot acrobatics [22], from legged locomotion [23] to surgical robotics [24], from free-floating robots [25] to manipulators with flexibility concentrated at the joints [26] or distributed along the links [27]. Another particularly interesting example is that of manipulators to be operated in spite of actuator failure [28]. In this latter case, in order to preserve active operation of the system, one needs to take into account the arising non-holonomic constraints at both the trajectory planning and the control level.

To some extent, the under-actuation structure is made possible for the robots to reduce the weight, energy consumption, and cost of manipulators. Application to the tasks involving an impact, e.g., hitting or hammering an object, will be useful since the impact causes no damage to the joint actuators. It can also contribute to fault tolerance of fully actuated manipulators when some of the joint actuators fail. However, the price of this benefit is that planning and controlling trajectories become more difficult than the holonomic cases [20]. Since these robots usually have nonholonomic second-order constraints, the control problems are challenging (see e.g., [1–5,7]). Therefore, it is necessary to combine the limited number of inputs skillfully in order to control all the coordinates. So how to efficiently control these kinds of nonholonomic systems becomes an interesting research area. However, most of these existing results applied the approach of partial feedback linearization or only considered the deterministic cases. As well as we know, the disadvantage of feedback linearization is that

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Nomenclature

q, q_a, q_b	the coordinate vectors of under-actuated robot, subsystems Σ_a and Σ_b, respectively
q_{am}, q_{bm}	the responses of given reference model of subsystems Σ_a and Σ_b, respectively
q_{ad}, q_{bd}	the desired responses of subsystems Σ_a and Σ_b, respectively
e_a, e_b	the tracking error of subsystems Σ_a and Σ_b
$\mathcal{M}, \mathcal{G}, \mathcal{C}$	the inertia, damper matrix and the Coriolis and centrifugal torque
$W^*, \hat{W}, \tilde{W}$	the NN ideal, estimated and estimation error weight vectors, respectively
$S(\cdot)$	NN Basis function vector
τ_a	the torque of the actuated joints
w	an n-dimensional independent standard Wiener process
$\mathcal{L}$	the infinitesimal generator
$(B)_i$	the ith column of matrix B
$0_{[m]}$, $0_{[m,n]}$	zero column vector of dimension m, and zero matrix of m rows and n columns, respectively
$\|\cdot\|$	vector's and matrix's L^2-norm

input constraints are not considered explicitly as part of the controller design [49]. At the same time, the stochastic noises exist almost everywhere (see, e.g., [42–45]). Sometimes engineers and researchers just ignored them for the simplification of the research when these disturbances are ignorable compared to the research problem.

Motivated by the above discussion, in this paper we consider the stochastic control problem for an n-joints under-actuated system with a passive last joint. It is worth noting that there is a critical problem in the uncertainties of the system dynamics, i.e., either external unpredictable disturbances or internal uncertain dynamics. From this point of view, the development of parameter estimation based adaptive control or function approximation based NN control becomes an important issue (see, e.g., [4,5,29–37]). It is worth to mention that novel adaptive controller that compensates for both parametric and nonparametric uncertainties has been designed in [6]. In [4] and [5], NN has been effectively employed for control design of underactuated wheeled robot. In this paper, we use NN to generate both the adaptive model reference controller and a reference trajectory generator. The main contributions of this paper lie on the following:

(i) A reference model for the first $n-1$ joints subsystems of the n-joints under-actuated system is derived by using the LQR optimization approach which guarantees the motion tracking and achieves the minimized moving accelerations.
(ii) Instead of leaving the unactuated joint dynamics uncontrolled, the reference trajectory for the last joint is designed to indirectly affect the movements such that the desired trajectory can be achieved.
(iii) RBFNNs have been employed to design the adaptive reference control in order to make the controlled dynamics to match the reference model dynamics in finite time. At the same time, RBFNNs have been used to construct a reference trajectory generator for the last joint as well.

The rest of this paper is organized as follows. In Section 2, preliminary knowledge of RBFNN approximation, LQR optimization and stochastic stability and stochastic finite-time attractiveness is presented. In Section 3, the under-actuated system is decomposed into two subsystems with the first $n-1$ joints subsystem fully actuated while the last joint subsystem unactuated. In Section 4, a reference model for the first $n-1$ joints subsystems is derived by using the Linear Quadratic Regulator (LQR) optimization approach which guarantees the motion tracking and achieves the minimized moving accelerations. A reference trajectory generator using RBFNN for the last joint is designed in Section 5 such that the forward velocity is indirectly manipulated to follow its desired trajectory. In Section 6, simulation studies are carried out to verify the effectiveness of the proposed method. Concluding remarks are given in Section 7.

2. Preliminaries

2.1. RBFNN approximation

In this paper, an unknown smooth nonlinear function $\phi(z): R^m \to R$ will be approximated on a compact set D by the following RBF neural network [40]

$$\phi(z) = W^{*T} S(z) + \delta(z), \tag{1}$$

where $z \in D \subset R^m$ is the input vector of dimension m; $\delta(z)$ denotes the NN inherent approximation error; $S(z) = [s_1(z), \ \ldots, s_l(z)]^T : D \to R^l$ is a known smooth vector function with the NN node number $l > 1$; Basis function $s_i(z)(1 \le i \le l)$ is chosen as the commonly used Gaussian function with the form $s_i(z) = \exp[-(z-\mu_i)^T (z-\mu_i)/\eta_i^2]$, where $\mu_i = [\mu_{i1}, \ldots, \mu_{im}]^T \in D$ is the center of the receptive field and $\eta_i > 0$ is the width of the Gaussian function; the ideal weight vector $W^* = [W_1^*, \ldots, W_l^*]^T$ is defined as the optimal value of $\hat{W}$ that could minimize the approximation error $\delta(z)$ for all $z \in D$, i.e.,

$$W^* \stackrel{\text{def}}{=} \arg \min_{\hat{W} \in R^l} \left\{ \sup_{z \in D} |\phi(z) - \hat{W}^T S(z)| \right\}, \tag{2}$$

In many previous published works, the approximation error $\delta(z)$ is assumed to be bounded by a fixed constant, i.e.,

Assumption 1. There exists an unknown positive constant δ such that

$$|\delta(z)| \le \epsilon. \tag{3}$$

Remark 1. From the universal approximation results for neural networks [38], it is known that the constant ϵ can be made arbitrarily small by increasing the NN nodes number l. Also it should be noticed that the optimal weight W^* is an unknown artificial value. In practice, we use estimated weight $\hat{W}$ instead of W^* to approximate a continuous nonlinear function, where $\hat{W}$ is derived from a learning law [4].

Lemma 1 (*Munkres [39]*). *Consider a C^r function $f: R^{k+n} \to R^n$ with $f(a,b) = \mathbf{0}_{[n]}$ and $\mathrm{rank}(Df(a,b)) = n$ where $Df(a,b) = (\partial f(x,y)/\partial y)|_{(x,y)=(a,b)} \in R^{n \times n}$. Then, there exist a neighborhood A of a in R^k and a unique C^r function $g: A \to R^n$ such that $g(a) = b$ and $f(x, g(x)) = \mathbf{0}_{[n]}, \ \forall x \in A$.*

Remark 2. This lemma is called the implicit function theorem, which will be used to warrant the existence of the implicit function f^* later (refer to Section 5).

To easily approximate a nonlinear matrix function with each element a unknown scalar function, we use the following block matrix operators $\langle T \rangle$ and $\langle \cdot \rangle$ as introduced in [4] and [40].