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Fully fuzzy polynomial regression with fuzzy neural networks

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ABSTRACT

In this paper a polynomial fuzzy regression model with fuzzy independent variables and fuzzy parameters is discussed. Within this paper the fuzzy neural network model is used to obtain an estimate for the fuzzy parameters in a statistical sense. Based on the extension principle, a simple algorithm from the cost function of the fuzzy neural network is proposed, in order to find the approximate parameters. Finally, we illustrate our approach by some numerical examples.

1. Introduction

Regression is a very powerful methodology for forecasting, which is considered as an essential component of successful OR applications. It is applied to evaluate the functional relationship between the dependent and independent variables. Fuzzy regression analysis is an extension of the classical regression analysis in which some elements of the model are represented by fuzzy numbers. Fuzzy regression methods have been successfully applied to various problems such as forecasting [6,7,34,17,35] and engineering [15]. Thus, it is very important to develop numerical procedures that can appropriately treat fuzzy regression models.

In the literature, several papers have addressed the issue of regression under a fuzzy environment. Tanaka et al. [32] first formulated a problem of fuzzy regression. They considered the fuzzy linear regression model

$$Y(x) = A_0 + A_1 x_1 + \cdots + A_n x_n \tag{1}$$

with symmetric triangular fuzzy parameters A_i chosen to match given n-dimensional input vector $x^j = (x_1^j, x_2^j, \ldots, x_n^j)$ with fuzzy output $y^j, j = 1, 2, \ldots, m$. The parameters in Eq. (1) were chosen, through a linear programming solution method to meet for each input–output pair (x^j, y^j)

$$[\overline{y}^j]_h \subset [\overline{Y}(x^j)]_h, \quad j = 1, 2, \ldots, m, \tag{2}$$

where $[\overline{Y}(x)]_h$ is the h-cut for a specified level h. The objective was to minimize the total spread of the fuzzy parameters subject to the support of the estimated values that cover the support of the observed values for a certain h-level. This technique was extended by Tanaka and Ishibuchi to fuzzy numbers with quadratic membership function [30] and fuzzy numbers defined by a more general shape function $L(\cdot)$ [29].

The problem was simplified and recast as linear interval regression by Ishibuchi and Tanaka in [12]. These interval regression models are closely connected to standard linear fuzzy regression in Eqs. (1) and (2). In interval regression, the linear programming problem is [4]

$$\begin{aligned} &\text{Minimize} \quad y_w(x^1) + y_w(x^2) + \cdots + y_w(x^m) \\ &\text{subject to} \quad [\overline{y}^j]_h \subset [\overline{Y}(x^j)]_h, \quad j = 1, 2, \ldots, m, \end{aligned} \tag{3}$$

where $y_w(x)$ is the width of the interval

$$Y(x) = A_0 + A_1 x_1 + \cdots + A_n x_n \tag{4}$$

and Y and A_i are interval variables. Fuzzy models with trapezoidal membership functions are easily derived from these interval models [13].

Later, Tanaka [29], Tanaka and Watada [33] and Tanaka et al. [31] made some improvements. Kao and Chyu [14] concept of least squares which was widely applied in the classical regression analysis was adopted to determine the regression coefficients. Ishibuchi et al. [10] proposed a learning algorithm of fuzzy neural networks with triangular fuzzy weights and Hayashi et al. [9] fuzzified the delta rule. Buckley and Eslami [5] consider neural net solutions to fuzzy problems. Recently, the fuzzy neural network model (FNNM) successfully used for solving fuzzy polynomial equation and systems of fuzzy polynomials [1,2], approximate solution of fuzzy linear systems and fully fuzzy linear systems [19,25,26] and fuzzy differential [18,21]. Recently, Mosleh et al. [22,20] proposed a learning algorithm

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of fuzzy neural network with crisp inputs, fuzzy weights and fuzzy output for adjusting fuzzy weights of the fuzzy linear regression model of the form

$$\overline{Y}_i = A_0 + A_1 x_{i1} + \cdots + A_n x_{in},$$

where i indexes the different observations, $x_{i1}, x_{i2}, \ldots, x_{in} \in \mathbb{R}$, all coefficients and $\overline{Y}_i$ are fuzzy numbers. Then, Mosleh et al. [23] proposed a learning algorithm of fuzzy neural network with crisp inputs, fuzzy weights and fuzzy output for adjusting fuzzy weights of the fuzzy polynomial regression model. If we have fuzzy inputs, fuzzy weights and fuzzy output, how can we approximate the fuzzy weights in this problem?

In this paper, we first propose an architecture of FNN with fuzzy weights for fuzzy input vectors and fuzzy targets to find approximate coefficients to the fully fuzzy polynomial regression model (fuzzy polynomial regression with fuzzy weights, fuzzy output signal and fuzzy inputs)

$$\overline{Y}_i = A_{l0} + \sum_{j=1}^{n} A_{lj} X_{ij} + \sum_{j=1}^{n} \sum_{k=1}^{n} A_{ljk} X_{ij} X_{ik} + \cdots$$

where i indexes the different observations, $X_{i1}, X_{i2}, \ldots, X_{in}$, all coefficients and $\overline{Y}_i$ are fuzzy numbers. The input–output relation of each unit is defined by the extension principle of Zadeh [36]. Output from the fuzzy neural network, which is also a fuzzy number, is numerically calculated by interval arithmetic [3] for fuzzy weights and real inputs. Next, we define a cost function for the level sets of fuzzy outputs and fuzzy targets. Then, a crisp learning algorithm is derived from the cost function to find the fuzzy coefficients of the fuzzy polynomials regression models. The advantages of the proposed method over some other methods are discussed in this paper. The remaining part of the paper is organized as follows. In Section 2, we discuss some basic definitions. Section 3 gives details of problem formulation and the way to construct the fuzzy trial function and training of a fuzzy neural network for finding the unknown adjustable coefficients and the algorithm is proposed in Section 4. Then, we compare this method with other methods in Section 5. Numerical examples are discussed in Section 6 and conclusion is in final section.

2. Preliminaries

In this section the basic notations used in fuzzy calculus are introduced. Let $\mathbb{R}$ be a universal real number set. Then a fuzzy subset A of $\mathbb{R}$ is defined by its membership function $\mu_A : \mathbb{R}^1 \longrightarrow I = [0,1]$. We denote by $[A]_h = \{x \in \mathbb{R} | \mu_A(x) \geq h\}$ the h-level set of A, where A_0 is the closure of the set $\{x \in \mathbb{R} | \mu_A(x) \neq 0\}$. A is called a normal fuzzy set if there exists an x such that $\mu_A(x) = 1$. A is called a convex fuzzy set if $\mu_A(\lambda x + (1-\lambda) y) \geq \min\{\mu_A(x), \mu_A(y)\}$ for $\lambda \in [0,1]$. (That is, μ_A is a quasi-concave function.)

Let f be a real-value function defined on $\mathbb{R}$. f is said to be upper semicontinuous if $\{x \in \mathbb{R} | f(x) \geq h\}$ is a closed set for each h. Or equivalently, f is upper semicontinuous at y if and only if $\forall \epsilon > 0, \exists \delta > 0$ such that $|x-y| < \delta$ implies $f(x) < f(y) + \epsilon$.

Definition 1. A is called a fuzzy number if the following conditions are satisfied:

(i) A is a normal and convex fuzzy set.
(ii) Its membership function μ_A is upper semicontinuous.
(iii) The h-level set $[A]_h$ is bounded for each $h \in [0,1]$.

The set of all the fuzzy numbers is denoted by E^1.

A popular fuzzy number is the triangular fuzzy number $u = (u_m, u_l, u_r)$ where u_m denotes the modal value and the real values $u_l > 0$ and $u_r > 0$ represent the left and right fuzziness, respectively. The membership function of a triangular fuzzy number is defined by

$$\mu_u(x) = \begin{cases} \dfrac{x-u_m}{u_l} + 1, & u_m - u_l \leq x \leq u_m, \\ \dfrac{u_m - x}{u_r} + 1, & u_m \leq x \leq u_m + u_r, \\ 0 & \text{otherwise}. \end{cases}$$

Triangular fuzzy numbers are fuzzy numbers in LR representation where the reference functions L and R are linear. The set of all triangular fuzzy numbers on $\mathbb{R}$ is called $\hat{FZ}$.

From Zadeh [39], A is a convex fuzzy set if and only if its h-level set $[A]_h = \{x \in \mathbb{R} | \mu_A(x) \geq h\}$ is a convex set for all h. Therefore, if A is a fuzzy number, then the h-level set $[A]_h$ is a compact and convex set; that is, A is a closed interval. The h-level set of A is then denoted by $[A]_h = [[A]_h^L, [A]_h^U]$. We also see that $[A]_h^L$ and $[A]_h^U$ are continuous with respect to h, since its membership function is upper semicontinuous.

Proposition 1 (*Zadeh [36]*). *Let A be a fuzzy set with membership function μ_A and the h-level set $[A]_h$ be given. Then*

$$\mu_A(x) = \sup_{h \in [0,1]} h.1_{[A]_h}(x).$$

A is called a crisp number with value m if its membership function is

$$\mu_A(x) = \begin{cases} 1 & \text{if } x = m, \\ 0 & \text{otherwise}. \end{cases}$$

A is called a nonnegative fuzzy number if $\mu_A(x) = 0$ for all $x < 0$ and a nonpositive fuzzy number if $\mu_A(x) = 0$ for all $x > 0$. It is obvious that $[A]_h^L$ and $[A]_h^U$ are nonnegative real numbers for all $h \in [0,1]$ if A is a nonnegative fuzzy number, and $[A]_h^L$ and $[A]_h^U$ are nonpositive real numbers for all $h \in [0,1]$ if A is a nonpositive fuzzy number.

2.1. Operations on fuzzy numbers

We briefly mention fuzzy number operations defined by the extension principle [36]. Let $\odot$ be any binary operation $\oplus$ and $\otimes$ between two fuzzy numbers A and B. The membership function of $A \odot B$ is defined by

$$\mu_{A \odot B}(z) = \sup\{\mu_A(x) \wedge \mu_B(y) | z = x \circ y\}$$

where $\wedge$ is the minimum operator and $\odot = \oplus$ or $\otimes$ correspond to the operations $\circ = +$ or $\times$.

We denote by $\mathcal{F}$ the set of all fuzzy subsets of $\mathbb{R}$. Let $f(x_1, \ldots, x_n)$ be a nonfuzzy function from $\mathbb{R}^n$ into $\mathbb{R}$ and $A_1, \ldots, A_n$ be n fuzzy subsets of $\mathbb{R}$. By the extension principle in [36–38], we can induce a fuzzy-valued function $f : \mathcal{F}^n \longrightarrow \mathcal{F}$ from the nonfuzzy function $f(x_1, \ldots, x_n)$. That is to say, $f(A_1, \ldots, A_n)$ is a fuzzy subset of $\mathbb{R}$. The membership function of $f(A_1, \ldots, A_n)$ is defined by

$$\mu_{f(A_1, \ldots, A_n)}(z) = \sup\{\mu_{A_1}(x_1) \wedge \cdots \wedge \mu_{A_n}(x_n) | z = f(x_1, \ldots, x_n)\}.$$

Proposition 2 (*Wu [35]*). *Let $f(x_1, \ldots, x_n)$ be a real-value function and $A_1, A_2, \ldots, A_n$ be n fuzzy subsets of $\mathbb{R}$. Let $F : \mathcal{F}^n \longrightarrow \mathcal{F}$ be a fuzzy-value function induced by $f(x_1, \ldots, x_n)$ via the extension principle. Suppose that each membership function μ_{A_i} is upper semicontinuous for all $i = 1, \ldots n$ and $\{(x_1, \ldots, x_n) | z = f(x_1, \ldots, x_n)\}$ is a compact set (it will be a closed and bounded set in $\mathbb{R}^n$ for all z. Then the h-level set of $F(A_1, \ldots, A_n)$ is*

$$[F(A_1, \ldots, A_n)]_h = \{f(x_1, \ldots, x_n) | x_1 \in [A_1]_h, \ldots, x_n \in [A_n]_h\}.$$

Proposition 3 (*Wu [35]*). *Let $f(x_1, \ldots, x_n)$ be a continuous real-value function and $A_1, A_2, \ldots, A_n$ be n fuzzy numbers. Let $F : \mathcal{F}^n \longrightarrow \mathcal{F}$ be a fuzzy-value function induced by $f(x_1, \ldots, x_n)$ via the extension principle. Suppose that $\{(x_1, \ldots, x_n) | z = f(x_1, \ldots, x_n)\}$ is a compact set*