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Model-based control of redundantly actuated parallel manipulators in redundant coordinates

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ABSTRACT

Model-based control of parallel kinematics machines (PKM) relies on computationally efficient formulations in terms of a set of independent joint coordinates. Since PKM models are commonly expressed in terms of actuator or end-effector coordinates the models are not valid at input- or output-singularities, respectively. Moreover input-singularities limit the motion range of PKM. Actuation redundancy is a means to increase the singularity-free range of motion. However, due to the redundancy only a subset of the actuator coordinates constitute independent coordinates. This subset corresponds to the actuator coordinates of the non-redundant PKM, which does generally not constitute proper minimal coordinates for the entire workspace. Hence a redundantly actuated PKM (RA-PKM) controlled by a model-based controller in terms of minimal coordinates would exhibit the same limitations as the non-redundant PKM. One approach to tackle this problem is to switch between different minimal coordinates, i.e., different motion equations are used within the controller.

In this contribution a computed torque and augmented PD control scheme in redundant coordinates is proposed, as an alternative to coordinate switching, and applied to the control of redundantly actuated PKM. That is, no minimal coordinates are selected. This novel formulation is numerically robust and does not suffer from input- or output-singularities. Even more the formulation is always valid except at configuration space singularities. For the redundancy resolution within the inverse dynamics the pseudoinverse of a rank deficient matrix is required, for which an explicit formulation is presented. For both controllers exponential trajectory tracking is shown. Experimental results are reported for a planar 2 DOF RA-PKM.

1. Introduction

Model-based control of parallel kinematics machines (PKM) calls for a computationally efficient formulation of motion equations applicable in real-time controllers. This is achieved by expressing the dynamic model in terms of a set of δ independent joint coordinates, referred to as minimal coordinates, where δ is the DOF. Minimal coordinate formulations have been widely used for non-linear motion control of PKM [1]. To this end the n joint coordinates of a PKM, that are subject to kinematic loop constraints, are split into a set of dependent and independent coordinates. The latter constitute (local) parameters for describing the PKM configuration. Such a parameterization is generally not valid for all PKM configurations. Configurations where the parameterization fails are called parameterization-singularities. For modeling non-redundantly actuated PKM often either the coordinates of actuated joints or the end-effector (EE) coordinates, as in [2], are used as independent parameters. Both choices suffer from singularities where the parameterization fails. When using the actuator coordinates these parameterization-singularities are identical with the input-singularities where the instantaneous motion is not uniquely determined by the actuator motion. If EE coordinates are used as independent coordinates, the parameterization-singularities are identical with the output-singularities, where there exists instantaneous PKM motions even if the EE is fixed. Such phenomena have been studied to a great extent in the past, e.g. [3–9]. Singularities are commonly avoided by appropriate motion planning which consequently limits the admissible range of motion.

Redundant actuation of PKM is deemed a promising approach to improve their kinematic and dynamic properties. It can in particular eliminate the aforementioned input-singularities and thus increase the usable range of motion.

For motion control of RA-PKM the established model-based augmented PD and computed torque control schemes have been adopted [10–13]. Further the classical model predictive control

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(MPC) concept (ref. to [14] for an overview) was pursued for controlling serial robots [15] as well as non-redundant PKM [16]. In recent contributions [17,18] the MPC was generalized to RA-PKM. In particular a novel MPC concept was proposed in [18] where redundancy is exploited for backlash avoidance by generating permanent internal prestress.

The basis for these control methods is a PKM model in terms of minimal coordinates. Now in case of RA-PKM there are more actuator coordinates than the DOF δ, and only a subset of the m actuator coordinates can be used as minimal coordinates. That is, δ actuator coordinates must be selected to parameterize the motion equations, which resembles the situation of non-redundantly actuated PKM. Consequently, the model of a RA-PKM in minimal coordinates exhibits exactly the same parameterization-singularities as that of the PKM without actuation redundancy. This is merely a mathematical artifact that has no correspondence in the physical PKM system, but it severely impairs the stability of the model-based controller, and thus limits again the admissible motion range. Hence although the RA-PKM may not posses input-singularities (it can always be controlled by the m actuator coordinates) a particular set of δ actuator coordinates is only valid within a limited range of motion, and at any time another set of δ actuator coordinates could be selected that represents a valid parameterization. It is thus possible to switch between different sets of minimal coordinates. Such a switching method was proposed in [19] where the feasibility was shown and a real-time implementation presented. Also reported was the numerical complexity and the necessary implementation effort. It requires a complete change of the dynamic model. Moreover, the switching condition must be monitored numerically since in general no analytic expression is available. Another critical point for the actual model-based control scheme is that the feedback control forces do not change smoothly with the transition to different sets of minimal coordinates.

An alternative approach to deal with the lack of a globally valid parameterization is to express the dynamical PKM model in terms of the n dependent (redundant) coordinates. This method will be described in this paper. It builds upon a robust inverse dynamics formulation of RA-PKM in terms of redundant coordinates introduced in [20]. This formulation does not need a set of independent minimal coordinates so that *the motion equations are valid in the full range of motion*. In this redundant coordinate formulation the control matrix (that relates actuator coordinates to generalized forces) is singular, and the crucial part of the inverse dynamics formulation is the resolution of the actuator redundancy. To this end an explicit formulation of the pseudoinverse of the singular control matrix is presented that does require selecting valid minimal coordinates from the set of actuator coordinates, however.

The paper is structured as follows. In Section 2 the PKM motion equations in minimal coordinates are recalled. It is pointed out that there are no generalized coordinates that give rise to a global parameterization of the PKM configuration space (c-space). A formulation in redundant coordinates is taken up in Section 3. This formulation does not need a set of independent minimal coordinates. As a prerequisite for the model-based control the skew symmetry property of the motion equation is established that is well-known for the minimal coordinate formulation. In Section 4 the inverse dynamics is solved as a basis for the model-based control. Therein actuation redundancy is resolved by means of a pseudoinverse of the singular control matrix. An augmented PD and computed torque control scheme is introduced in Section 5 upon the dynamics formulation in Section 3. Both schemes are shown to achieve exponentially stable trajectory tracking. Experimental results are reported in Section 6 for a redundantly actuated planar 2$\underline{\text{R}}$RR/$\underline{\text{R}}$R PKM. The paper closes with a short summary and discussion of future directions in Section 7. The symbols used in this paper are summarized in Appendix A.

2. Minimal coordinate formulation

2.1. Equations of motion

The PKM dynamics is governed by the Lagrangian motion equations. PKM are kinematically characterized by kinematic loops imposing certain loop closure conditions. The PKM configuration is represented by a vector $\mathbf{q} \in \mathbb{V}^n$ comprising n generalized coordinates $q^a, a = 1, \ldots, n$. The conditions give rise to a system of r geometric and kinematic constraints

$$\mathbf{0} = \mathbf{h}(\mathbf{q}), \quad \mathbf{h}(\mathbf{q}) \in \mathbb{R}^r \tag{1}$$

$$\mathbf{0} = \mathbf{J}(\mathbf{q})\,\dot{\mathbf{q}}, \quad \mathbf{J}(\mathbf{q}) \in \mathbb{R}^{r,n}. \tag{2}$$

These constraints are accounted for by the Lagrange multipliers $\boldsymbol{\lambda}$ in the Lagrangian motion equations of the PKM

$$\mathbf{G}(\mathbf{q})\,\ddot{\mathbf{q}} + \mathbf{C}(\mathbf{q},\dot{\mathbf{q}})\,\dot{\mathbf{q}} + \mathbf{Q}(\mathbf{q},\dot{\mathbf{q}},t) + \mathbf{J}^T(\mathbf{q})\,\boldsymbol{\lambda} = \mathbf{u}. \tag{3}$$

Therein $\mathbf{G}$ is the generalized mass matrix of the unconstrained system, $\mathbf{C}\dot{\mathbf{q}}$ represents generalized Coriolis and centrifugal forces, $\mathbf{Q}$ represents all remaining forces (possibly including EE loads), and $\mathbf{u}(t)$ are the generalized control forces. The set of admissible configurations, defined by the geometric constraints (1), is the c-space of the PKM model:

$$V := \left\{\mathbf{q} \in \mathbb{V}^n | \mathbf{h}(\mathbf{q}) = \mathbf{0}\right\}. \tag{4}$$

Presuming $\mathbf{J}$ has full rank, $\delta := n - r$ joint variables can be selected as independent coordinates that constitute a vector $\mathbf{q}_2$. They represent generalized (minimal) coordinates for the PKM, and δ is the DOF of the PKM. If the rank of $\mathbf{J}$ is constant, the c-space is a smooth δ-dimensional manifold. A configuration $\mathbf{q}$ is called a c-space singularity if rank $\mathbf{J}$ changes in $\mathbf{q}$ [21,6,22,23,9].

Denote with $\mathbf{q}_1$ and $\mathbf{q}_2$ the vector of dependent and independent coordinates, respectively, so that $\mathbf{q} = (\mathbf{q}_1, \mathbf{q}_2)$. Then (2) can be written as

$$\mathbf{J}_1\dot{\mathbf{q}}_1 + \mathbf{J}_2\dot{\mathbf{q}}_2 = \mathbf{0}, \tag{5}$$

where $\mathbf{J} = (\mathbf{J}_1, \mathbf{J}_2)$, with $\mathbf{J}_1(\mathbf{q}) \in \mathbb{R}^{r,r}, \mathbf{J}_2(\mathbf{q}) \in \mathbb{R}^{r,\delta}$. With $\mathbf{q}_2$ being local coordinates, $\mathbf{J}_1$ is full rank so that

$$\dot{\mathbf{q}} = \mathbf{F}\dot{\mathbf{q}}_2, \quad \text{where } \mathbf{F} := \begin{pmatrix} -\mathbf{J}_1^{-1}\mathbf{J}_2 \\ \mathbf{I}_\delta \end{pmatrix}. \tag{6}$$

Since $\mathbf{JF} \equiv \mathbf{0}$ the matrix $\mathbf{F}$ is an orthogonal complement of $\mathbf{J}$. The accelerations follow with $\ddot{\mathbf{q}} = \mathbf{F}\ddot{\mathbf{q}}_2 + \dot{\mathbf{F}}\dot{\mathbf{q}}_2$.

A PKM comprises passive joints so that $\mathbf{u}$ only comprises m non-zero control forces. By definition, for RA-PKM the number of actuator coordinates exceeds the DOF of the PKM, i.e. $m > \delta$. Denote with $\mathbf{c} \equiv (c_1, \ldots, c_m)$ the vector of generalized control forces in the actuated joints, and let $\mathbf{A}$ be that part of $\mathbf{F}$ so that $\mathbf{F}^T\mathbf{u} = \mathbf{A}^T\mathbf{c}$. Denoting with $\mathbf{q}_{\mathrm{a}}$ the vector of actuator coordinates, this implies $\dot{\mathbf{q}}_{\mathrm{a}} = \mathbf{A}\dot{\mathbf{q}}_2$. Projecting the motion equations (3) of the tree-system to the configuration space V, with the help of the orthogonal complement $\mathbf{F}$, yields

$$\overline{\mathbf{G}}(\mathbf{q})\,\ddot{\mathbf{q}}_2 + \overline{\mathbf{C}}(\mathbf{q},\dot{\mathbf{q}})\,\dot{\mathbf{q}}_2 + \overline{\mathbf{Q}}(\mathbf{q},\dot{\mathbf{q}},t) = \mathbf{A}^T(\mathbf{q})\,\mathbf{c} \tag{7}$$

where

$$\overline{\mathbf{G}} := \mathbf{F}^T\mathbf{G}\mathbf{F}, \qquad \overline{\mathbf{C}} := \mathbf{F}^T(\mathbf{C}\mathbf{F} + \mathbf{G}\dot{\mathbf{F}}), \qquad \overline{\mathbf{Q}} := \mathbf{F}^T\mathbf{Q}. \tag{8}$$

The Eqs. (7) have a long tradition and can be traced back to the work of Voronets [24] at least, and will in the following be called Voronets equations. They have been used for PKM modeling [10,6,11,12]. They are a particular type of Maggi equations [25]. These δ ODEs, together with the r dynamic constraints $\mathbf{J}\ddot{\mathbf{q}} + \dot{\mathbf{J}}\dot{\mathbf{q}} = \mathbf{0}$, yield n ordinary differential equations in $\mathbf{q} \in \mathbb{V}^n$ that