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Pattern preserving path following of unicycle teams with communication delays ☆

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ABSTRACT

This paper examines the problem of pattern-preserving path following control for unicycle teams with time-varying communication delay. A key strategy used here introduces a virtual vehicle formation such that each real vehicle has a corresponding virtual vehicle as its pursuit target. Under an input-driven consensus protocol, the virtual vehicle formation is forced to stay close to the desired vehicle formation; and a novel controller design is proposed to achieve virtual leader tracking for each vehicle with constrained motion. It is shown that, by the proposed strategy, the pattern can be preserved if the formation speed is less than some computable value that decreases with increasing size of delay, and the exact desired formation pattern can be eventually achieved if this speed tends to zero.

1. Introduction

Formation control design for multi-vehicle systems continues to attract great attention due to the needs in many industrial and military applications such as surveillance, search, rescue and terrain mapping. Trajectory tracking and path following with formation maintenance are two problems under wide study. In trajectory tracking (resp. path following), one vehicle or some geometric characteristics of a vehicle team is required to track a virtual vehicle moving on the given trajectory (resp. to follow a given path). With the formation maintenance requirement, the geometric pattern of a vehicle team needs to be (globally) asymptotically stabilized at a desired one, which either is given by the relative positions among the vehicles, or maps to a value (e.g. global or local minimum) of some given function (e.g. artificial potential function). For real-world applications, there may be some extra control objectives which need to be achieved. For instance, inter-vehicle and vehicle–obstacle collisions should be avoided in the transient of the tracking or path following.

Several types of strategies have been proposed for the control purposes stated above during the past few years. The authors of [1–9] investigated leader–follower structure based strategies, where the vehicle group is layered and each vehicle in some layer has a vehicle in the upper layer as the local leader to follow; and the only vehicle in the top layer is required to track a given trajectory or follow a given path when the group is performing formation tracking or a path following task. Artificial potential function (APF) based approaches were firstly employed for the swarming and flocking control of multiple vehicles with holonomic dynamics [10–14]. These approaches have recently proved useful also for nonholonomic vehicle teams [15–19]. By the APF based strategy, each vehicle in a team tries to follow the direction specified by the negative gradient of corresponding APF component, and the geometric pattern of the team almost converges to the one that maps to a local minimum of the collective APF. For holonomic vehicles, this following can be exactly realized at any time; but it may only be achieved asymptotically for nonholonomic vehicles. The main difficulty of the APF based method is to design an APF without local minima which correspond to undesirable patterns. Another important method for formation control of multi-vehicle systems is based on the use of the so called virtual structure, which is composed of virtual leaders playing the role of reference targets for the real vehicles. These virtual leaders can be in rigid configurations, interact with each other for some formation control purposes, or interconnect their motions with those of the real vehicles. Early work along this line can be found in [20,21]. Recent years have witnessed much effort in applying this strategy to trajectory tracking and path following of multiple vehicles with various types of dynamics [22–25,19,26,27]. See [28] for more references on the subject of formation control.

Although many previous works have studied the formation control of nonholonomic vehicles, very few of them dealt with data transmission delay in inter-vehicle communication channels. Among the latter, [25] showed a decentralized strategy in which

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each vehicle follows a path traced out by a virtual leader, and the desired pattern of the vehicles can be eventually achieved if the individual coordination states reach consensus. In [29], the authors designed distributed control laws based on backstepping techniques such that a team of vehicles are forced to asymptotically form a desired pattern, with respect to a global coordinate system, whose centroid moves along a desired trajectory. An appealing feature of the approach is that the desired trajectory only needs to be available to a portion of the vehicles. Note that in both the papers the communication delay is considered time-invariant, and the topology of the communication graph of the vehicle team is not related to its geometric pattern.

In this paper, we address, for the first time, the problem of how to drive a nonholonomic vehicle team to move along some given path with a preserved pattern and a specified formation speed profile in the presence of time-varying communication delay. The pattern of the vehicle team is said to be preserved if, roughly speaking, the error between it and the desired one remains in a small range. Considering distance-dependent communication capability of the vehicles, we assume that the communication network of the vehicle team is fixed, bidirectional and connected if the pattern is preserved. Our main contribution is to give a control strategy by which the pattern is preserved if the desired formation speed is upper bounded, where the bound is shown inversely proportional to the size of delay. The strategy can deal with both the cases that the desired formation speed is bounded away from zero and converges to zero asymptotically.

Applying a virtual structure framework, we assign each vehicle with a path reference point (PRP) on the given path, based on which a virtual lead is defined. The formation of the virtual leaders, called the virtual formation, coincides with the desired vehicle formation if all the PRPs reach agreement. The proposed strategy can be outlined as follows. On one hand, an input-driven consensus protocol is employed to keep PRPs having small differences and proceeding roughly with the desired formation speed. In other words, the resulting virtual formation remains close to the desired vehicle formation. On the other hand, under the action of a control law derived from the APF based approach, each vehicle asymptotically tracks its virtual leader with the tracking error constrained inside a pre-defined range. As a consequence, the real vehicle formation is always close to the virtual formation. The combination of the above two points guarantees that the actual vehicle formation has a small error with respect to the desired one. Last but not least, by our approach, when the desired formation speed converges zero (e.g., in the scenario of point-to-point migration) the team of vehicles eventually form exactly the desired formation.

2. Preliminaries

Throughout this paper, we use $\mathbb{N}, \mathbb{R}_+, \mathbb{Z}_+$ to denote the sets of natural numbers, nonnegative real numbers and nonnegative integers. $\|x\|$ denotes the Euclidean norm of the vector $x \in \mathbb{R}^n$ for any $n \in \mathbb{N}$. In addition, we use $\mathbf{0}_E$ to represent the function mapping any point in an interval $E \subseteq \mathbb{R}$ to $0 \in \mathbb{R}^N$, where the dimension $N \in \mathbb{N}$ can be identified from the context.

2.1. Graph theory

A directed graph $\mathcal{G}(\mathcal{V}, \mathcal{E})$ consists of a vertex set $\mathcal{V}$ and an arc, or directed edge, set $\mathcal{E} \subset \mathcal{V} \times \mathcal{V}$. For any $i, j \in \mathcal{V}$, the ordered pair $(i, j) \in \mathcal{E}$ if and only if i is a neighbor of j. Vertex i is said to have a self edge if $(i, i) \in \mathcal{E}$. A directed path, with length $n-1$, from vertex i to j is a sequence of distinct vertices $v_1, v_2, \ldots, v_n$, where $n \geq 1$, $v_1 = i$, $v_n = j$ and $(v_1, v_2), \ldots, (v_{n-1}, v_n) \in \mathcal{E}$. A directed graph is said to have a spanning tree if and only if there exists a vertex $i \in \mathcal{V}$, called the root, such that there is a directed path from i to any other vertex. A graph $\mathcal{G}(\mathcal{V}, \mathcal{E})$ is undirected if and only if for any $i, j \in \mathcal{V}$, $(i, j) \in \mathcal{E}$ implies $(j, i) \in \mathcal{E}$. A path in an undirected graph is defined analogously as a directed path in a directed graph. An undirected graph is said to be connected if and only if there is a path between any pair of vertices. The *degree* d_i of vertex $i \in \mathcal{V}$ in an undirected graph $\mathcal{G}(\mathcal{V}, \mathcal{E})$ is defined as $d_i = \text{Card}(\{j : (i, j) \in \mathcal{E}\})$; and the value of $\max_i d_i$ is called the *maximum degree* of the graph. The *diameter* of a connected undirected graph $\mathcal{G}(\mathcal{V}, \mathcal{E})$ is defined to be $\max_{i,j \in \mathcal{V}} L_{ij}$, where L_{ij} is the minimum length of any path from vertex i to vertex j. In addition, for the graph $\mathcal{G}(\mathcal{V}, \mathcal{E})$ with time-dependent edge set $\mathcal{E}(t)$, we use $\biguplus_t \mathcal{G}(\mathcal{V}, \mathcal{E}(t))$ to represent the graph composed of node set $\mathcal{V}$ and edge set $\bigcup_t \mathcal{E}(t)$. See [30] for more basics in graph theory.

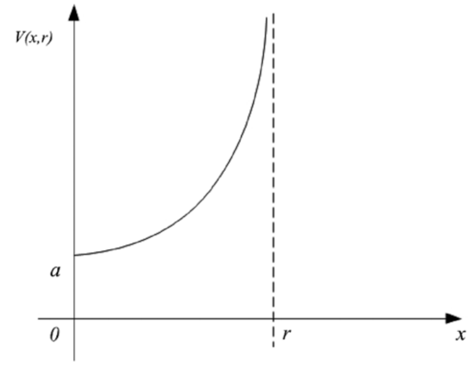

Fig. 1. An example of artificial potential functions.

2.2. Artificial potential function

An artificial potential function $V(\cdot, r) : [0, r) \to [a, \infty)$, with $r > 0$ and $a \in \mathbb{R}_+$, used in this paper has the following properties:

(a) $V \in C^2$;
(b) $\lim_{x \to r} V(x, r) = \infty$;
(c) $\forall \epsilon \in (0, r), \exists \delta > 0$ such that $V'(x, r) \geq \delta, \forall x \in [\epsilon, r)$.

In the following, we use V_x' and V_x'' to denote the first and second derivatives of the function $V(x, r)$ with respect to x. An example of artificial potential functions is depicted in Fig. 1.

3. Pattern preserving path following

3.1. Problem statement and description of the control strategy

The path to be followed by the vehicle team is a plane curve represented by the function $q : \mathbb{R} \to \mathbb{R}^2$. We use $x_q(s), y_q(s)$ to denote the first and second components of $q(s)$ respectively, i.e., $q(s) = (x_q(s), y_q(s))$. Physically, $x_q(s), y_q(s)$ are the x- and y-coordinates of the point corresponding to s on the path with respect to some right-handed Cartesian coordinate system Σ_g. Regarding the smoothness of the path, we make the following Assumption 1:

Assumption 1. The derivatives $x_q'(s)$, $y_q'(s)$, $x_q''(s)$, $y_q''(s)$, $x_q'''(s)$, $y_q'''(s)$ exist and are bounded, and there exists a positive real constant c such that $\sqrt{(x_q'(s))^2 + (y_q'(s))^2} \geq c$, for any $s \in \mathbb{R}$.

An example of such a path is $x_q(s) = 10\cos(0.1s)$ and $y_q(s) = 10\sin(0.1s)$. Assumption 1 will be mainly used to ensure some properties of the motion of the virtual leaders that the real vehicles are supposed to track.