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A direct variational method for planning monotonically optimal paths for redundant manipulators in constrained workspaces

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ABSTRACT

This paper proposes a path planner for serial manipulators with a large number of degrees of freedom, working in cluttered workspaces. Based on the variational principles, this approach involves formulating the path planning problem as constrained minimization of a functional representing the total joint movement over the complete path. We use modified boundary conditions at both ends of the trajectory to find more suitable start and end configurations. The concept of monotonic optimality is introduced in order to optimize the manipulator paths between the resulting end configurations. For obstacle avoidance, volume and proximity based penalizing schemes are developed and used. The presented planner uses a global approach to search for feasible paths and at the same time involves no pre-processing task. A variety of test cases have been presented to establish the efficacy of the presented scheme in providing good quality paths. The extent of advantage accruing out of the measures of free end-configurations and monotonic optimality are also analyzed quantitatively.

1. Introduction

Service manipulators in most commercial applications are expected to perform complicated tasks in cluttered environments. This necessitates the development of techniques for autonomous path planning in arbitrary 3-dimensional workspaces. In order to increase the manipulability and dexterity of a manipulator working in such environments, it has been widely accepted to increase the number of links of the manipulator and utilize the extra degrees of freedom (refer to [1–3]). However, it complicates the motion planning problem due to increasing complexity of the inverse kinematics of redundant manipulators with each added degree of freedom (DOF). In this paper, we address this problem using a direct variational approach.

The problem of generating collision-free paths for manipulators with increasingly large numbers of links has attracted considerable attention in the last two decades [4–8]. Broadly, the strategies proposed and pursued in this period can be classified as *local* and *global* (refer to [9,10]). *Local* methods start from a given initial configuration and step towards the goal configuration using localized information of the workspace. On the other hand, the *global* path planning methods typically apply search algorithms on the precomputed connectivity graphs of the free regions of C-space (refer to [11,12]). Some recent works [13,14] present improvised methodologies which have been proven better than completely randomized techniques.

The local approaches towards path planning involve searching a grid placed across the robot's configuration space [15] around the current configuration. Artificial collision measures computed from partial information of the geometry of the configuration space are used to guide the search, which in most cases turn out to be "greedy" in nature. Most proposed heuristics are in the form of articulated potential-field functions, guiding the search along the flow of its negative gradient vector field [16–19]. These are known as the *potential field* methods. Although these methods are found to be effective for local obstacle avoidance, in highly constrained workspaces, they suffer from the drawback of eventually leading the search to dead ends, i.e. local minima of the proposed potential field. Moreover, with each added DOF to the manipulator, the non-linearity of the problem increases progressively, making the technique more prone to such dead ends and, in turn, impractical for redundant manipulators in highly constrained workspaces. A few researchers [20–22] have presented neural network based techniques for trajectory control of redundant manipulators. They emphasize the importance of motion planning for manipulators with large numbers of degrees of freedom working in constrained environments.

The global techniques have made this highly complex problem tractable by employing complete[1] methods, such as those using *Roadmap* [23,24] and *Cell-Decomposition* [25,26], through a

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[1] An algorithm is called *complete* if it either finds a solution or reports the non-existence of the same.

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Nomenclature

i:	Configuration number
n:	Number of configurations (control points)
j:	Joint number
N:	Number of degrees of freedom
k:	B-spline order
$q_j(t)$:	j-th joint variable
$\mathbf{q}(t)$:	General configuration
$\mathbf{p}_i$:	i-th control configuration
$\mathbf{x}$:	Combined vector of control configurations: variable vector to be optimized
l:	Inequality constraint number
m:	Equality constraint number.

probabilistic framework. In general, roadmap based methods consist of generating a network (called *roadmap*) of one dimensional paths in the configuration space [23,24,27]. Such a roadmap consists of free configurations connected to one another. Path planning is thus reduced to searching for a path from the start node to the end node. On the other hand, the cell-decomposition methods aim at representing the set of collision-free configurations as a collection of cells and search the graph representing the adjacency relation among these cells [25,26]. One basic disadvantage of the C-space based global techniques is that they require an exhaustive pre-computation exercise in order to develop the connectivity graphs. Since the computation time required by this construction is typically exponential in the dimension 'N' of the configuration space (refer [28]), the approach is impractical even for reasonably large values of 'N'.

Recently, some researchers have used the calculus of variations to develop another class of methods in the *global* approach to path planning. Cecil and Marthaler [29,30] used level set methods for minimizing energy-based functionals to get locally optimal paths. However, their approach is shown to be applicable for only 2- and 3-dimensional C-spaces. Dasgupta et al. [31] used action minimization by reducing the functional to Euler's equations and solving them using the relaxation method. They achieved relatively complicated manipulation tasks in higher dimensional C-spaces, but only for planar manipulators. Moreover, their algorithm requires a feasible initial path which they obtain from the *probabilistic roadmap* (PRM) technique and subsequently optimize over the same. In contrast, our approach in this paper works for spatial manipulators as effectively as that for planar manipulators and does not require any feasible initial path.

Trade-off between speed and completeness of local and global techniques respectively, led to the development of hybrid methods, utilizing the desirable features of both global and local methods. Krogh and Thorpe [32] and Tournassoud [33] used geometrical solutions for global planning and potential fields for local planning in the same planner. However, their applications are limited to mobile robots in low dimensional C-spaces. Warren [34] considered operating upon the entire path at each iteration, while using a repulsive potential on obstacles. The results shown are limited to planar manipulators and to mobile robots operating in 2-D. Barraquand and Latombe [19] presented a potential based approach used in conjunction with Brownian movement (random walk) to escape from local minima. While this approach works for simple geometries in 2-D, it suffers from the need of application-specific potential functions and exhibits performance degradation with more complex geometry. Attempts have also been made to construct a potential field with only one minimum (refer to [35–38]), but the analytical definition of such a potential turns out to be mathematically involved and computationally expensive [36,38], similar in drawback to the construction of a connectivity graph in global methods. Furthermore, even if a definition is available, the applicability of such methods has only been shown for planar manipulators and low-dimensional C-spaces. A recent work (refer to [39]) presents a stochastic technique for planning paths with the objectives of maximum smoothness, but ignores the issue of obstacle avoidance. The strategy is expected to work successfully even for higher degrees of freedom.

All the aforesaid techniques use prescribed initial and final configurations generally computed by a separate inverse kinematics routine. However, it is to be noted that when the start and end configurations are deemed fixed, a degradation from optimality is inherited by the problem definition itself.

1. The chosen start and goal configurations may not be the best choice in terms of the path length in joint space, increasing the total joint movement associated with the trajectory.
2. The chosen start and goal configurations may not be at all reachable from each other.

These issues have been addressed partially by Bertram et al. [40]. They have incorporated the calculation of the goal configuration within the planner, keeping only the start configuration fixed. It is also shown that a randomly chosen pair of configurations may lie in disconnected regions of free C-space, rendering the chosen configurations unreachable from each other. However, their technique remains a local one, building a *rapidly-exploring random tree* (RRT) starting from the provided initial configuration, and hence inherits the ever-present drawbacks of a local search technique discussed earlier.

In this paper, we present a path planner for spatial redundant manipulators working in arbitrary 3-D environments. The presented method uses a global approach without resorting to the need for developing the complete C-space connectivity graph. It involves defining an energy-based functional in such a way that its minimization over the function space of all the possible paths results in the definition of a feasible path, together with the initial and final configurations. A partially random initial guess, which may be infeasible, is generated using a proximity based heuristic, ensuring that the initial path sweeps over roughly the same Cartesian region as the given start and end points. The problem is formulated in a non-linear optimization scheme and the augmented Lagrangian optimization technique of constrained optimization (refer to [41,42]) is used to numerically minimize the functional. We introduce the concept of monotonically optimal paths and incorporate a mild penalty term in the original objective function, to improve the quality of the solution obtained. The efficiency of the approach in handling spatial manipulators in higher dimensional C-spaces is demonstrated through various case studies in cluttered workspaces.

In the next section, we propose a B-spline formulation for the manipulator path. We recognize a set of variables, which collectively define the complete path in joint space. Section 3 elaborates the collision detection and quantification scheme. In Section 4, we formulate the energy based functional and use it as a part of the objective function in the subsequently formulated non-linear programming (NLP) problem. In Section 5, we formulate the optimization problem and discuss the solution strategy using the augmented Lagrangian technique. Results have been shown in Section 6 and a comparison has been made with the method presented in an earlier work. Finally, some conclusions are drawn in Section 7.

2. Path modeling

For a manipulator with 'N' joints, a path expressed in joint space is a collection of 'N' scalar functions of time ($q_i(t)$), with each function q_i representing the motion of the i-th joint in time. Hence,