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New global robust stability condition for uncertain neural networks with time delays

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ABSTRACT

In this paper, we investigate the robust stability problem for the class of delayed neural networks under parameter uncertainties and with respect to nondecreasing activation functions. Firstly, some new upper bound values for the elements of the intervalized connection matrices are obtained. Then, a new sufficient condition for the existence, uniqueness and global asymptotic stability of the equilibrium point for this class of neural networks is derived by constructing an appropriate Lyapunov–Krasovskii functional and employing homeomorphism mapping theorem. The obtained result establishes a new relationship between the network parameters of the neural system and it is independent of the delay parameters. A comparative numerical example is also given to show the effectiveness, advantages and less conservatism of the proposed result.

1. Introduction

In the recent years, the analysis of dynamical behaviors of neural networks has been an important issue due to their potential and important applications in many engineering areas such as image and signal processing, associative memory, pattern recognition, parallel computation, control theory and optimization. In particular, the focus has been on the stability problem in neural networks since stability analysis is of prime importance in applications of neural networks for solving practical engineering problems. However, when studying stability properties of neural networks, some critical issues must be taken into consideration. One of these issues is the effect of time delays which are due to the finite switching speed of neurons and amplifiers in the process of implementation of neural networks. It is known that the existence of time delays may lead to oscillation and instability of neural networks, which may cause a negative effect on the results of the intended applications of neural networks. Another critical issue to be considered in the stability analysis of neural systems is the deviations in the values of the electronic components during the electronic implementation of neural networks. Therefore, stability analysis of neural networks at the presence of time delays and parameter uncertainties becomes an important task to be achieved. In order to establish the desired stability properties of neural networks with time delays under the parameter uncertainties, one must ensure the robust stability of delayed neural networks. Robust stability analysis of different models of neural networks has been carried out by many researchers and various stability results have been reported in the past literature (see e.g. [1–31] and the references therein). The previously reported literature results have mainly employed Lyapunov–Krasovskii functionals and used different methods such as delay decomposition approach, convex combination techniques, free-weighting matrix approach, fixed point theorem, homeomorphism mapping theorem, linear matrix inequalities and M-Matrix condition in the derivation of the robust stability of various classes of neural networks with respect to different classes of activation functions (see e.g. [1–31]and the references therein). The main question to be addressed in the analysis of robust stability of neural networks is the problem of finding some upper bound parameters relying on the values of the elements of the intervalized interconnection matrices of neural networks, which requires to get involved in the analysis of some certain properties of interval matrices. Some of the previously reported results have defined different upper bound norms for the intervalized interconnection matrices, which have been successfully applied to derive new sufficient conditions for robust stability of delayed neural networks [2–5]. Motivated by the results of [2–5], we carry out an analysis of interval matrices whose elements are defined in a certain interval and derive a new upper bound for the norm of the intervalized interconnection matrices of neural networks. Then, by employing this new result, and using a generalized Lyapunov–Krasovskii functional and homeomorphism

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mapping theorem, we study the robust stability problem for the class of neural networks with discrete time delays with respect to nondecreasing activation functions and derive a new easily verifiable delay-independent sufficient condition for the existence, uniqueness and global robust asymptotic stability of this class of neural networks. A comparative numerical example is also given to demonstrate the effectiveness of the obtained result.

We use the notations: Throughout this paper, the superscript T represents the transpose. I stands for the identity matrix of appropriate dimension. For the vector $v=(v_1,v_2,\dots,v_n)^T$, $|v|$ will denote $|v|=(|v_1|,|v_2|,\dots,|v_n|)^T$. For any real matrix $Q=(q_{ij})_{n\times n}$, $|Q|$ will denote $|Q|=(|q_{ij}|)_{n\times n}$, and $\lambda_m(Q)$ and $\lambda_M(Q)$ will denote the minimum and maximum eigenvalues of Q, respectively. If $Q=(q_{ij})_{n\times n}$ is a symmetric matrix, then, $Q>0$ will imply that Q is positive definite, i.e., Q has all real and positive eigenvalues. We also recall the following vector and matrix norms:

$$\|v\|_1=\sum_{i=1}^n |v_i|,\quad \|v\|_2=\sqrt{\sum_{i=1}^n v_i^2},\quad \|v\|_\infty=\max_{1\le i\le n}|v_i|$$

$$\|Q\|_1=\max_{1\le i\le n}\sum_{j=1}^n |q_{ji}|,\quad \|Q\|_2=[\lambda_{\max}(Q^TQ)]^{1/2},$$

$$\|Q\|_\infty=\max_{1\le i\le n}\sum_{j=1}^n |q_{ij}|$$

2. Model description and preliminaries

In this paper, we will consider delayed neural network model whose dynamical behavior is governed by the following sets of nonlinear differential equations:

$$\frac{dx_i(t)}{dt}=-c_ix_i(t)+\sum_{j=1}^n a_{ij}f_j(x_j(t))+\sum_{j=1}^n b_{ij}f_j(x_j(t-\tau_j))+u_i,\quad i=1,2,\dots,n \tag{1}$$

where n is the number of the neurons, $x_i(t)$ denotes the state of the neuron i at time t, $f_i(\cdot)$ denote activation functions, a_{ij} and b_{ij} denote the strengths of connectivity between neurons j and i at time t and $t-\tau_j$, respectively; τ_j represents the time delay required in transmitting a signal from the neuron j to the neuron i, u_i is the constant input to the neuron i, c_i is the charging rate for the neuron i.

The matrix–vector form of the neural network model (1) is the following:

$$\dot{x}(t)=-Cx(t)+Af(x(t))+Bf(x(t-\tau))+u \tag{2}$$

where $x(t)=(x_1(t),x_2(t),\dots,x_n(t))^T\in R^n$, $A=(a_{ij})_{n\times n}$, $B=(b_{ij})_{n\times n}$, $C=diag(c_i>0)$, $u=(u_1,u_2,\dots,u_n)^T\in R^n$, $f(x(t))=(f_1(x_1(t)), f_2(x_2(t)),\dots,f_n(x_n(t)))^T\in R^n$ and $f(x(t-\tau))=(f_1(x_1(t-\tau_1)), f_2(x_2(t-\tau_2)),\dots,f_n(x_n(t-\tau_n)))^T\in R^n$.

The parameters $A=(a_{ij})_{n\times n}$, $B=(b_{ij})_{n\times n}$, $C=diag(c_i>0)$ in neural system (1) are assumed to be norm-bounded and satisfy the following conditions:

$$C_I=[\underline{C},\overline{C}]=\{C=diag(c_i):0<\underline{c}_i\le c_i\le\overline{c}_i,i=1,2,\dots,n\}$$
$$A_I=[\underline{A},\overline{A}]=\{A=(a_{ij})_{n\times n}:\underline{a}_{ij}\le a_{ij}\le\overline{a}_{ij},i,j=1,2,\dots,n\}$$
$$B_I=[\underline{B},\overline{B}]=\{B=(b_{ij})_{n\times n}:\underline{b}_{ij}\le b_{ij}\le\overline{b}_{ij},i,j=1,2,\dots,n\} \tag{3}$$

We will assume that the functions f_i satisfy the following condition:

$$0\le\frac{f_i(x)-f_i(y)}{x-y}\le k_i,\quad i=1,2,\dots,n,\quad \forall x,y\in R, x\ne y$$

where the k_i's are some positive constants. This class of functions will be denoted by $f\in\mathcal{K}$.

In the light of (3), the robust stability of the equilibrium point of system (1) is stated as follows:

Definition 1. The neural network defined by (2) with the parameter ranges defined by (3) is globally asymptotically robust stable if the unique equilibrium point $x^*=(x_1^*,x_2^*,\dots,x_n^*)^T$ of the neural system (2) is globally asymptotically stable for all $C\in C_I$, $A\in A_I$ and $B\in B_I$.

The following fact is stated in [1] :

Fact 1 (*Arik [1]*). If $A=(a_{ij})_{n\times n}$ and $B=(b_{ij})_{n\times n}$ satisfy the parameter ranges defined by (3) and have bounded norms, then, there exist some positive constants $\sigma(A)$ and $\sigma(B)$

$$\|A\|_2\le\sigma(A)\quad\text{and}\quad\|B\|_2\le\sigma(B)$$

The following results that define different upper bound norms for the interval matrices will be needed in the context of comparison of our result with previously reported results in the literature:

Lemma 1 (*Faydasicok and Arik [2]*). *Let B be any real matrix defined by*

$$B\in B_I=[\underline{B},\overline{B}]=\{B=(b_{ij})_{n\times n}:\underline{b}_{ij}\le b_{ij}\le\overline{b}_{ij},i,j=1,2,\dots,n\}$$

Define $B^*=\frac{1}{2}(\overline{B}+\underline{B})$ *and* $B_*=\frac{1}{2}(\overline{B}-\underline{B})$. *Let*

$$\sigma_1(B)=\sqrt{\||B^{*T}B^*|+2|B^{*T}|B_*+B_*^TB_*\|_2}$$

Then, the following inequality holds:

$$\|B\|_2\le\sigma_1(B).$$

Lemma 2 (*Chen et al. [3]*). *Let B be any real matrix defined by*

$$B\in B_I=[\underline{B},\overline{B}]=\{B=(b_{ij})_{n\times n}:\underline{b}_{ij}\le b_{ij}\le\overline{b}_{ij},i,j=1,2,\dots,n\}$$

Define $B^*=\frac{1}{2}(\overline{B}+\underline{B})$ *and* $B_*=\frac{1}{2}(\overline{B}-\underline{B})$. *Let*

$$\sigma_2(B)=\|B^*\|_2+\|B_*\|_2$$

Then, the following inequality holds:

$$\|B\|_2\le\sigma_2(B)$$

Lemma 3 (*Ensari and Arik [4]*). *Let B be any real matrix defined by*

$$B\in B_I=[\underline{B},\overline{B}]=\{B=(b_{ij})_{n\times n}:\underline{b}_{ij}\le b_{ij}\le\overline{b}_{ij},i,j=1,2,\dots,n\}$$

Define $B^*=\frac{1}{2}(\overline{B}+\underline{B})$ *and* $B_*=\frac{1}{2}(\overline{B}-\underline{B})$. *Let*

$$\sigma_3(B)=\sqrt{\|B^*\|_2^2+\|B_*\|_2^2+2\|B_*^T|B^*|\|_2}$$

Then, the following inequality holds:

$$\|B\|_2\le\sigma_3(B).$$

Lemma 4 (*Singh [5]*). *Let B be any real matrix defined by*

$$B\in B_I=[\underline{B},\overline{B}]=\{B=(b_{ij})_{n\times n}:\underline{b}_{ij}\le b_{ij}\le\overline{b}_{ij},i,j=1,2,\dots,n\}$$

Define $\hat{B}=(\hat{b}_{ij})_{n\times n}$ *with* $\hat{b}_{ij}=\max\{|\underline{b}_{ij}|,|\overline{b}_{ij}|\}$. *Let*

$$\sigma_4(B)=\|\hat{B}\|_2$$

Then, the following inequality holds:

$$\|B\|_2\le\sigma_4(B).$$

In what follows, we present two lemmas whose results will play an important role in determining our main result for the existence, uniqueness and global robust asymptotic stability of the equilibrium point of neural system (1).