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Closed-form redundancy solving of serial chain robots with a weak generalized inverse approach

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HIGHLIGHTS

- A pseudoinverse weaker than the Moore–Penrose pseudoinverse is used for redundancy solving.
- Formal compact solution including projector onto null space is developed.
- The method can be applied for optimizing a criterion or multiple task performance.
- A redundancy degree of 1 induces very simple expression of the projector onto null space.
- A general inverse model is given for any nR-planar robot whatever n.

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ABSTRACT

Redundancy solving of robot-limbs is generally made at the velocity level by means of sophisticated matrix tools. We attempt to develop an alternative approach based on a right-inverse of the robot Jacobian which is not the Moore–Penrose inverse with, as a goal, to generate closed-form and compact expressions of the joint velocities. Considering the definition of the determinant of a rectangular $m \times n$ matrix ($m < n$) as the sum of its $m \times m$ minors, and using the weak inverse proposed by the Indian mathematician Joshi we show that it is possible to derive a general expression of its projector onto null space. These mathematical tools are applied to a new approach for redundancy solving including optimization criteria and multiple task performance. It is also shown that the combinatorial explosion peculiar to the method for highly redundant robots can be controlled in the case of some classes of modular robotic structures. Simulation results are reported for the regional redundant structure of a 7R-robot arm and a 30 d.o.f. planar elephant trunk-like robot.

1. Introduction

Most of the methods for solving redundancy of robot limbs are applied at a velocity level and, because this approach is mainly based on the Moore–Penrose pseudoinverse of the robot Jacobian and the possibility to arbitrarily choose the associated projection operator for avoiding singularities or obstacles [1,2], the solution to the problem is essentially expressed in terms of operation on matrices. The increasing computing power of robot controllers has made possible the on-line implementation of such methods although mathematical forums sometimes emphasized the computational burden involved by the use of 'pinv' or 'pinv2'-type algorithms [3]. Without renouncing to solve the problem at the velocity level, we would like, in this study, to analyze the possible relevance of solving the problem with a pseudoinverse "weaker" than the Moore–Penrose pseudoinverse. To the best of our knowledge, no attempt has still been made to consider weak inverses for solving redundancy in robotics although, as noted by Lovass-Nagy, Miller and Powers [4], the use of generalized inverses weaker than the Moore–Penrose pseudoinverse can lead to simpler solutions in inverse linear problems. We expect, by this new mean, to be able to exhibit closed-form joint velocity expressions for solution to the inverse problem instead of a numerical matrix formulation. This look for new ways in the determination of compact formal solutions to the inverse kinematic problem for serial-chain robot-limbs is motivated by two ideas. First it is by a theoretical—and it could even be said esthetical aim. Because the determinant of a square matrix is independent on the choice of the base in which are expressed its column vectors, it is possible to choose a "good" frame and then to derive the "optimal" compact form for the robot Jacobian with respect to this frame. In their

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classic book, Gorla and Renaud [5] can so write the Jacobian of main 6R-robot structures in a very compact form. Even after choosing the "right" frame, the general Moore–Penrose expression for a redundant robot, $\boldsymbol{J}^{+} = \boldsymbol{J}^{T}(\boldsymbol{J}\boldsymbol{J}^{T})^{-1}$ where $\boldsymbol{J}$ is the robot Jacobian, generally, prevents such a possibility due to matrix operations its computation imposes. But this look for compactness for the inverse velocity solution has also a practical goal. It is well known that solving the inverse problem at the velocity level suffers an integration problem: because joint position is integrated from joint velocity, numerical errors are summed up at each sampling time. If we were able to solve the joint velocity problem with compact real expressions instead of sophisticated methods like singular value decomposition, an obvious gain in computing time could be expected and, without the velocity integration problem disappears, it would be reduced by the possibility to choose a shorter sampling time. This last technical problem will not be addressed in this paper where we want, essentially, to discuss the possibility of a relevant alternative to the Moore–Penrose pseudoinverse. Section 2 is devoted to the mathematical aspect of the problem. After a first problem analysis in the frame of generalized inverses, we introduce an intuitive definition of the determinant of a rectangular $m \times n$ matrix ($m < n$) and we show how the right inverse proposed by the Indian mathematician Joshi is a good candidate for formal computation of a rectangular inverse and can lead to a very interesting expression for its associated projector onto null space. In Section 3, these mathematical tools are applied for solving redundancy including criteria optimization and multiple tasks performance. The scope of application of the proposed method is analyzed, especially with respect to the degree of redundancy of the robot.

2. Solving the redundancy by means of a weak inverse

2.1. Right inverse versus left inverse

Let us start recalling the classical definition of a generalized inverse of any $m \times n$ matrix $\boldsymbol{M}$ as being the matrix $\boldsymbol{X}$ satisfying at least the first or the second of the four following equations:

$$\begin{cases} \boldsymbol{MXM} = \boldsymbol{M} \quad \text{(a)} \\ \boldsymbol{XMX} = \boldsymbol{X} \quad \text{(b)} \\ (\boldsymbol{MX})^{T} = \boldsymbol{MX} \quad \text{(c)} \\ (\boldsymbol{XM})^{T} = \boldsymbol{XM} \quad \text{(d).} \end{cases} \tag{1}$$

The Moore–Penrose pseudoinverse is the unique matrix satisfying the four equations. A matrix satisfying only some of these four equations is said to be a $\{i_1, i_2, i_3\}$-inverse where $1 \le i_1 < i_2 < i_3 \le 4$ and any solution of this class will be noted $\boldsymbol{M}^{(i_1,\, i_2,\, i_3)}$. Let us consider a linear equation $\boldsymbol{v} = \boldsymbol{Mu}$ where $\boldsymbol{M}$ is a $m \times n$-matrix, $\boldsymbol{u}$ is a vector of $\boldsymbol{R}^{n}$ and $\boldsymbol{v}$ is a vector of $\boldsymbol{R}^{m}$. The Moore–Penrose pseudoinverse $\boldsymbol{M}^{+}$ solving this equation is called the best approximate solution or minimum norm least-squares solution. This means it is the only solution sharing the property of least-squares approximation peculiar to all $\{1, 3\}$-inverses and the property of minimum-norm peculiar to all $\{1, 4\}$-inverses [6]. In the case of a redundant robot with n degrees of freedom, moving in a m-dimensional operational space with $m < n$, the Moore–Penrose pseudoinverse of the robot Jacobian is so able to generate the minimum-norm solution when the robot task has a multiplicity of solutions and the least-squares solution when the datum of an additional task makes the linear system inconsistent. If we give the Moore–Penrose pseudoinverse up, the choice of an alternative pseudoinverse can be either based on the choice of a $\{1, 3\}$-inverse or a $\{1, 4\}$-inverse with, respectively, looking for a least-squares solution or a minimum-norm solution. Moreover, it is important to remember the following characterization of the sets of $\{1, 3\}$ and $\{1, 4\}$-inverses respectively noted $\boldsymbol{M}\{1, 3\}$ and $\boldsymbol{M}\{1, 4\}$ [6, Chapter 2]—we voluntarily limit our approach to real matrices:

$$\begin{aligned} \boldsymbol{M}\{1, 3\} &= \{\boldsymbol{M}^{(1,3)} + (\boldsymbol{I}_n - \boldsymbol{M}^{(1,3)}\boldsymbol{M})\boldsymbol{Z}, \\ &\qquad \text{with } \boldsymbol{Z} \text{ is any } n \times m \text{ matrix}\} \\ \boldsymbol{M}\{1, 4\} &= \{\boldsymbol{M}^{(1,4)} + \boldsymbol{Z}(\boldsymbol{I}_m - \boldsymbol{M}\boldsymbol{M}^{(1,4)}), \\ &\qquad \text{with } \boldsymbol{Z} \text{ is any } n \times m \text{ matrix}\}. \end{aligned} \tag{2}$$

It is also possible to distinguish right inverses from left inverses: a right inverse of $\boldsymbol{M}$ is any $n \times m$-matrix $\boldsymbol{M}^{r}$ verifying $\boldsymbol{M}\boldsymbol{M}^{r} = \boldsymbol{I}_m$; it is easy to check that $\boldsymbol{M}^{r}$ is a $\{1, 2, 3\}$-inverse and so a $\{1, 3\}$-inverse. Alternatively, a left-inverse of $\boldsymbol{M}$ is any $n \times m$-matrix $\boldsymbol{M}^{l}$ verifying $\boldsymbol{M}^{l}\boldsymbol{M} = \boldsymbol{I}_n$; it is easy to check that $\boldsymbol{M}^{l}$ is a $\{1, 2, 4\}$-inverse and so a $\{1, 4\}$-inverse. Because our challenge consists in looking for a new class of weak inverses able to compete in some extent the classic formula $\boldsymbol{J}^{+} = \boldsymbol{J}^{T}(\boldsymbol{J}\boldsymbol{J}^{T})^{-1}$ we propose to approach the issue using the distinction we just made between right and left inverses. And because we have in mind the important role of least-squares solution property peculiar to $\{1, 3\}$-inverses, we focus our look on a right-inverse with the hope that it will also be able to generate a solution close to the minimum-norm solution. On the other hand, because our work was initially inspired by the search for determinant of rectangular matrices, we propose to write the looked right-inverse as follows:

$$\boldsymbol{M}^{r} = \mathrm{Co}(\boldsymbol{M})^{T} / \det(\boldsymbol{M}) \tag{3}$$

where $\det(\boldsymbol{M})$ is a determinant-like function equal to zero if and only if $\mathrm{rank}(\boldsymbol{M}) < m$ and $\mathrm{Co}(\boldsymbol{M})$ is like a matrix of cofactors. This is this right inverse that we now define.

2.2. Joshi's weak inverse and its use in linear systems solving

Matrix determinant is a fundamental algebra notion essentially associated to square matrices. Attempts have been made to extend this notion to non-square matrices. If no global theory exists and, may be, would be meaningless in the general case of a rectangular matrix, the problem can more easily be approached if it is limited to $m \times n$ matrices with $m \le n$. In this case, the matrix $\boldsymbol{M}$ can be read as a sequence of n vectors $(\boldsymbol{a}_1, \boldsymbol{a}_2, \ldots, \boldsymbol{a}_n)$ belonging to a m-dimensional vector space. An intuitive way to define a determinant associated to this vector sequence consists to consider all $m \times m$ minors by a sign, computed from their components. Radic [7] proposed at his time a first definition of the determinant of such rectangular matrices. We will however prefer a more recent definition proposed by Joshi [8] for which it is easy to show that it is equivalent to the simple form:

$$\det(\boldsymbol{M}) = \sum_{1 \le i_1 < i_2 < \cdots < i_m \le n} \det(\boldsymbol{a}_{i_1},\ \boldsymbol{a}_{i_2}, \ldots, \boldsymbol{a}_{i_m}) \tag{4}$$

i.e. the sum of all $\binom{n}{m}$ $m \times m$ minors of the ordered sequence of the matrix n column vectors of the matrix.[1] In the remaining, we will adopt this definition which, obviously, includes the case of a square

[1] It is possible to geometrically justify this definition of the determinant of a rectangular matrix by means of zonotope theory. Indeed, each considered minor corresponds to the determinant of a set of m vectors defined in a m-dimensional vector space. According to the theory of square matrices, any determinant can be geometrically interpreted as the signed volume of the corresponding m-parallelepiped whose generating vectors are the vectors of the minor. In this same m-dimensional space, the n vectors of the rectangular matrix can be viewed as the generating vectors of a geometric figure called a zonotope [9] i.e. a set of m-parallelepipeds gathered in such a way that all "faces" are parallel two by two. The volume of this zonotope is not equal to the considered determinant but the volume of each "cell" of the zonotope is equal to one of its $m \times m$ minor by a sign.