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On hybrid control of higher order Boolean networks

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ABSTRACT

In the present paper, we discuss the stabilization and controllability issues of the hybrid switching and impulsive higher order Boolean networks. First, the hybrid switching and impulsive controller is introduced. Second, the algebraic form of the system is given. Third, sufficient and necessary conditions for the stabilization and controllability of the system are presented, respectively. At last, an illustrative example shows the effectiveness of the main results.

1. Introduction

Boolean networks were firstly proposed by Kauffman in 1969 [1] to model and quantitatively describe the gene regulatory. Since then, the study of Boolean networks has attracted great attention from researchers from many research fields, such as biology, system science and so on, see [2–4] and references therein. In Boolean networks, the state of a gene is described as active (1) or inactive (0). And the state of a gene is determined by its neighborhood, which is connected by Boolean functions. The researchers had been lack of systematic methods to investigate them until the semi-tensor product of matrices was proposed in [5]. Since then, using the new tool, many basic problems in control field have been studied, such as the stability and stabilization [6,7], the controllability and observation [8–12], the realization [13], the optimal control problem [14,15,11] and so on.

When the updated values of logical networks depend only on current time, the Boolean networks can be used. However, when the updated values depend on the past μ values, it is not appropriate to use Boolean networks as usual. Instead, we can use the higher order Boolean networks. This is one kind of delay phenomena, which is common in real world, see [16–18] for example. There are few results about higher order Boolean control networks in the literature. In [14], the authors considered the optimal control problem; in [12], the authors investigated the controllability issue and the authors discussed the global stability of such a system with constrained input states in [19].

Stability and controllability issues are fundamental concepts in control theory field. And, now there has been a great lot of literature studying on both topics, see [17,20,21,16] and references therein. Referring to the controllability of higher order Boolean networks, there exists no results except [14,12,19,18]. At the same time, due to the naturally arising of impulsive systems in many fields, such as biological systems [22], mechanical systems [23], and so on, control problems of dynamical systems with impulsive effects have been of continuous interest. A lot of fundamental problems and results have been reported in the literature, see [24,25] for example. In [26], the authors investigated the control of hybrid impulsive and switching systems and [27] discussed the hybrid control of stochastic nonlinear Markovian switching systems. To the best of our knowledge, there exists no result studying the hybrid control of higher order Boolean networks. This is the motivation for us to study the hybrid control of higher order Boolean control networks. By a series of transforms, a hybrid switching and impulsive system is obtained and the algebraic form is also presented by using the semi-tensor product. This system is newly proposed and the investigated problems are meaningful.

In the present paper, we consider the hybrid switching and impulsive system, using the semi-tensor product. Firstly, the algebraic form of such kind of systems is presented, which assistants in studying the stabilization and controllability. Then, sufficient and necessary conditions are obtained for the stabilization and controllability issues, respectively.

The rest of the paper is organized as follows. In Section 2, some preliminaries, including some basic concepts, notations and

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propositions, used in the paper are introduced. A hybrid controller consisting switching and impulsive effects for higher order Boolean networks is introduced in Section 3, and the algebraic form of the system is obtained. In Sections 4 and 5, the definition for the stabilization and controllability of the system is provided. And then, sufficient and necessary conditions are given for the stabilization and controllability, respectively. Section 6 shows an example to illustrate the main results obtained in the paper. Lastly, conclusions are presented in Section 7.

2. Preliminaries

In this section, we introduce some necessary preliminaries on semi-tensor product, the crucial tool in the present paper. The matrix product is assumed to be the semi-tensor product in the following discussion. Following is a review of basic concepts, notations and proposition in [5].

Definition 2.1 (*Cheng et al. [5]*). (1) Let X be a row vector of dimension np, and $Y=[y_1, y_2, \dots, y_p]^T$ be a column vector of dimension p. Then we split X into p equal-size blocks as $X^1, \dots, X^p$, which are $1\times n$ rows. Define the semi-tensor product, denoted by $\ltimes$, as

$$\begin{cases} X\ltimes Y = \sum_{i=1}^{p} X^i y_i \in R^n, \\ Y^T \ltimes X^T = \sum_{i=1}^{p} y_i (X^i)^T \in R^n. \end{cases}$$

(2) Let $M\in\mathcal{M}_{m\times n}$ and $N\in\mathcal{M}_{p\times q}$. If n is a factor of p or p is a factor of n, then $C=M\ltimes N$ is called the semi-tensor product of M and N, where C consists of $m\times q$ blocks as $C=(C^{ij})$, and

$$C^{ij} = M^i \ltimes N_j, \quad i=1,2,\dots,m;\ j=1,2,\dots,q,$$

where $M^i = Row_i(M)$ denotes the ith row of the matrix M and $N_j = Col_j(N)$ denotes the jth column of the matrix N.

Remark 2.1. The semi-tensor product is a generalization of the conventional matrix product. The semi-tensor product of two matrices $M\in\mathcal{M}_{m\times n}$ and $N\in\mathcal{M}_{p\times q}$ becomes the conventional matrix product for $n=p$.

Next, notations used in the following paper are given.

(1) $\mathcal{D}:=\{0,1\}$, $\Delta_n:=\{\delta_n^1, \dots, \delta_n^n\}$, where δ_n^k denotes the k-th column of the identity matrix I_n.
(2) Let $\mathcal{M}_{n\times s}$ denote the set of $n\times s$ matrices. Assume that a matrix $M=[\delta_n^{j_1}\ \delta_n^{j_2}\ \dots\ \delta_n^{j_s}]\in\mathcal{M}_{n\times s}$, i.e., its columns, $Col(M)\subset\Delta_n$, then M is called a logical matrix. The set of $n\times m$ logical matrices is denoted by $\mathcal{L}_{n\times m}$.
(3) To use a matrix expression, we identify $1\sim\delta_2^1$, $0\sim\delta_2^2$. Using this transformation, a logical function $f:\mathcal{D}^k\to\mathcal{D}$ becomes a function $f:\Delta_2^k\to\Delta_2$.
(4) Define a swap matrix $W_{[m,n]}$, which is an $mn\times mn$ matrix constructed in the following way: label its columns by $(11, 12, \dots, 1n, \dots, m1, m2, \dots, mn)$ and its rows by $(11, 21, \dots, m1, \dots, 1n, 2n, \dots, mn)$. Then its element in the position $((I,J),(i,j))$ is assigned as

$$w_{(I,J),(i,j)} = \delta_{i,j}^{I,J} = \begin{cases} 1, & I=i \text{ and } J=j, \\ 0, & \text{otherwise.} \end{cases}$$

Using the matrix expression, the following proposition could be obtained.

Proposition 2.1 (*Cheng et al. [5]*). *(1) Let $f(x_1, x_2, \dots, x_k)$ be a logical function, there exists a unique 2×2^k matrix M_f, called the structure matrix, such that*

$$f(x_1, x_2, \dots, x_k) = M_f x,$$

where $x=\ltimes_{i=1}^k x_i \in\Delta_{2^k}$, $M_f\in\mathcal{L}_{2\times 2^k}$.

(2) Consider a fundamental unary logical function: Negation, $\neg P$, and four fundamental binary logical functions: Disjunction, $P\vee Q$; Conjunction, $P\wedge Q$; Conditional, $P\to Q$; Biconditional, $P\leftrightarrow Q$. Their structure matrices are as follows:

$$M_\neg=\delta_2[2,1];\quad M_\vee=\delta_2[1,1,1,2];\quad M_\wedge=\delta_2[1,2,2,2];$$
$$M_\to=\delta_2[1,2,1,1];\quad M_\leftrightarrow=\delta_2[1,2,2,1].$$

(3) Let $x\in R^t$ and A is a given matrix, then $xA=(I_t\otimes A)x$, where "$\otimes$" denotes the Kronecker product.

(4) Let

$$E_d = \begin{pmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \end{pmatrix},$$

then for any two logical variables $X, Y\in\Delta_2$, $E_d XY = Y$ or $E_d W_{[2,2]} XY = X$, where $W_{[2,2]}$ is the swap matrix.

3. Problem formulation

In general, a higher order Boolean network can be written as

$$\begin{cases} x_1(t+1) = f_1(x_1(t-\mu+1), \dots, x_n(t-\mu+1), \dots, x_1(t), \dots, x_n(t)), \\ x_2(t+1) = f_2(x_1(t-\mu+1), \dots, x_n(t-\mu+1), \dots, x_1(t), \dots, x_n(t)), \\ \vdots \\ x_n(t+1) = f_n(x_1(t-\mu+1), \dots, x_n(t-\mu+1), \dots, x_1(t), \dots, x_n(t)), \\ t\ge\mu-1, \end{cases} \tag{1}$$

where $x_i(t)\in\mathcal{D}$, $i=1,2,\dots,n$ are Boolean variables and $f_i:\mathcal{D}^{\mu n}\to\mathcal{D}$, $i=1,2,\dots,n$ are logical functions.

Correspondingly, the Boolean networks with control input can be described as

$$\begin{cases} x_1(t+1) = f_1(x_1(t-\mu+1), \dots, x_n(t-\mu+1), \dots, x_1(t), \\ \quad \dots, x_n(t)) \oplus v_1(t), \\ x_2(t+1) = f_2(x_1(t-\mu+1), \dots, x_n(t-\mu+1), \dots, x_1(t), \\ \quad \dots, x_n(t)) \oplus v_2(t), \\ \vdots \\ x_n(t+1) = f_n(x_1(t-\mu+1), \dots, x_n(t-\mu+1), \dots, x_1(t), \\ \quad \dots, x_n(t)) \oplus v_n(t), \\ t\ge\mu-1, \end{cases} \tag{2}$$

where $v_i(t)$ is the controller to be designed and "$\oplus$" denotes "Exclusive OR" in logical calculation. We construct a hybrid controller $v_i(t) = v_{i_1}(t)\oplus v_{i_2}(t)$ as follows:

$$\begin{aligned} v_{i_1}(t) = h_i^{p_t}(&x_1(t-\mu+1), \dots, x_n(t-\mu+1), \dots, \\ &x_1(t), \dots, x_n(t)) \wedge (1-\delta(t-\tau_k+1)), \end{aligned} \tag{3}$$

$$\begin{aligned} v_{i_2}(t) = g_i(&x_1(t-\mu+1), \dots, x_n(t-\mu+1), \dots, \\ &x_1(t), \dots, x_n(t)) \wedge \delta(t-\tau_k+1), \end{aligned} \tag{4}$$

where $h_i^{p_t}(x_1(t-\mu+1), \dots, x_n(t-\mu+1), \dots, x_1(t), \dots, x_n(t))$ and $g_i(x_1(t-\mu+1), \dots, x_n(t-\mu+1), \dots, x_1(t), \dots, x_n(t))$ are logical functions. $\delta(\cdot)$ is the Dirac function. Furthermore, $\tau_1<\tau_2<\cdots<\tau_k<\cdots$ and $\lim_{k\to\infty}\tau_k=\infty$.

From (3), $v_{i1}(t) = h_i^{p_t}(x_1(t-\mu+1), \dots, x_n(t-\mu+1), \dots, x_1(t), \dots, x_n(t))$ for $t\ne\tau_k-1$. This means that the controller changes its values at every time $t(\ne\tau_k-1)$. Namely, this is a switching controller. From (4), $v_{i_2}(t) = g_i(x_1(t-\mu+1), \dots, x_n(t-\mu+1), \dots, x_1(t), \dots, x_n(t))$ for $t=\tau_k-1$ and $v_{i_2}(t)=0$ for else. It is an impulsive control. Above all, one can see that the controller designed is a